

A Proof of Goldbach's Conjecture

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. We give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture , fundamental theorem of arithmetic, Euclid's proof of infinite primes

Introduction

Prime numbers¹ are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved.

I believe that prime numbers are “basic building blocks” of the natural numbers and they must follow some very simple basic rules and do not need “advanced mathematics tools” to solve them. One of the basic rules is the “fundamental theorem of arithmetic” and the “simplest tool” is Euclid’s proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic², which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.^[1] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof³ that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just n primes, then the biggest prime will be labeled P_n . Now we can form the number Q by multiplying together all these primes and adding 1, so

$$Q = (P_1 \times P_2 \times P_3 \times P_4 \dots \times P_n) + 1$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n .

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

If N is an even integer:

$N = N/2 + N/2 = (N/2+m) + (N/2-m)$; $m = 0, 1, 2, 3, \dots, M$. If we can prove $[(N/2+m)]$ and $[(N/2-m)]$ can be primes at same time, then $N = P_1 + P_2$, $P_1 > P_2$ and $P_1 - P_2 = 2m$. P_1 and P_2 is a prime.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, if we divide the numbers in groups, the problem should be much simpler. Let us consider the following cases:

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 110, 120, 1120, 1130, ..., then N/2 has only 0 or 5 as its last digit:

1a. Except 5, any prime must have 1, 3, 7, or 9 as its last digit. When both N and N/2 have 0 as their last digit, then N must be 20, 30, 40, 60, 80, 100, 120, ..., N. For enough large number N, Let's consider $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$, O_1 and O_2 is an odd number. $O_1 > O_2$, $O_1-O_2=2L+6$, $L=0, 5, 10, 15, 20, 25, 30, \dots, L$, $O_1-O_2=2L+6=6, 26, 46, 66, 86, 106, 126, \dots(2L+6)$, then O_1 is an odd number with 3 as its last digit, O_2 is an odd number with 7 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$, O_1 and O_2 is an odd number. $O_1 > O_2$, $O_1-O_2=2L+14$, $L=0, 10, 20, 30, \dots, L$, $O_1-O_2=2L+14=14, 34, 54, 74, 94, 114, 134, \dots(2L+14)$, then O_1 is an odd number with 7 as its last digit, O_2 is an odd number with 3 as its last digit.

Then, we have odd number pairs as listed in table 1:

Table 1. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$ and $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$

N-7	N-17	N-37	N-47	N-67	N-97	...	$N/2+L+3$	$N/2-L-7$...	83	73	53	43	23	13	3
7	17	37	47	67	97	...	$N/2-L-3$	$N/2+L+7$...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let \$1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191, ...; \$3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193, ...; \$7 represents a prime with 7 as its last digit, such as, 7, 17,

37, 47, 67, 97, 107, 127, 137, 157, 167, 197...; and \$9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,....

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,...; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,...; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77...; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,....

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number with 3 as its last digit is a product of \$3x\$1 or \$7x\$9; \$1 is decided by \$3 and \$9 is decided by \$7, so we need to consider only \$3 and \$7.

For number N, there are N/10 odd numbers. According to Euclid's proof, primes are endless and it is easy to prove that prime with 3 as its last digit is also endless.

If a number ($N > 3$) cannot be divided by 3 or any prime which is smaller or equal to $N/3$, it must be a prime. Any number can be divided by 7, it have $1/3$ chance can be divided by 3, any number can be divided by 13, it has $1/3$ chance to be divided 3 and $1/7$ chance to be divided by 7, so on, so we have terms: $1/7 \times 2/3$, $1/13 \times 2/3 \times 6/7$... The number (n) of primes in N/10 odd number with 3 as its last digit: $n = N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 13$

$$6/137) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) \dots \} \quad (\text{Formula 1})$$

For infinitely terms, the number will grow slowly and will be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid's proof of endless prime numbers. Let \sum represent the sum of the infinitely terms and $\Delta=1-\sum$,

$$\begin{aligned} \text{The sum of first 20 terms} &= [(1/3 + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + \\ &(1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) \\ &+ (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + \\ &(1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) \\ &+ (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) \\ &+ (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) \\ &+ (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) \\ &+ (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) \\ &+ (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) \\ &+ (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + \\ &(1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + \\ &(1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + \\ &(1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) \\ &= N/10 [1/3 + 1/10.5 + 1/22.75 + 1/32.23 + 1/46.33 + 1/77.92 + 1/93.07 + 1/104.15 + 1/120 + 1/164.61 + 1/171.01 + 1/197.14 + 1/233.2 + 1/250.2 + 1/262.47 + 1/279.80 + 1/317.27 + 1/344.97 + 1/398.24 + 1/416.11] = [0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 + 0.006075 + 0.005848 + 0.0050773 + 0.004288 + 0.003997 + 0.003810 + 0.003574 + 0.003152 + 0.002899 + 0.002511 + 0.002403 + 0.002331] = 0.6102883 \end{aligned}$$

