Using metaballs to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes

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Abstract

In this paper, Blinn's metaballs are used to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes. An analytical solution is provided.

1 Metaballs

Metaballs have been used in computer graphics ever since their discovery by Jim Blinn. They were originally used to visualize electron density [1]. In this paper we will model the pre-ringdown phase of the merger of $n = 2$

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Schwarzschild black holes using the simple mathematics of metaballs [2]. Analogously, we will be visualizing graviton density.

Where $G = c = \hbar = k = 1$, there is a metaball-based solution for the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes travelling directly toward each other:

$$f(l) = \sum_{i=1}^{n=2} \frac{2M_i}{r_i} = \frac{2M_1}{r_1} + \frac{2M_2}{r_2}. \tag{1}$$

Here M_i is the mass of the ith black hole (metaball), and

$$r_i = \sqrt{(l.x - v_i.x)^2 + (l.y - v_i.y)^2 + (l.z - v_i.z)^2}, \tag{2}$$

where l is a sample location, and v_i is the centre of the ith black hole. [1] The event horizon (the isosurface) is given by $f(l) = 1$.

Included are figures of the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes. It is shown that the event horizon resembles the Cassini ovals (or some generalization thereof), which makes for an analytical solution.

The C++/OpenGL code for this paper can be found at [3].

[1] Here the '.' operator is the 'member of' operator, like in the C++ programming language – that is, both l and v_i are instantiations that each encapsulate three variables: x, y, and z.

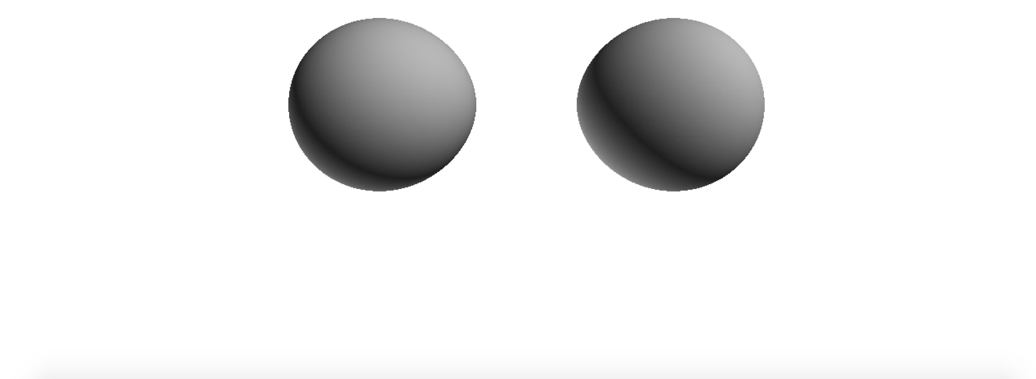

Figure 1: Two black holes of unit mass each, at a distance of 9. Here the unit mass is the Planck mass $M_P = \sqrt{\hbar c/G} = 1$, and the unit distance is the Planck length $\ell_P = \sqrt{\hbar G/c^3} = 1$. Note that the event horizon resembles Cassini ovals.

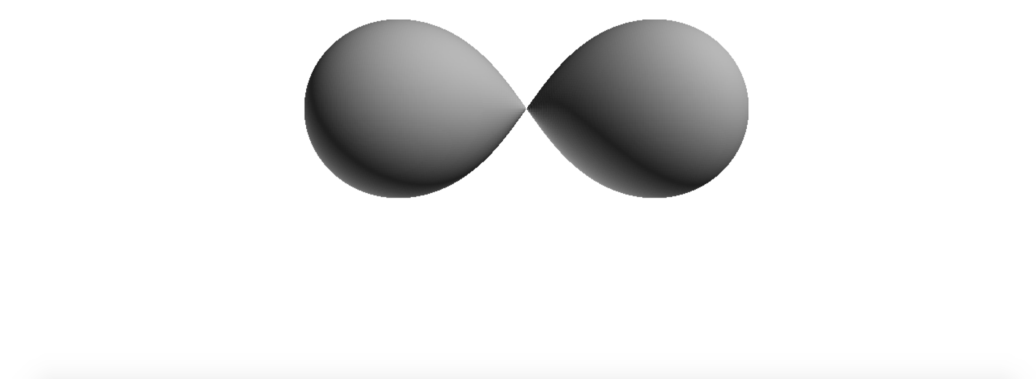

Figure 2: Two black holes of unit mass each, at a distance of 8. Note that the event horizon resembles the lemniscate of Bernoulli, which is a special case of Cassini ovals.

