Using metaballs to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes

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Abstract

In this paper, Blinn's metaballs are used to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes.

1 Metaballs

Metaballs have been used in computer graphics ever since their discovery by Jim Blinn. They were originally used to visualize electron density [1]. In this paper we will model Schwarzschild black holes using the simple mathematics of metaballs, as was first hinted at in [2]. Analogously, we will be visualizing graviton density.

Where $G = c = \hbar = k = 1$, there is an analytical solution for the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes travelling directly toward each other:

$$f(l) = \sum_{i=1}^{n=2} \frac{2M_i}{r_i} = \frac{2M_1}{r_1} + \frac{2M_2}{r_2}. \tag{1}$$

Here M_i is the mass of the ith black hole, and

$$r_i = \sqrt{(l.x - v_i.x)^2 + (l.y - v_i.y)^2 + (l.z - v_i.z)^2}, \tag{2}$$

where l is the sample location, and v_i is the centre of the ith black hole. The event horizon is given by

$$f(l) = 1. \tag{3}$$

Included are figures of the pre-ringdown phase of Schwarzschild black hole mergers. The event horizon was tessellated using the Marching Cubes algorithm [3].

The C++ code for this paper can be found at [4].

Another method for generating the event horizon is given in [5].

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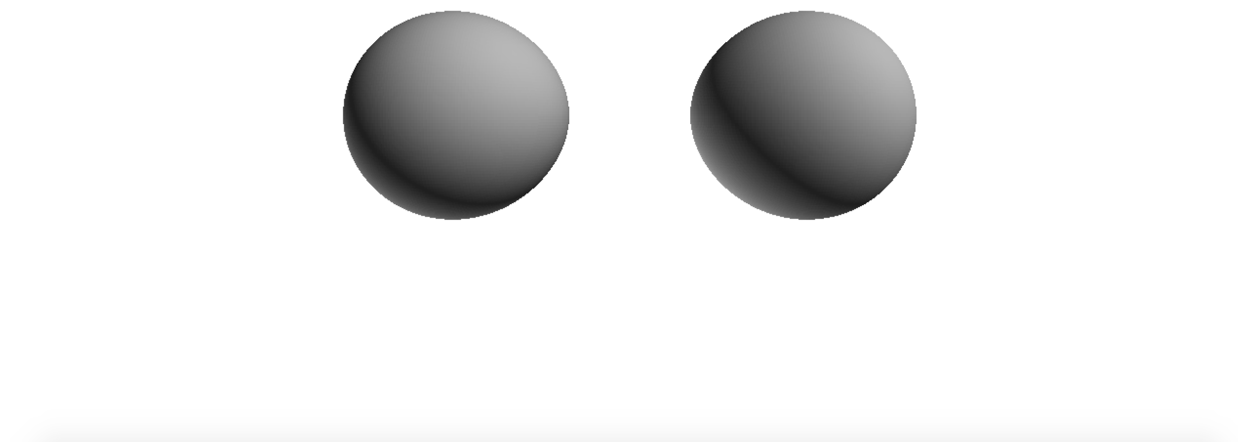

Figure 1: Two black holes of unit mass each, at a distance of 9. Here the unit mass is the Planck mass $M_P = \sqrt{\hbar c/G} = 1$, and the unit distance is the Planck length $\ell_P = \sqrt{\hbar G/c^3} = 1$. Note that the event horizon resembles Cassini ovals.

