A Simple Method of Determining the Mass of Hadrons

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Abstract

Determining the mass of hadrons was a predicament in Quantum Chromodynamics, suggestions and attempts of using a theory of lattice QCD for such determination has not provided satisfying results. This paper will suggest a new method that offers 99% accuracy therefore much higher than lattice QCD, as well as simplicity. The methodology provided in this paper is relatively simple which makes it easier to do the calculus without unnecessarily losing time on extremely complex equations that serve not practical purpose since the accuracy of lattice QCD in determining hadrionic mass is approximately 10% which is underwhelming. The aforementioned new method will be applied for protons, neutrons and pions.

Introduction

In the method for theoretically determining the mass of hadrons we will use two dimensionless constants: the electromagnetic constant α, also known as the fine structure constant [1], which has a value of $\alpha = 0.00729735256$ [2] and the strong nuclear constant α_S. The value of α_S varies with distance, on the scale of 1 fm it has been determined to be $\alpha_S \approx 1$.

The constant alpha can be determined through various methods, from the anomalous magnetic dipole moment

$$a_e = A_1 \frac{\alpha}{\pi} + A_2 \left(\frac{\alpha}{\pi}\right)^2 + A_3 \left(\frac{\alpha}{\pi}\right)^3 + \cdots + a\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}, \ldots\right) \tag{1}$$

Where, for electrons we have:

$$\delta F_1(q^2) \to \delta F_1(q^2) - \delta F_1(0) \tag{2}$$

Where $F_1(0) = 1$ and δF_1 is the first order correction to F_1. We define for electrons:

$$F_1(q^2) = 1 + \frac{\alpha}{\pi}\int_0^1 dx\, dy\, dz\, \delta(x + y + z + 1)\left[\log\left(\frac{m^2(1-z)^2}{m^2(1-z)^2 - q^2xy}\right) + \left(\frac{m^2(1-4z+z^2) + q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2xy + \mu^2 z}\right) - \left(\frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + \mu^2 z}\right)\right] + \mathcal{O}\{\alpha^2\} \tag{3}$$

After the calculus the equation is reduced to

$$F_2(q^2) = \frac{\alpha}{2\pi}\int_0^1 x\, dy\, dz\, \delta(x + y + z - 1)\left[\frac{2m^2(1-z)}{m^2(1-z)^2 - q^2xy}\right] + \mathcal{O}\{\alpha^2\} \tag{4}$$

Therefore:

$$F_2(q^2 = 0) = \frac{\alpha}{2\pi}\int_0^1 x\, dy\, dz\, \delta(x + y + z - 1)\frac{2m^2(1-z)}{m^2(1-z)^2} = \frac{\alpha}{\pi}\int_0^1 dz \int_0^{1-z} dy \frac{z}{1-z} = \frac{\alpha}{2\pi} \tag{5}$$

The anomalous magnetic dipole moment is:

$$a_e = \frac{\alpha}{2\pi} = 0.001161409695 \tag{6}$$

which is the one loop result. Knowing the g factor of the electron, see [2], allows for a simple equation.

Using the electron g factor, we can calculate the anomalous magnetic dipole moment:

$$a_e \equiv \frac{g_e - 2}{2} \tag{7}$$

This method is often used to measure the anomalous magnetic moments of electrons, muons, taus etc.

It should also be mentioned that on low q^2 due to large M_W the constant $\alpha_S \approx 0.2$ on the scale 0.002 fm when the weak nuclear dimensionless constant is $\alpha_W \approx 0.03$.

Proton QED Number

The method for theoretical determination of the proton mass is:

$$E_0^p = \frac{hc}{\lambda_p} \tag{8}$$

Where E_0^p is the rest energy of the proton. Using this we postulate that:

$$\lambda_p = \sum \lambda_q \cdot 10^{-И} \tag{9}$$

Where $\sum \lambda_q$ is the sum of quark wavelengths and $И$ is the proton QED number. We define that:

$$\sum \lambda_q = \frac{hc}{\sum E_0^q} \tag{10}$$

Where $\sum E_0^q$ is the sum of quark rest energies. Since protons consist of two up quarks and one down quark $\sum E_0^q = 9.4$ MeV. The proton QED number is:

$$И = (n_q - 1) - \frac{\alpha}{n_q \pi} = 2 - \frac{\alpha}{3\pi} \tag{11}$$

Where n_q is the number of quarks, in this case 3. The number $И$ has a value 1.999225726845727 and is clearly dimensionless. Finally we constitute the equation:

$$E_0^p = \frac{hc}{\frac{hc}{\sum E_0^q} \cdot 10^{-И}} = \sum E_0^q \cdot 10^{И} \approx 938.3 \text{ MeV} \tag{12}$$

Which agrees with experimental results that m_p is approximately 938.3 MeV/c^2 [3]. The accuracy of this methodology depends on how accurate the values of quark masses are [4], which is known to a satisfactory level for up quarks and down quarks.