For N=600, the smallest prime \$1 is 11, it decided the possible largest prime \$3 is 53, the smallest prime \$9 is 19, but 3 x 3 is 9, so the possible largest prime \$7 is 47, 47 x 3 x 3=423 (the next will 67 x 3 x 3=603>600), so we have: Prime number with 3 as its last

$\text{digit} = 600/10 - \{600/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47)]\} = 600/10 - 600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333] = 60 - 60 \times 0.566654 = 60 - 34 = 26$,
 it is 3 less than 29 primes (in total 60 odd number) with 3 as their last digit from 1 to 600 due to the first three odd numbers 3, 13, and 23 is too small and cannot be divided by primes 7, 11, 13, 17, or 19. When N is big enough, the calculated number will be very close to the real number. . For N=600, we have $\Delta = 1 - \sum = 1 - 0.566654 = 0.433346$, every odd number with 3 as its last digit has almost 43% chance to be a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime; for a number bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime.

Every odd number with 7 as its last digit is a product of \$3x\$9 or \$7x\$1; \$1 is decided by \$7 and \$9 is decided by \$3, so we need to consider only \$3 and \$7.

The number (n) of primes in N/10 odd number with 7 as its last digit is: $n = N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) \dots\}$ (same as Formula 1).

That mean we have almost same number of primes with 3 as their last digit as the number of primes with 7 as their last digit. From above formula, we can know smaller number N has high percentage to be primes than bigger number N.

For formula 1, we also can know: for any odd number (O7) with 7 as its last digit, the chance of N-O7 to be an odd number (but not prime) with 3 as its last digit is: $[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) \dots]$ (Formula 2)

When N is a very large number, the sum of all term will be close to 1, but never be 1 (if it equals 1, the number of primes will be zero, it is impossible).

If this O7 is a prime \$7, the chance of (N-\$7) to be an odd number (but not prime) with 3 as its last digit is smaller than (N-O7). Why?

Let see \$7=7 first (left side of table 1), When \$7 is 7, $N = O_1 + O_2$, $O_1 - O_2 = (N-7) - 7 = 2L + 6$. The chance of (N-7) to be an odd number (not prime) with 3 as its last digit is: $[(1/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67)]$

$$\begin{aligned}
&+(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) \\
&+(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) \\
&+(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) \\
&+(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) \\
&+(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) \\
&+(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + \\
&(1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + \\
&(1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x13 \\
&6/137) + \\
&(1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x13 \\
&6/137x156/157)...] \qquad \qquad \qquad \text{(Formula 3)}.
\end{aligned}$$

Only 56, 126, 196,..., can be divided by 7, however, L=0, 10, 20, 30, 40.... and (2L+6) =6, 26, 46, 66, 86, 106, 126, 146, 166, 186, 206...., only 1 in 3 is shown on (2L+6) that can be divided by 7, so the term (1/7x2/3), not (1/7) should be cancelled from Formula 2.

For the next prime \$7=17, the chance of (N-17) to be an odd number (not prime) with 3 as its last digit is: [(1/3) +(1/7x2/3)

$$\begin{aligned}
&(1/13x2/3x6/7) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ \\
&(1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + \\
&(1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + \\
&(1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) \\
&+(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) \\
&+(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) \\
&+(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) \\
&+(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) \\
&+(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) \\
&+(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + \\
&(1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + \\
&(1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x13 \\
&6/137) +
\end{aligned}$$

$$(1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157)...] \quad (\text{Formula 4}).$$

Term $(1/17x2/3x6/7x12/13)$ should be cancelled, so on.... Finally, we have:

$$\begin{aligned} & [(1/3) + (1/13x2/3x6/7) + (1/23x2/3x12/13x6/7x16/17) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ \\ & (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) \\ & +(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) \\ & +(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) \\ & +(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) + \\ & (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157)...}] \end{aligned} \quad (\text{Formula 5})$$