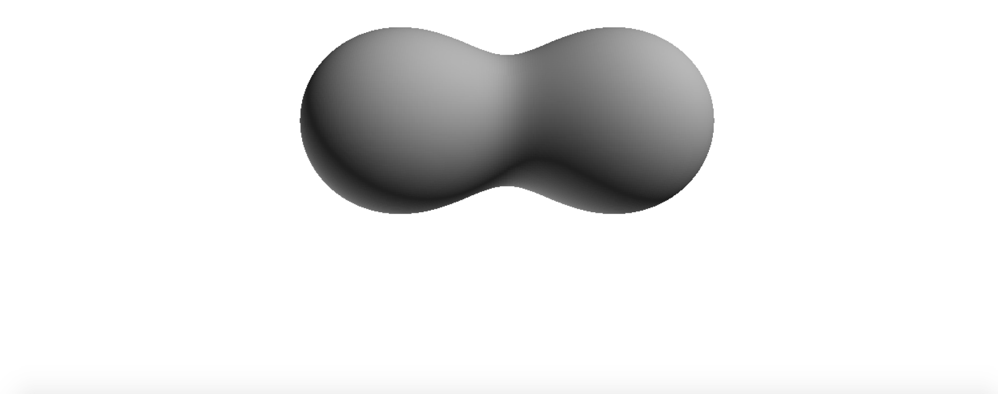

Figure 3: Two black holes of unit mass each, at a distance of 7.

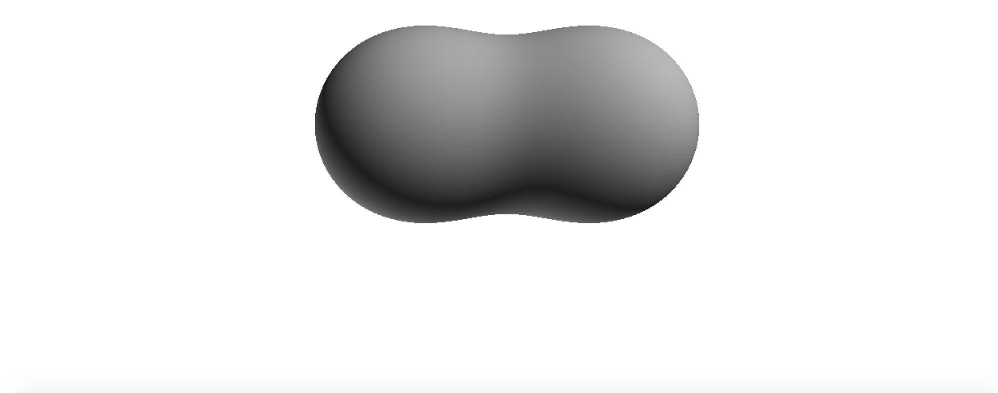

Figure 4: Two black holes of unit mass each, at a distance of 6.

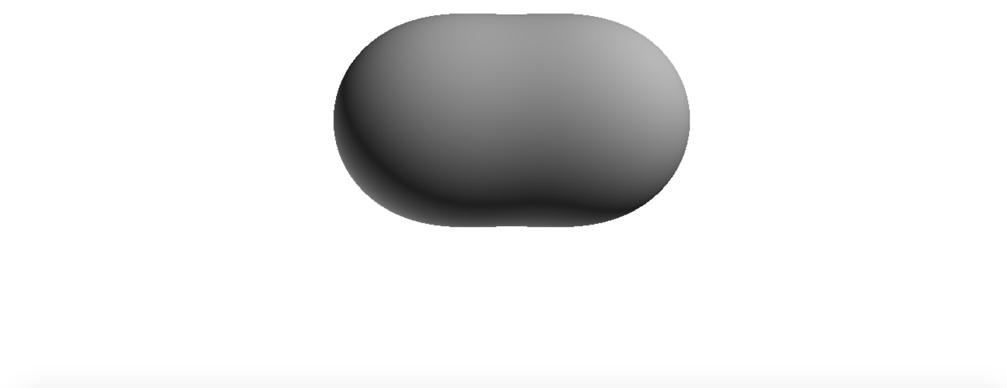

Figure 5: Two black holes of unit mass each, at a distance of 5.

