Using metaballs to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes

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Abstract

In this paper, Blinn's metaballs are used to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes.

1 Metaballs

Metaballs have been used in computer graphics ever since their discovery by Jim Blinn [1]. They were originally used to visualize electron density. In this paper we will model Schwarzschild black holes using the simple mathematics of metaballs. Analogously, we will be visualizing graviton density.

Where $G = c = \hbar = k = 1$, there is an analytical solution for the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes travelling directly toward each other:

$$f(l) = \sum_{i=1}^{n=2} \frac{2M_i}{r_i} = \frac{2M_1}{r_1} + \frac{2M_2}{r_2}. \quad (1)$$

Here M_i is the mass of the ith black hole, and

$$r_i = \sqrt{(l.x - v_i.x)^2 + (l.y - v_i.y)^2 + (l.z - v_i.z)^2}, \quad (2)$$

where l is the sample location, and v_i is the centre of the ith black hole. The event horizon is given by

$$f(l) = 1. \quad (3)$$

Included are figures of the pre-ringdown phase of Schwarzschild black hole mergers. The event horizon was tessellated using the Marching Cubes algorithm [2].

The C++ code for this paper can be found at [3].

Another method for generating the event horizon is given in [4].

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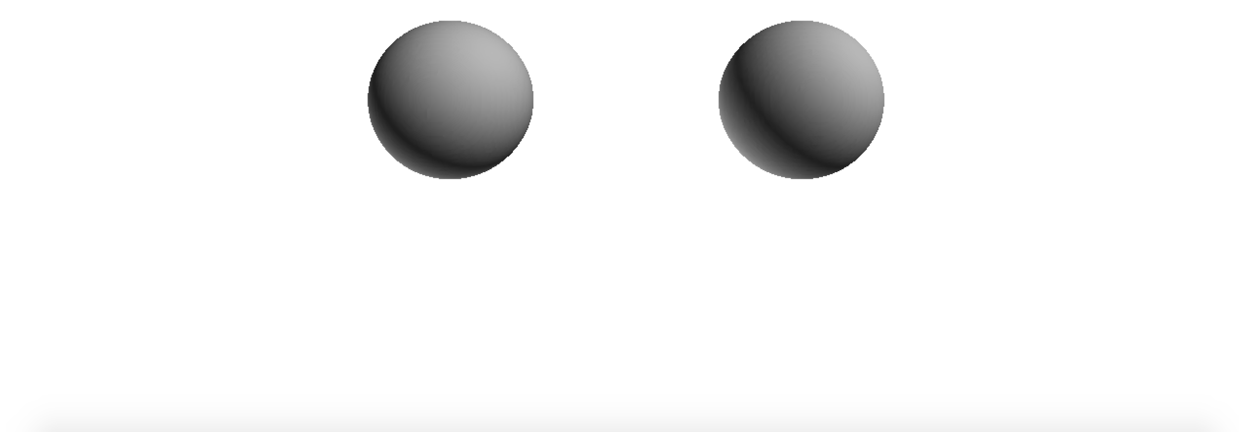

Figure 1: Two black holes of unit mass each (where unit mass is the Planck mass, e.g. the minimum black hole mass, $M_P = \sqrt{\hbar c/G} = 1$), at a distance of 11 (where unit distance is the Planck length $\ell_P = \sqrt{\hbar G/c^3} = 1$).