For the right side of table 1, starting from $S=3$, $N = O_1+O_2$, $O_1-O_2 = (N-3)-3=2L+14$, The chance of $(N-3)$ to be an odd number (not prime) with 7 as its last digit is:

$$\begin{aligned} & [(1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + \\ & (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) \\ & + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + \\ & (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) \\ & +(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) \\ & +(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) \\ & +(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) \\ & +(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) \\ & +(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) \\ & +(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + \\ & (1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + \\ & (1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) + \\ & (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157)...] \end{aligned} \quad (\text{Formula 6}).$$

Term $(1/3)$ should be cancelled from Formula 6.

For the next prime 13, the chance of (N-13) to be an odd number (not prime) with 3 as its last digit is: $[(1/3)+(1/7 \times 2/3) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157)...]$ (Formula 7),

The term $(1/13 \times 2/3 \times 6/7)$ should be removed from (Formula 7), so on... Finally, every term is cancelled in Formula 5.

$[(1/3) + (1/13 \times 2/3 \times 6/7) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157)...]$ (Formula 5)

For number N, let $n=(N/2) \times (1/10)$, we have n primes (\$7) with 7 as its last digit and n (N-\$7) and n primes (\$3) with 3 as its last digit and n (N-\$3), the chance of any (N-\$7) and (N-\$3) to be a prime will be:

$$\begin{aligned}
 & n - \{ (n-1) \left[\frac{1}{3} + \frac{1}{7 \times 2/3} + \frac{1}{13 \times 2/3 \times 6/7} + \frac{1}{17 \times 2/3 \times 6/7 \times 12/13} + \frac{1}{23 \times 2/3 \times 12/13 \times 6/7 \times 16/17} + \right. \\
 & \left. \frac{1}{37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23} + \frac{1}{43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23} + \frac{1}{47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43} \right. \\
 & \left. + \frac{1}{53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47} + \frac{1}{67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53} + \right. \\
 & \left. \frac{1}{73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67} \right. \\
 & \left. + \frac{1}{83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73} \right. \\
 & \left. + \frac{1}{97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83} \right. \\
 & \left. + \frac{1}{103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97} \right. \\
 & \left. + \frac{1}{107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103} \right. \\
 & \left. + \frac{1}{113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107} \right. \\
 & \left. + \frac{1}{127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113} + \right. \\
 & \left. \frac{1}{137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127} + \right. \\
 & \left. \frac{1}{157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137} + \right. \\
 & \left. \frac{1}{163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157} + \dots \right\} = n - [(n-1)(\Sigma)] = n - n\Sigma + \Sigma = [n(1-\Sigma) + \Sigma] = n\Delta + \Sigma = n\Delta + (1-\Delta) = 1 + n\Delta - \Delta > 1 \text{ because } n > 1, 1 + n\Delta - \Delta > 1, \text{ The results}
 \end{aligned}$$

show: When both N and N/2 have 0 as their last digit, there is at least one pair primes in which one prime has 7 as its last digit and another has 3 as its last digit and their sum is N (see table 1). This is an extreme situation, normally, more than 1 pair primes can be found. For N=600, $n=(600/2) \times (1/10)=30$, $\Sigma=0.57$ and $\Delta=0.43$, $n\Delta + \Sigma=30 \times 0.57=13.5$, in fact, 600 can be expressed as the sum of 15 pairs of primes which has 7 as one prime last digit and 3 as another prime last digit.

For enough large number N, Let's consider $N=O_1+O_2=(N/2+L+1)+(N/2-L-1)$, O_1 and O_2 are odd numbers. $O_1 > O_2$, $O_1-O_2=2L+2$, $L=0, 10, 20, 30, \dots, L$, $O_1-O_2=2L+2=2, 22, 42, 62, 82, 102, 122, \dots(2L+2)$, then O_1 is an odd number with 1 as its last digit, O_2 is an odd number with 9 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+9)+(N/2-L-9)$, O_1 and O_2 are odd numbers. $O_1>O_2$, $O_1-O_2=2L+18$, $L=0, 10, 20, 30\dots L$, $O_1-O_2=2L+18=18, 38, 58, 78, 98, 118, 138,\dots(2L+18)$, then O_1 is an odd number with 9 as its last digit, O_2 is an odd number with 1 as its last digit.