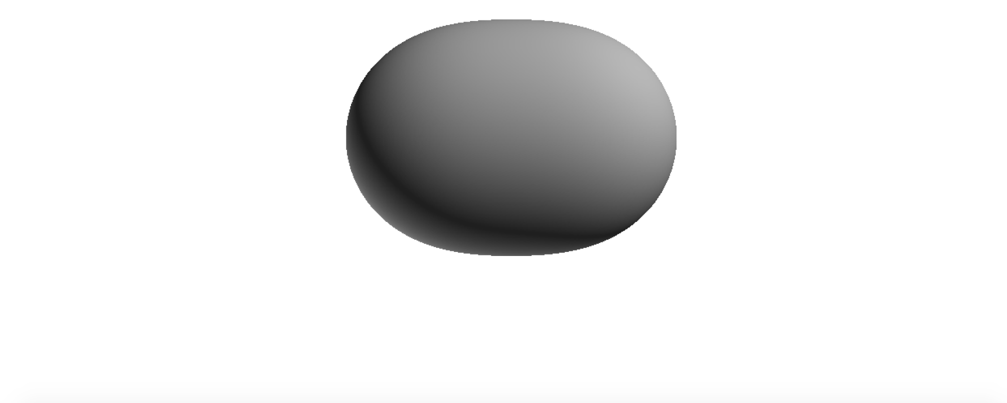

Figure 6: Two black holes of unit mass each, at a distance of 4.

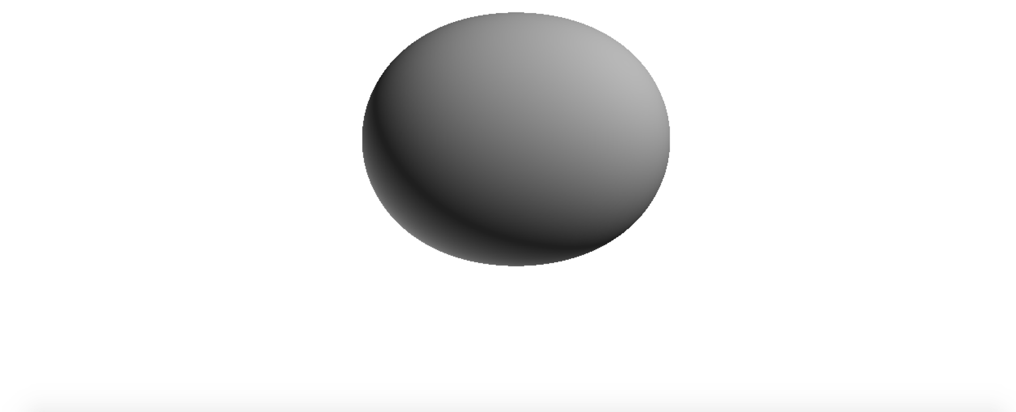

Figure 7: Two black holes of unit mass each, at a distance of 3.

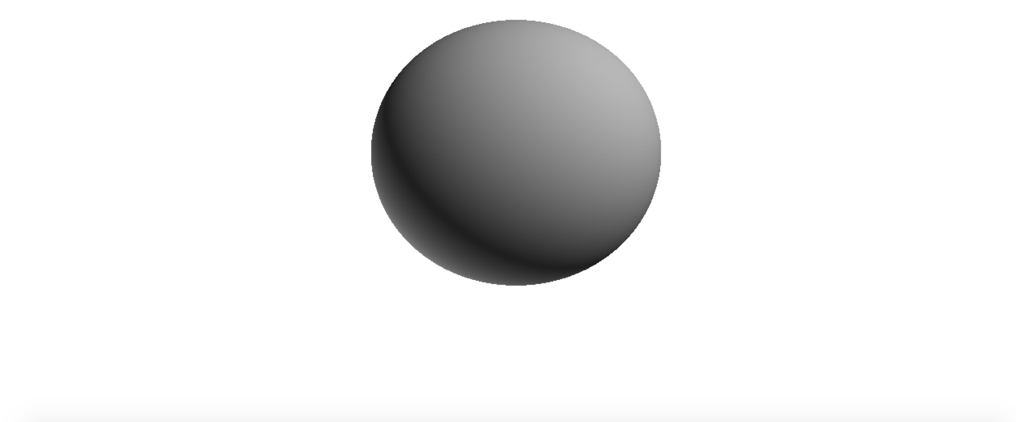

Figure 8: Two black holes of unit mass each, at a distance of 2.

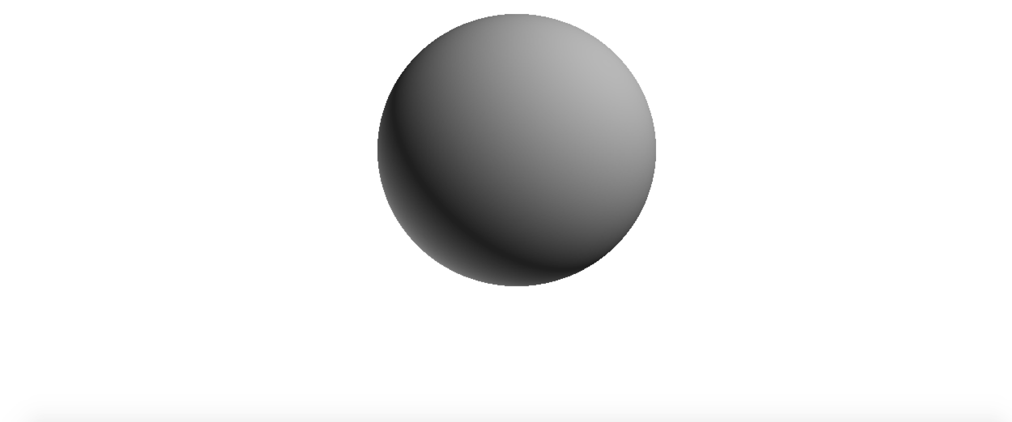

Figure 9: Two black holes of unit mass each, at a distance of 1.