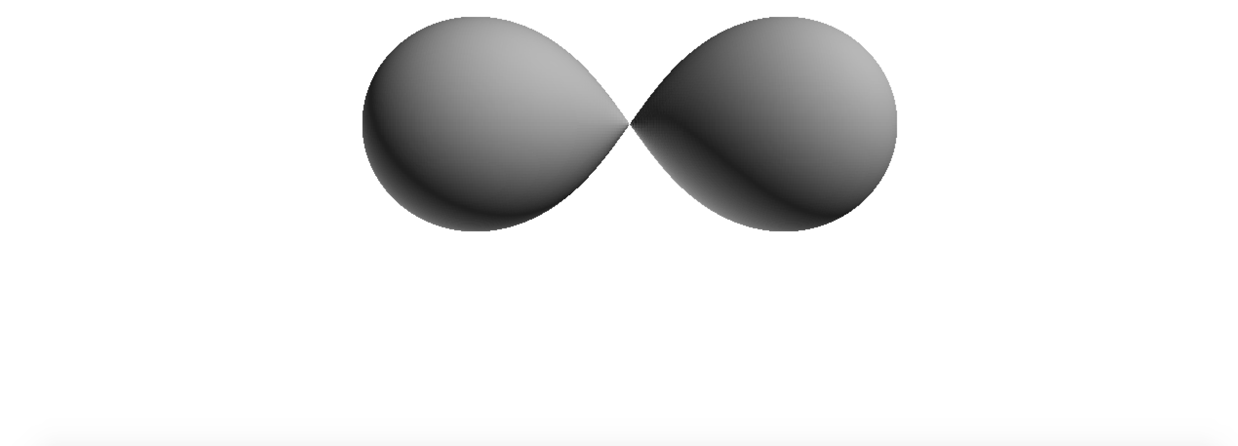

Figure 2: Two black holes of unit mass each, at a distance of 8. Note that the event horizon resembles the lemniscate of Bernoulli, which is a special case of Cassini ovals.

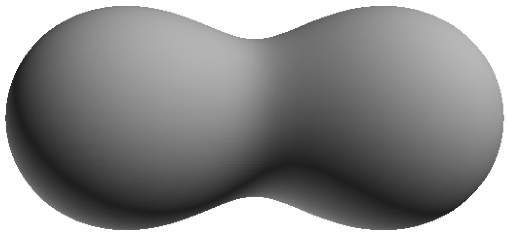

Figure 3: Two black holes of unit mass each, at a distance of 7.

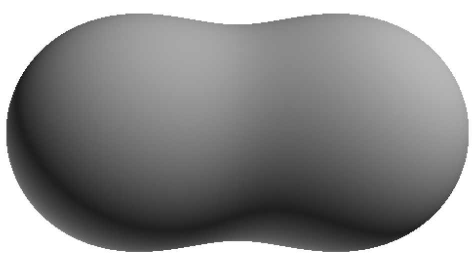

Figure 4: Two black holes of unit mass each, at a distance of 6.

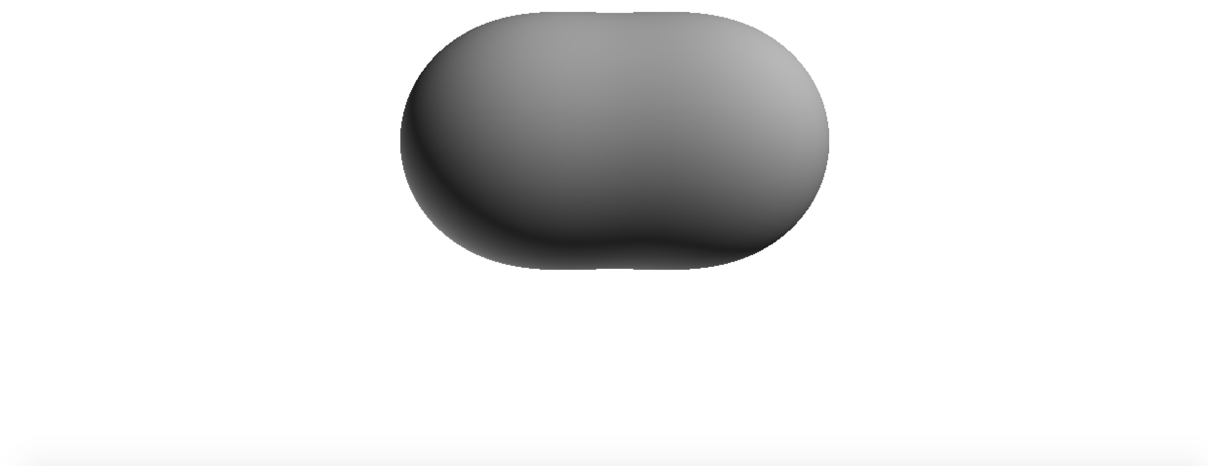

Figure 5: Two black holes of unit mass each, at a distance of 5.