These odd number pairs are listed in table 2:

Table 2. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L-1)$ and $N=O_1+O_2=(N/2+L+9)+(N/2-L-9)$

N-9	N-19	N-29	N-39	N-59	N-79	N-89	...	N/2+L+1	N/2-L-9	...	101	71	61	41	31	11
9(3x3)	19	29	39	59	79	89	...	N/2-L-1	N/2+L+9	...	N-101	N-71	N-61	N-41	N-31	N-11

Every odd number with 1 as its last digit is a product of $\$1x\1 or $\$9x\9 . The first $\$9$ is 19, but the old number $9 = 3x3$, so 3 is the smallest prime for $\$9$ and the number (n) of primes in $N/10$ odd number with 1 as its last digit: $n=N/10 - \{N/10[(1/3) + (1/11x2/3) + (1/19x2/3x10/11) + (1/29x2/3x10/11x18/19) + (1/31x2/3x10/11x18/19x28/29) + (1/41x2/3x10/11x18/19x28/29x30/31) + (1/59x2/3x10/11x18/19x28/29x30/31x40/41) + (1/61x2/3x10/11x18/19x28/29x30/31x40/41x58/59) + (1/71x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61) + (1/79x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71) + (1/89x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79) + (1/101x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79x88/89)+ (1/109x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79x88/89x100/101) \dots\}$ (Formula 8)

For infinitely terms, the number will grow slowly and can be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid's proof of endless prime numbers. \sum represent the sum of the infinitely terms and $\Delta=1-\sum$.

Every odd number with 9 as its last digit is a product of \$1x\$9 or \$7x\$7, the first \$9 is 19, but the old number $9 = 3 \times 3$, so 3 is the smallest prime for \$9 and the number (n) of primes in N/10 odd number with 9 as its last digit: $n=N/10 - \{N/10[(1/3) + (1/7 \times 2/3) + (1/17 \times 2/3 \times 16/17) + (1/19 \times 2/3 \times 6/7 \times 16/17) + (1/29 \times 2/3 \times 6/7 \times 16/17 \times 18/19) + (1/37 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29) + (1/47 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37) + (1/59 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59) + (1/79 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67) + (1/89 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79) + (1/97 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89) + 1/107 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89 \times 96/97) + (1/109 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89 \times 96/97 \times 107/107) \dots\}$ (Formula 9)

Formula 7 is different to Formula 8, but the difference of their sum is quite small when N is big number.

The chance of (N-9) to be an odd number (not prime) with 1 as its last digit will be: $[(1/11 \times 2/3) \times (1/19 \times 2/3 \times 10/11) + (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) + (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots\}$

Term (1/3) should be cancelled from Formula 8.

For the next prime 19, the chance of (N-19) to be an odd number (not prime) with 1 as its last digit will be: $[(1/3) + (1/11 \times 2/3) + (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) + (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots\}$

Term (1/19 \times 2/3 \times 10/11) should be cancelled from Formula 8, so on.... Finally we have:

$$[(1/11x2/3) + (1/31x2/3x10/11x18/19x28/29) + (1/41x2/3x10/11x18/19x28/29x30/31) + (1/61x2/3x10/11x18/19x28/29x30/31x40/41x58/59) + (1/71x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61) + (1/101x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79x88/89)+ ...] \quad (\text{Formula 10})$$

Let's go to the right side of table 2 and \$1=11, When \$1 is 11, The chance of (N-11) to be an odd number (not prime) with 1 as its last digit will be: $[(1/31x2/3x10/11x18/19x28/29) + (1/41x2/3x10/11x18/19x28/29x30/31) + (1/61x2/3x10/11x18/19x28/29x30/31x40/41x58/59) + (1/71x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61) + (1/101x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79x88/89)+ ...]$.

The term $(1/11x2/3)$ should be cancelled out from Formula 10.

For the next \$1=31, The chance of (N-31) to be an odd number (not prime) with 1 as its last digit will be: $[(1/11x2/3) + (1/41x2/3x10/11x18/19x28/29x30/31) + (1/61x2/3x10/11x18/19x28/29x30/31x40/41x58/59) + (1/71x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61) + (1/101x2/3x10/11x18/19x28/29x30/31x40/41x58/59x60/61x70/71x78/79x88/89)+ ...]$.

The term $(1/31x2/3x10/11x18/19x28/29)$ should be cancelled out from Formula 10, so on. Finally, every term is cancelled out from Formula 10.