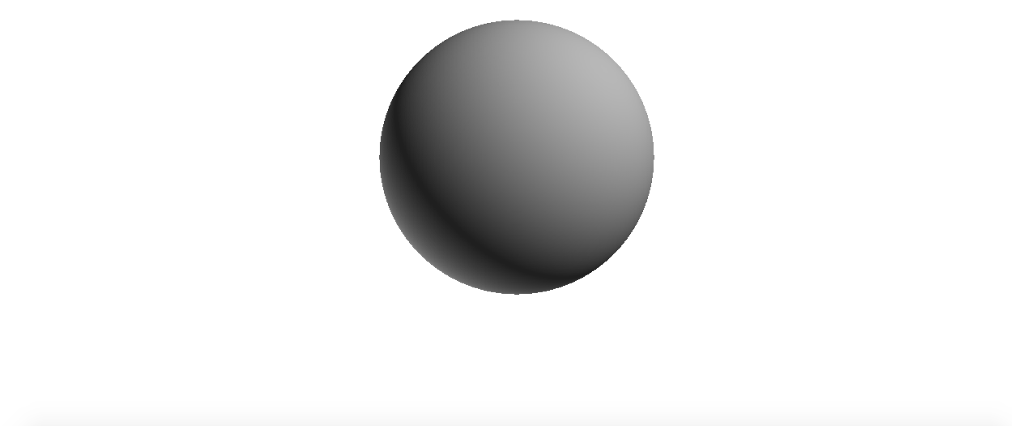

Figure 10: *One* black hole of mass = 2.

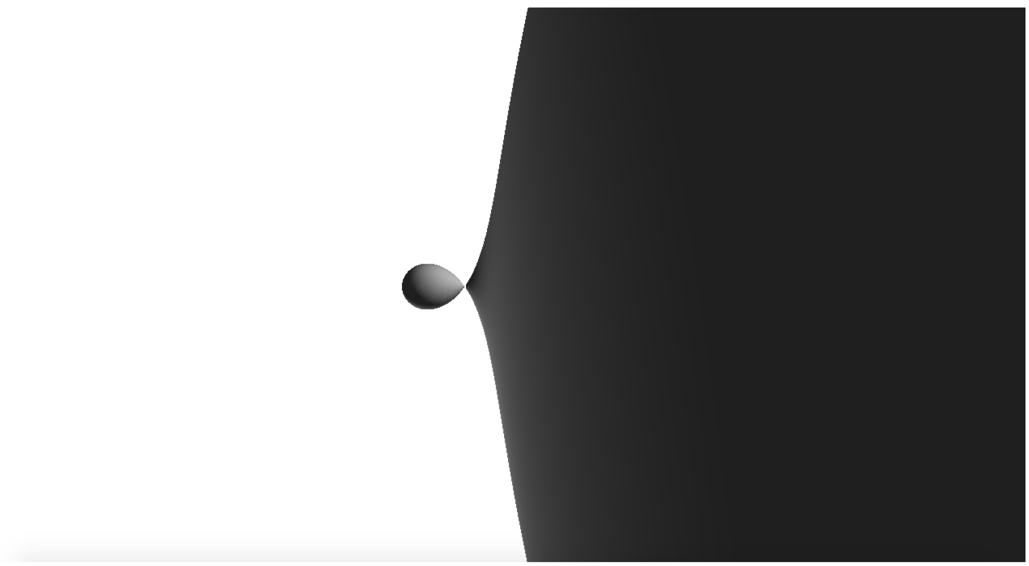

Figure 11: A unit mass black hole merging with a black hole of mass = 1000. Note that the event horizon is likely to be given by some generalization of Cassini ovals, where the torus in question is lopsided like a ring cyclide (Dupin cyclide).

References

[1] Blinn J. *A generalization of algebraic surface drawing* ACM Transactions on Graphics, Vol. 1, No. 3

[2] Daoust M. http://physics.stackexchange.com/questions/18769/black-hole-collision-and-the-event-horizon

[3] Halayka S. *Code* https://github.com/sjhalayka/bhmerger