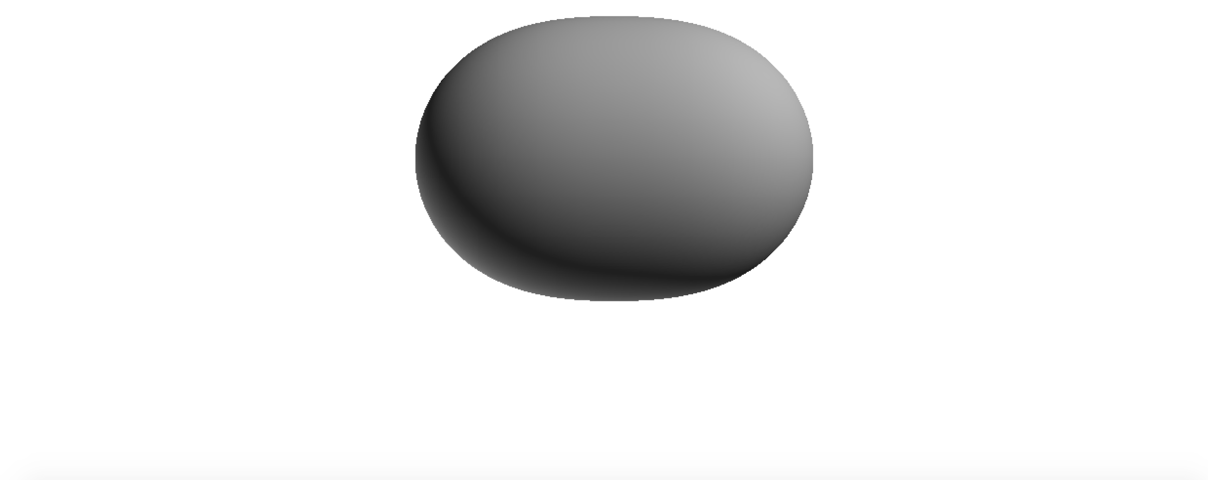

Figure 6: Two black holes of unit mass each, at a distance of 4.

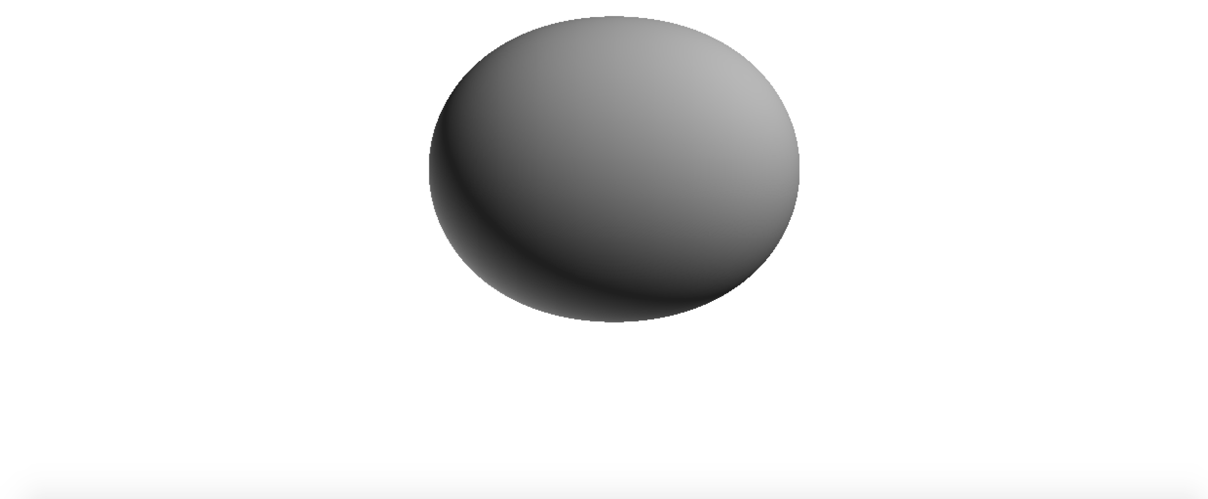

Figure 7: Two black holes of unit mass each, at a distance of 3.