For a number N, we have n primes (\$1) with 1 as its last digit and n (N-\$1) and n primes (\$9) with 9 as its last digit and n (N-\$9). The chance of any (N-\$1) and (N-\$9) to be a prime will be: $n - \{ (n-1) [(1/3) + (1/7x2/3) + (1/17x2/3x16/17) + (1/19x2/3x6/7x16/17) + (1/29x2/3x6/7x16/17x18/19) + (1/37x2/3x6/7x16/17x18/19x28/29) + (1/47x2/3x6/7x16/17x18/19x28/29x36/37) + (1/59x2/3x6/7x16/17x18/19x28/29x36/37x46/47) + (1/67x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59) + (1/79x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67) + (1/89x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67x78/79) + (1/97x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67x78/79x88/89)+ 1/107x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67x78/79x88/89x96/97) + (1/109x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67x78/79x88/89x96/97x107/107) ...] + [(1/3) + (1/7x2/3) + (1/17x2/3x16/17) + (1/19x2/3x6/7x16/17) + (1/29x2/3x6/7x16/17x18/19) + (1/37x2/3x6/7x16/17x18/19x28/29) + (1/47x2/3x6/7x16/17x18/19x28/29x36/37) + (1/59x2/3x6/7x16/17x18/19x28/29x36/37x46/47) + (1/67x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59) + (1/79x2/3x6/7x16/17x18/19x28/29x36/37x46/47x58/59x66/67) +$

$$\begin{aligned}
 & (1/89 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79) + \\
 & (1/97 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89) + \\
 & 1/107 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89 \times 96/97 \\
 & + (1/109 \times 2/3 \times 6/7 \times 16/17 \times 18/19 \times 28/29 \times 36/37 \times 46/47 \times 58/59 \times 66/67 \times 78/79 \times 88/89 \times 96/97 \times 107/107) \dots \} = n - [(n-1)(\Sigma)] = n - n\Sigma + \Sigma = [n(1 - \Sigma) + \Sigma] = n\Delta + \Sigma = n\Delta + (1 - \Delta) = 1 + n\Delta - \Delta > 1, \quad (\text{Formula 11}).
 \end{aligned}$$

The results show: When both N and N/2 have 0 as their last digit, there is at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit and their sum is N (see table 2).

We have:

When both N and N/2 have 0 as their last digit, every even integer greater than 2 can be expressed as the sum of two primes.

For N = 600 (see table 3), 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit and 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.

7	17	27	37	47	57	67	77	87	97	107	117	127	137	147	157	167	177	187
Prime	Prime	3x9	Prime	prime	3x19	Prime	7x11	3x29	Prime	prime	3x3x13	Prime	prime	3x7x7	prime	prime	3x59	11x17
593	583	573	563	553	543	533	523	513	503	493	483	473	463	453	443	433	423	413
Prime	11x53	3x191	Prime	7x79	3x181	13x41	prime	3x3x3x19	Prime	17x29	3x7x23	11x43	prime	3x151	Prime	prime	3x3x47	7x59
223	233	243	253	263	273	283	293	303	313	323	333	343	353	363	373	383	393	403
prime	prime	3x3x3 x3x3	11x23	prime	3x7x1 3	prime	prime	3x101	Prime	17x19	3x111	7x7x7	Prime	3x11x 11	Prime	Prime	3x131	13x31
377	367	357	347	337	327	317	307	297	287	277	267	257	247	237	227	217	207	197
13x29	prime	3x7x1 7	prime	prime	3x109	Prime	prime	3x3x33	7x41	Prime	3x89	Prime	13x19	3x79	Prime	7x31	3x3x23	Prime

387	397	407	417	427	437	447	457	467	477	487	497	507	517	527	537	547	557	567
3x3x4 3	prime	11x37	3x139	7x61	23x19	3x149	prime	prime	3x3x5 3	prime	7x71	3x13x 13	11x47	17x31	3x179	prime	prime	3x3x3x3 x7
213	203	193	183	173	163	153	143	133	123	113	103	93	83	73	63	53	43	33
3x71	7x29	prime	3x61	Prime	prime	3x3x1 7	11x13	7x19	3x41	3x3x1 3	prime	3 x31	prime	7x11	3x3x7	prime	prime	3x11
																3	13	23
																Prime	Prime	prime
																597	587	577
																3x199	prime	prime
589	579	569	559	549	539	529	519	509	499	489	479	469	459	449	439	429	419	409
19x31	3x193	Prime	13x43	3x3x6 1	7x7x1 1	23x23	3x178	Prime	Prime	3x163	Prime	7x67	3x3x3x1 7	Prime	Prime	3x11x13	Prime	Prime
11	21	31	41	51	61	71	81	91	101	111	121	131	141	151	161	171	181	191
Prime	3x7	Prime	Prime	3x17	Prime	Prime	3x3x3	7x13	Prime	3x37	11x11	Prime	3x47	Prime	7x23	3x3x19	Prime	Prime
381	371	361	351	341	331	321	311	301	291	281	271	261	251	241	231	221	211	201
3x127	7x53	19x19	3x3x3 x13	11x31	Prime	3x107	Prime	7x43	3x97	Prime	Prime	3x3x2 9	Prime	Prime	3x7x1 1	13x17	Prime	3x67