Neutron QCD Number

Neutron mass is calculated in a similar way to that of a proton. We also have:

$$E_0^n = \frac{hc}{\lambda_n} \tag{13}$$

and similarly:

$$\lambda_n = \sum \lambda_q \cdot 10^{-Ш} \tag{14}$$

Where Ш is the neutron QCD number and the equation 10 also applies for neutrons with the difference that neutrons consist of two down quarks and one up quark.

The equation for the neutron QCD number is:

$$Ш = (n_q - 1) - \frac{\alpha_S}{n_q \pi} = 2 - \frac{\alpha_S}{3\pi} \tag{15}$$

Where α_S is the strong interaction constant and on the scale of 1 fm we define that it equals $\alpha_S = 0.9669$ which means that $Ш = 1.8974087236829643$.

Finally we form the equation:

$$E_0^n = \frac{hc}{\frac{hc}{\sum E_0^q} \cdot 10^{-Ш}} = \sum E_0^q \cdot 10^{Ш} \approx 939.6 \text{ MeV} \tag{16}$$

Which agrees with the experiments that m_n is approximately $939.6 \text{ MeV}/c^2$ [5]. Same as with the proton QED number, the neutron QCD number depends on the accuracy of measuring quark energies which are $\sum E_0^q = 11.9$ MeV for two down quarks and one up quark that form a neutron. However, unlike the proton QED number, the neutron QCD number also depends on the accuracy of the dimensionless constant. The dimensionless constant for strong nuclear interactions changes drastically on different scales and it has not been very accurately measured, unlike the accuracy of measures taken on the fine structure constant which is one of the best achievements in experimental and theoretical physics today.

Pion QCD number

Pions, unlike protons and neutrons, have two different methods on two different scales for determining their mass. This paper will only form methods for π^+ and π^- pions, the neutral π^0 pion will not be included in either of the two methods.

For $\pi^{\pm}$ we form a similar equation:

$$E_0^{\pi^{\pm}} = \frac{hc}{\lambda_{\pi^{\pm}}} \tag{17}$$

and the equation:

$$\lambda_{\pi^{\pm}} = \sum \lambda_q \cdot 10^{-Љ} \tag{18}$$

has similarity with proton and neutron equations. Finally, we have:

$$E_0^{\pi^{\pm}} = \frac{hc}{\frac{hc}{\sum E_0^q} \cdot 10^{-Љ}} = \sum E_0^q \cdot 10^{Љ} \approx 139.6 \text{ MeV} \tag{19}$$

which is in agreement with $m_{\pi^{\pm}} \approx 139.6 \text{ MeV/c}^2$ [6].

There are two different methods on two different scales to determine the pion QCD number Љ.

1) The smaller scale method

On the scale of 0.002 fm

$$Љ = \exp(\alpha_S) \tag{20}$$

where $\alpha_s = 0.2573795(9)$ meaning that the pion QCD number Љ = 1.293536.

2) The larger scale method

On the scale of 1 fm we have:

$$Љ = (n_q - 1) + \frac{2\alpha_S}{n_q \pi} = 1 + \frac{2\alpha_S}{2\pi} \tag{21}$$

where $\alpha_s = 0.9221705(1)$ where the running of the constant is clearly observable and the value of Љ is the same as in the former method.

Since a π^+ consists of an up quark and an anti-down quark and π^- consists of a down quark and an anti-up quark, the result for $\sum E_0^q \approx 7.1$ MeV.

Conclusions

The mass of pions was previously calculated with a version of the Gell-Mann-Oakes-Renner mass formula [7] which would be reduced to:

$$m^2_{\substack{\pm \\ 0}\pi} = \frac{(m_u + m_d)\rho}{f_\pi^2} \quad (22)$$

where [8] the integral $\rho = \langle q\bar{q} \rangle$ is the condensate parameter, for $N_c = 3$ we have:

$$\rho = N_c tr\big(G(x = 0)\big) = 12 \int \frac{d^4q}{(2\pi)^4} \sigma_s(q^2) \quad (23)$$

where f_π is the pion decay constant.

It is evident that this method is more complex and harder for one to apply while not offering high accuracy. The new method presented in this paper is simple and elegant, relatively easy to apply which makes our jobs a little bit easier.

Running of the fundamental constants has to be taken into account when the scale is changed or higher energy levels are applied, however this should not present a problem on scales used in this paper. Without the necessity of using super computers for calculations, this methodology is vastly superior in accuracy and more cost effective to the previous ones. Calculating masses for such hadrons as protons and neutrons with the methodology presented in the paper is especially simple and very accurate and requires no software or supercomputers.

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