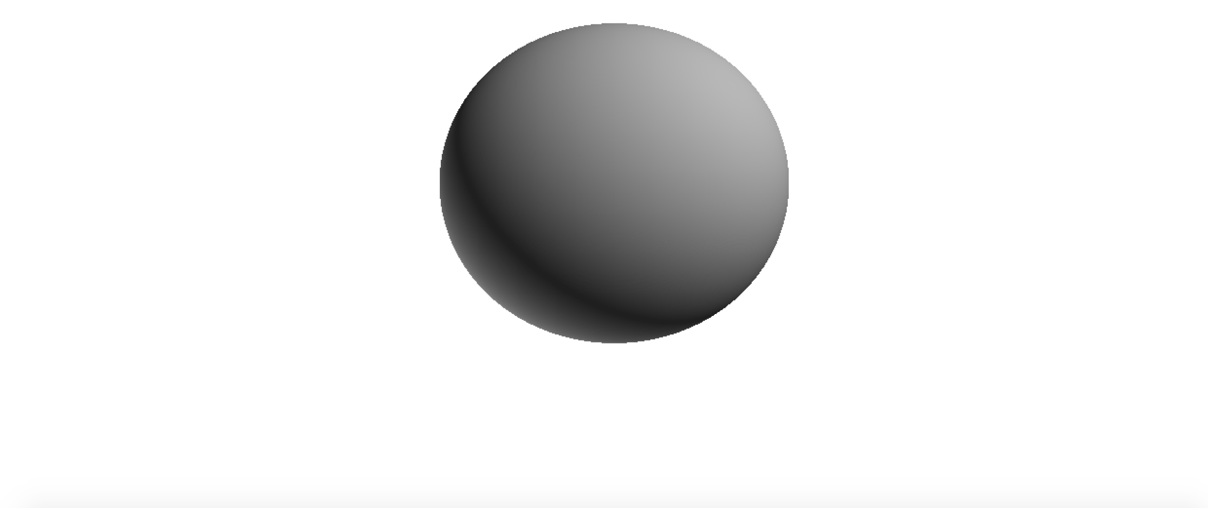

Figure 8: Two black holes of unit mass each, at a distance of 2.

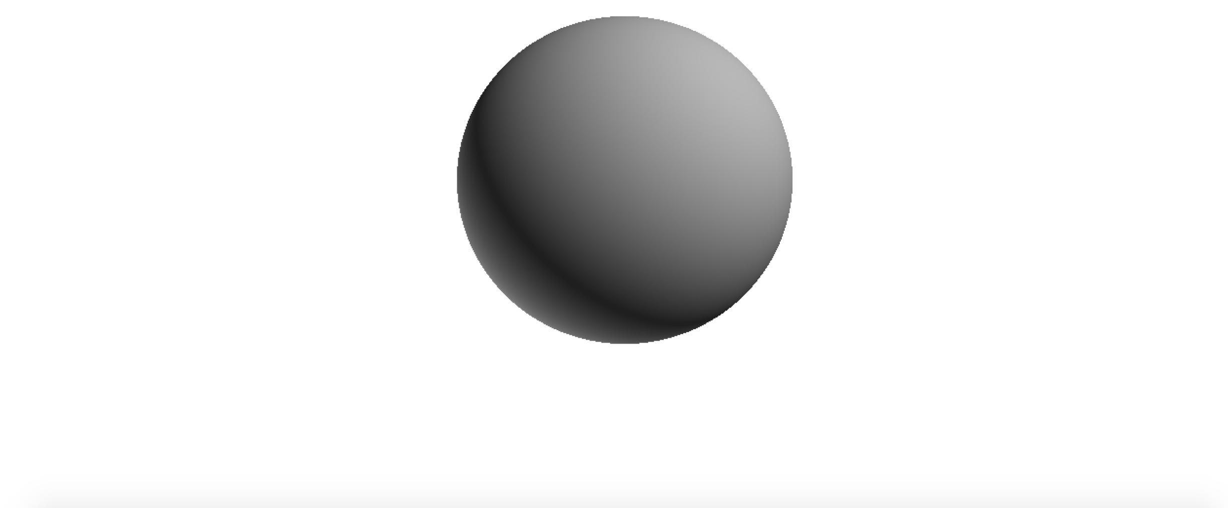

Figure 9: Two black holes of unit mass each, at a distance of 1.