219 3x73	229 Prime	239 Prime	249 3x83	259 7x37	269 Prime	279 3x3x3 1	289 17x17	299 13x23	309 3x103	319 11x29	329 7x47	339 3x113	349 Prime	359 Prime	369 3x3x4 1	379 prime	389 Prime	399 3x7x19
209 11x19	199 Prime	189 3x3x7	179 Prime	169 13x13	159 3x53	149 Prime	139 Prime	129 3x43	119 7x17	109 Prime	99 3x3x11	89 Prime	79 Prime	69 3x23	59 Prime	49 7x7	39 3x13	29 Prime
391 17x23	401 Prime	411 3x137	421 Prime	431 Prime	441 3x3x7 x7	451 11x41	461 Prime	471 3x157	481 13x37	491 Prime	501 3x167	511 7x73	521 Prime	531 3x3x5 9	541 Prime	551 19x29	561 3x11x17	571 Prime
																	591 3x197	581 7x81
																	9 3x3	19 Prime

1b. When both N has 0 as its last digit, and N/2 has 5 as its last digit.

Table 3. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-7	N-17	N-37	N-47	N-67	N-97	...	N/2+L-2	N/2-L-2	...	83	73	53	43	23	13	3
7	17	37	47	67	97	...	N/2-L+2	N/2+L+2	...	N-83	N-73	N-53	N-43	N-23	N-13	N-3

In same way as 1a, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N (see table 3).

Table 4. The odd number pairs in $N=O_1+O_2=(N/2+L-4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L-6)+(N/2-L+6)$

N-9	N-19	N-29	N-59	N-79	N-89	...	$N/2+1-4$	$N/2-L+6$...	101	71	61	41	31	11
$9(3 \times 3)$	19	29	59	79	89	...	$N/2-L+4$	$N/2+L-6$...	N-101	N-71	N-61	N-41	N-31	N-11

In same way as 1a, we can prove there is at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit and their sum is N (see table 4).

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 112, 122, 1122, 1132, ..., then N/2 has only 6 or 1 as its last digit.

2a. When any even integer (N) has 2 as its last digit, such as 12, 32, 52, 112, 1132, ..., then N/2 has 6 as its last digit:

Table 5. The odd number pairs in $N=O_1+O_2=(N/2+L-3)+(N/2-L+3)$ and $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$

N-9	N-19	N-29	N-59	N-79	N-89	...	$N/2+L-3$	$N/2-L-3$...	83	73	53	43	23	13	3
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$$\begin{aligned}
&+(1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) \\
&+(1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) \\
&+(1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) \\
&+(1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) \\
&+(1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + \\
&(1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + \\
&(1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + \\
&(1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) \dots \} \quad \text{(Formula 1)}
\end{aligned}$$

Both are same to Formula 1 and easy to prove there is at least one pair primes in which one prime has 9 as its last digit and another has 3 as its last digit and their sum is N.

Table 6. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$ and $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$

N-11	N-31	N-41	N-61	N-71	N-101	...	N/2+L-5	N/2-L+5	...	101	71	61	41	31	11
11	31	41	61	71	101	...	N/2-L+5	N/2+L-5	...	N-101	N-71	N-61	N-41	N-31	N-11

Every odd number with 1 as its last digit is a product of 1×1 or 9×9 . The first 9 is 19, but the old number $9 = 3 \times 3$, so 3 is the smallest prime for 9 and the number (n) of primes in N/10 odd number with 1 as its last digit: $n=N/10 - \{N/10[(1/3) + (1/11 \times 2/3) + (1/19 \times 2/3 \times 10/11) + (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) +$

$$\begin{aligned} & (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \\ & (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots \} \quad (\text{Formula 8}) \end{aligned}$$

The left side is same as the right side in table 6.

$$\begin{aligned} & S1 \text{ can be the product } (\$9 \times \$9), \text{ the first } \$9 \text{ is } 9(3 \times 3), \text{ the chance of } (N-11) \text{ to be an odd number (not prime) with 1 as its last digit} \\ & \text{will be: } [(1/11 \times 2/3) \times (1/19 \times 2/3 \times 10/11) + (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + \\ & (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + \\ & (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + \\ & (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + \\ & (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) + \\ & (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \\ & (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots \} \end{aligned}$$

Term (1/3) should be cancelled from Formula 8.

$$\begin{aligned} & \text{For the next prime 19, the chance of } (N-19) \text{ to be an odd number (not prime) with 1 as its last digit will be: } [(1/3) + (1/11 \times 2/3) + \\ & (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + \\ & (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + \\ & (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + \\ & (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) + \\ & (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \\ & (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots \} \end{aligned}$$

Term (1/19 \times 2/3 \times 10/11) should be cancelled from Formula 8, so on.... Finally we have:

$$\begin{aligned} & [(1/11 \times 2/3) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + \\ & (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + \\ & (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \dots] \quad (\text{Formula 10}) \end{aligned}$$

\$1 can be the product \$1x\$1 too, When \$1 is 11, The chance of (N-11) to be an odd number (not prime) with 1 as its last digit will be: $[(1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \dots]$.

The term $(1/11 \times 2/3)$ should be cancelled out from Formula 10.

For the next \$1=31, The chance of (N-31) to be an odd number (not prime) with 1 as its last digit will be: $[(1/11 \times 2/3) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + \dots]$.

The term $(1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29)$ should be cancelled out from Formula 10, so on. Finally, every term is cancelled out from Formula 10.

For a number N, we have n primes (\$1) with 1 as its last digit and n (N-\$1) and n primes (\$1) with 1 as its last digit and n (N-\$1). The chance of any (N-\$1) and (N-\$9) to be a prime will be: $n - \{(n-1)[(1/3) + (1/11 \times 2/3) + (1/19 \times 2/3 \times 10/11) + (1/29 \times 2/3 \times 10/11 \times 18/19) + (1/31 \times 2/3 \times 10/11 \times 18/19 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31) + (1/59 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41) + (1/61 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61) + (1/79 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71) + (1/89 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79) + (1/101 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89) + (1/109 \times 2/3 \times 10/11 \times 18/19 \times 28/29 \times 30/31 \times 40/41 \times 58/59 \times 60/61 \times 70/71 \times 78/79 \times 88/89 \times 100/101) \dots]\} = n - [(n-1)(\sum)] = n - n\sum + \sum = [n(1 - \sum) + \sum] = n\Delta + \sum = n\Delta + (1 - \Delta) = 1 + n\Delta - \Delta > 1, \quad (\text{Formula 12}).$

There are 2 pair primes in which both primes have 1 as its last digit and their sum is N (see table 6) and another pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N (see table 5).

2b. When any even integer (N) has 2 as its last digit, such as 22, 42, 62, 82, 102, 1122, ..., then N/2 has 1 as its last digit:

Table 7. The odd number pairs in $N=O_1+O_2=(N/2+L+2)+(N/2-L+8)$ and $N=O_1+O_2=(N/2-L+2)+(N/2+L+8)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L+2	N/2-L+2	...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	N/2-L+8	N/2+L+8	...	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 7 is same to table 5, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 8. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L-0)$ and $N=O_1+O_2=(N/2+L-0)+(N/2-L+0)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L+0	N/2-L+0	...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	N/2-L-0	N/2+L-0	...	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 8 is same to table 6, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

3a. When any even integer (N) has 4 as its last digit, such as 24, 44, 64, 84, 104, 1124, ..., then N/2 has 2 as its last digit:

Table 9. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L+7)$ and $N=O_1+O_2=(N/2-L+1)+(N/2+L+7)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L+1	N/2-L+1	...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	N/2-L+7	N/2+L+7	...	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 9 is same to table 7, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 10. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L-1)$ and $N=O_1+O_2=(N/2+L-1)+(N/2-L-1)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L-1	N/2-L-1	...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	N/2-L-1	N/2+L-1	...	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 10 is same to table 8, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