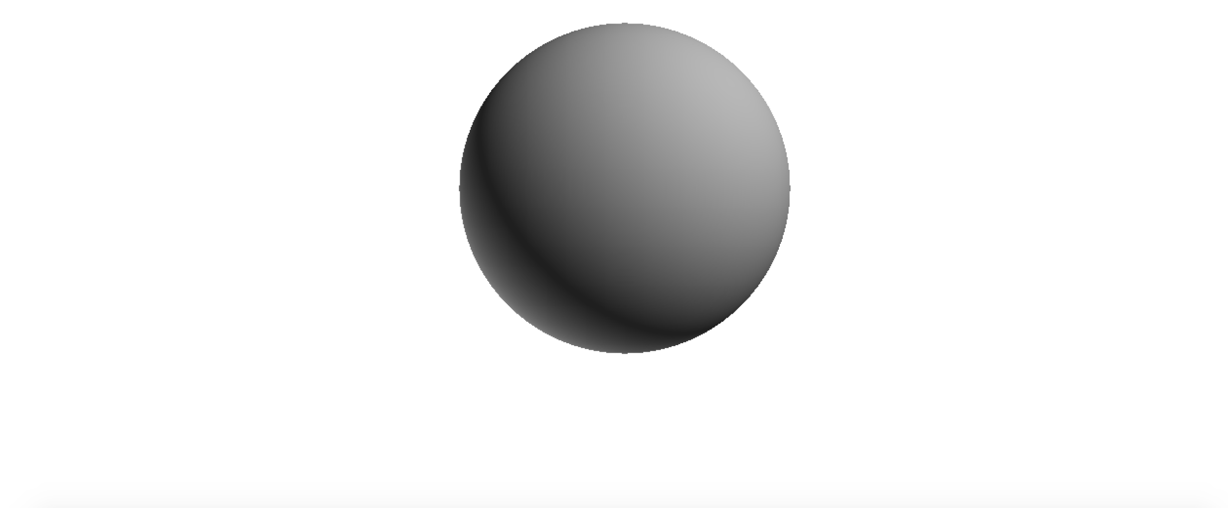

Figure 10: *One* black hole of mass = 2.

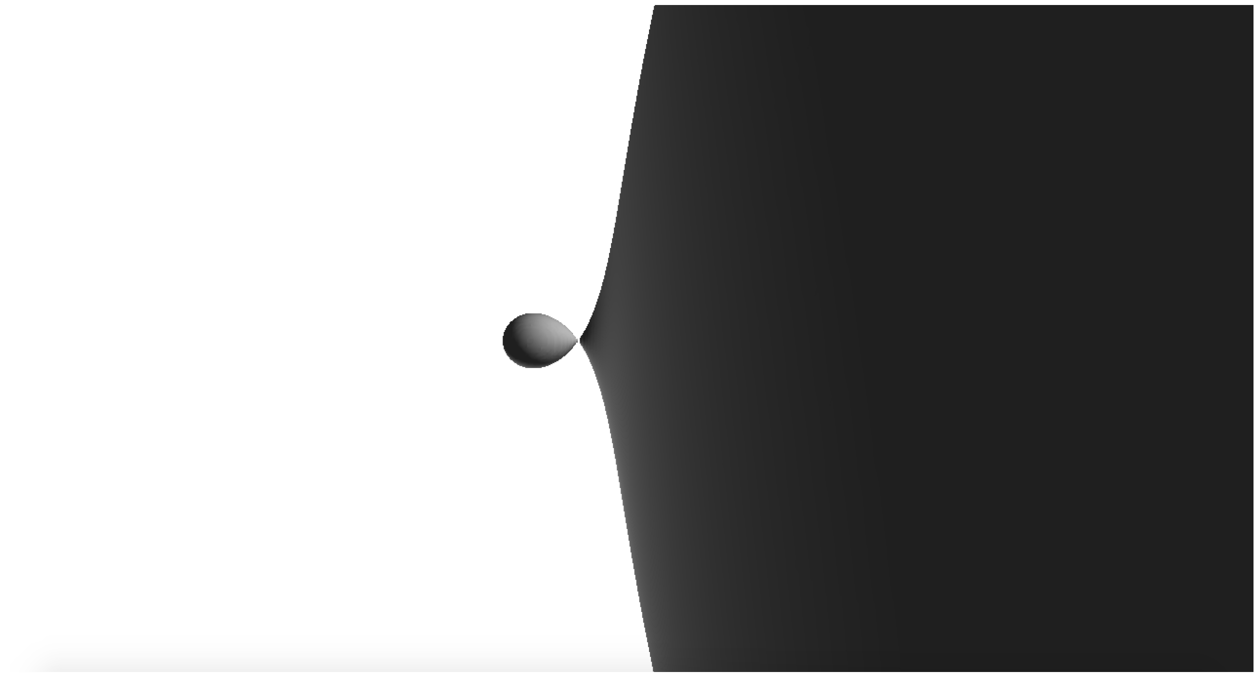

Figure 11: A unit mass black hole merging with a black hole of mass = 1000.

References

[1] Blinn J. *A generalization of algebraic surface drawing* ACM Transactions on Graphics, Vol. 1, No. 3 (1982)

[2] http://physics.stackexchange.com/questions/18769/black-hole-collision-and-the-event-horizon

[3] Lorensen W, Cline H. *Marching Cubes: A high resolution 3D surface construction algorithm* Computer Graphics, Vol. 21, No. 4 (1987)

[4] Halayka, S. *C++ code* (2017) https://github.com/sjhalayka/bhmerger

[5] Emparan R, Martinez M. *Exact event horizon of a black hole merger* (2016) arXiv:1603.00712 [gr-qc]