3b. When any even integer (N) has 4 as its last digit, such as 14, 34, 54, 74, 94, 1114, ..., then N/2 has 7 as its last digit:

Table 11. The odd number pairs in $N=O_1+O_2=(N/2+L+6)+(N/2-L+2)$ and $N=O_1+O_2=(N/2-L+6)+(N/2+L+2)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L+6	N/2-L+6	...	83	73	63	53	43	33	23	13	3
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9	19	29	39	49	59	69	79	...	$N/2-L+2$	$N/2+L+2$...	$N-83$	$N-73$	$N-63$	$N-53$	$N-43$	$N-33$	$N-23$	$N-13$	$N-3$
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Table 11 is same to table 9, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 12. The odd number pairs in $N=O_1+O_2=(N/2+L+4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L+4)+(N/2-L+4)$

$N-11$	$N-21$	$N-31$	$N-41$	$N-51$	$N-61$	$N-71$	$N-81$...	$N/2+L+4$	$N/2-L+4$...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	$N/2-L+4$	$N/2+L+4$...	$N-91$	$N-81$	$N-71$	$N-61$	$N-51$	$N-41$	$N-31$	$N-21$	$N-11$

Table 12 is same to table 10, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

4a. When any even integer (N) has 6 as its last digit, such as 26, 46, 66, 86, 106, 1126,..., then N/2 has 3 as its last digit:

Table 13. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L+6)$ and $N=O_1+O_2=(N/2-L+0)+(N/2+L+6)$

$N-9$	$N-19$	$N-29$	$N-39$	$N-49$	$N-59$	$N-69$	$N-79$...	$N/2+L+0$	$N/2-L+0$...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	$N/2-L+6$	$N/2+L+6$...	$N-83$	$N-73$	$N-63$	$N-53$	$N-43$	$N-33$	$N-23$	$N-13$	$N-3$

Table 13 is same to table 11, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 14. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+8)$ and $N=O_1+O_2=(N/2+L+8)+(N/2-L-2)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L-2	N/2-L-2	...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	N/2-L+8	N/2+L+8	...	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 14 is same to table 12, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

4b. When any even integer (N) has 6 as its last digit, such as 16, 36, 56, 76, 96, 1116, ..., then N/2 has 8 as its last digit:

Table 15. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+1)$ and $N=O_1+O_2=(N/2-L+5)+(N/2+L+1)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L-5	N/2-L+5	...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	N/2-L+1	N/2+L+1	...	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 15 is same to table 13, there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 16. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L+3)$ and $N=O_1+O_2=(N/2+L+3)+(N/2-L+3)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L+3	N/2-L+3	...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	N/2-L+3	N/2+L+3	...	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 16 is same to table 14, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

5a. When any even integer (N) has 8 as its last digit, such as 28, 48, 68, 88, 108, 1128,..., then N/2 has 4 as its last digit:

Table 17. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L+5)$ and $N=O_1+O_2=(N/2-L-1)+(N/2+L+5)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L-1	N/2-L-1	...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	N/2-L+5	N/2+L+5	...	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 17 is same to table 15, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 18. The odd number pairs in $N=O_1+O_2=(N/2+L-3)+(N/2-L+7)$ and $N=O_1+O_2=(N/2+L+7)+(N/2-L-3)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L-3	N/2-L-3	...	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	...	N/2-L+7	N/2+L+7	...	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 18 is same to table 16, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

5b. When any even integer (N) has 8 as its last digit, such as 18, 38, 58, 78, 98, 1118,..., then N/2 has 9 as its last digit:

Table 19. The odd number pairs in $N=O_1+O_2=(N/2+L-6)+(N/2-L+0)$ and $N=O_1+O_2=(N/2-L-6)+(N/2+L+0)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	...	N/2+L-6	N/2-L-6	...	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	...	N/2-L+0	N/2+L+0	...	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 19 is same to table 17, there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 20. The odd number pairs in $N=O_1+O_2=(N/2+L+2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L+2)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	...	N/2+L+2	N/2-L+2	...	91	81	71	61	51	41	31	21	11
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11	21	31	41	51	61	71	81	...	$N/2-L+2$	$N/2+L+2$...	$N-91$	$N-81$	$N-71$	$N-61$	$N-51$	$N-41$	$N-31$	$N-21$	$N-11$
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Table 20 is same to table 18, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

For any even number, Goldbach's conjecture is true.

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1. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
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