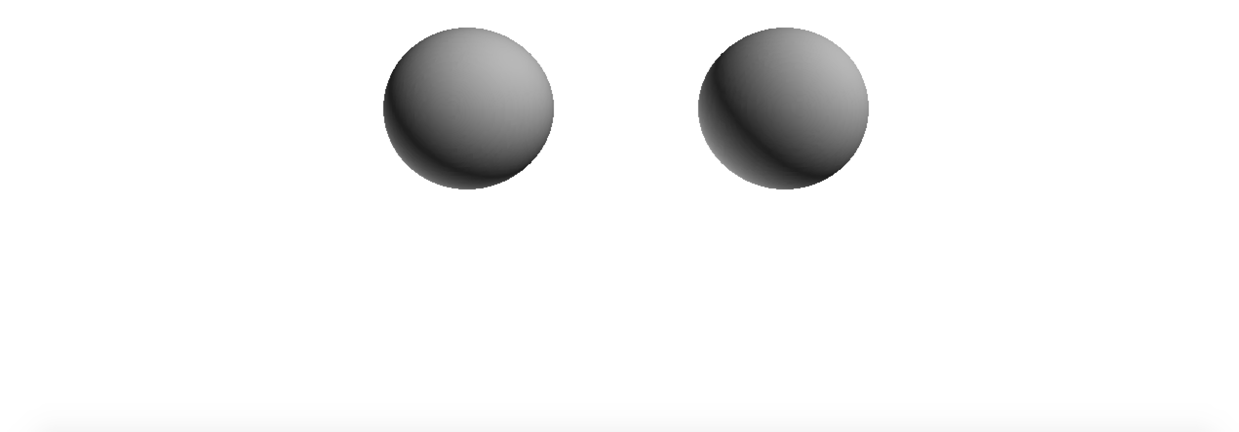

Figure 2: Two black holes of unit mass each, at a distance of 10.

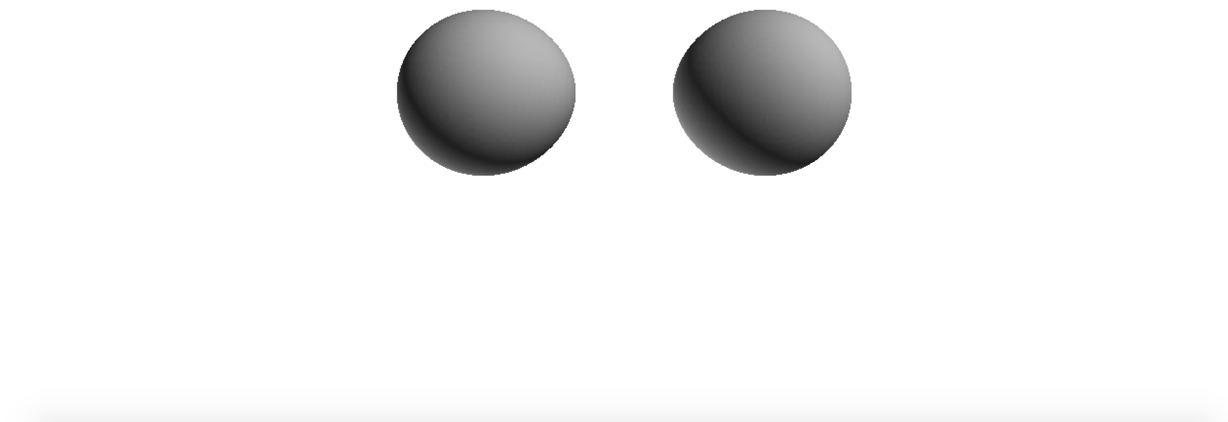

Figure 3: Two black holes of unit mass each, at a distance of 9.

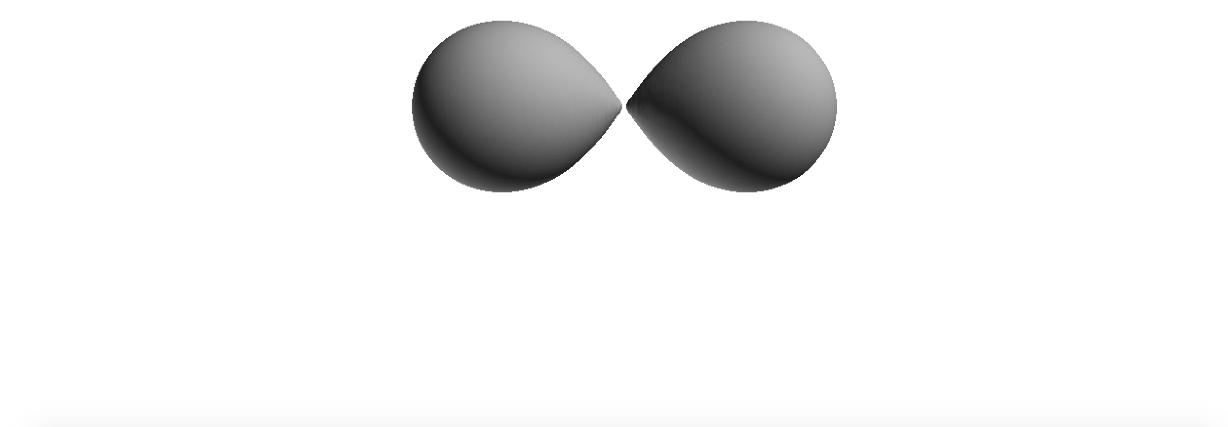

Figure 4: Two black holes of unit mass each, at a distance of 8. Note that this is the distance where the two black holes are kissing.

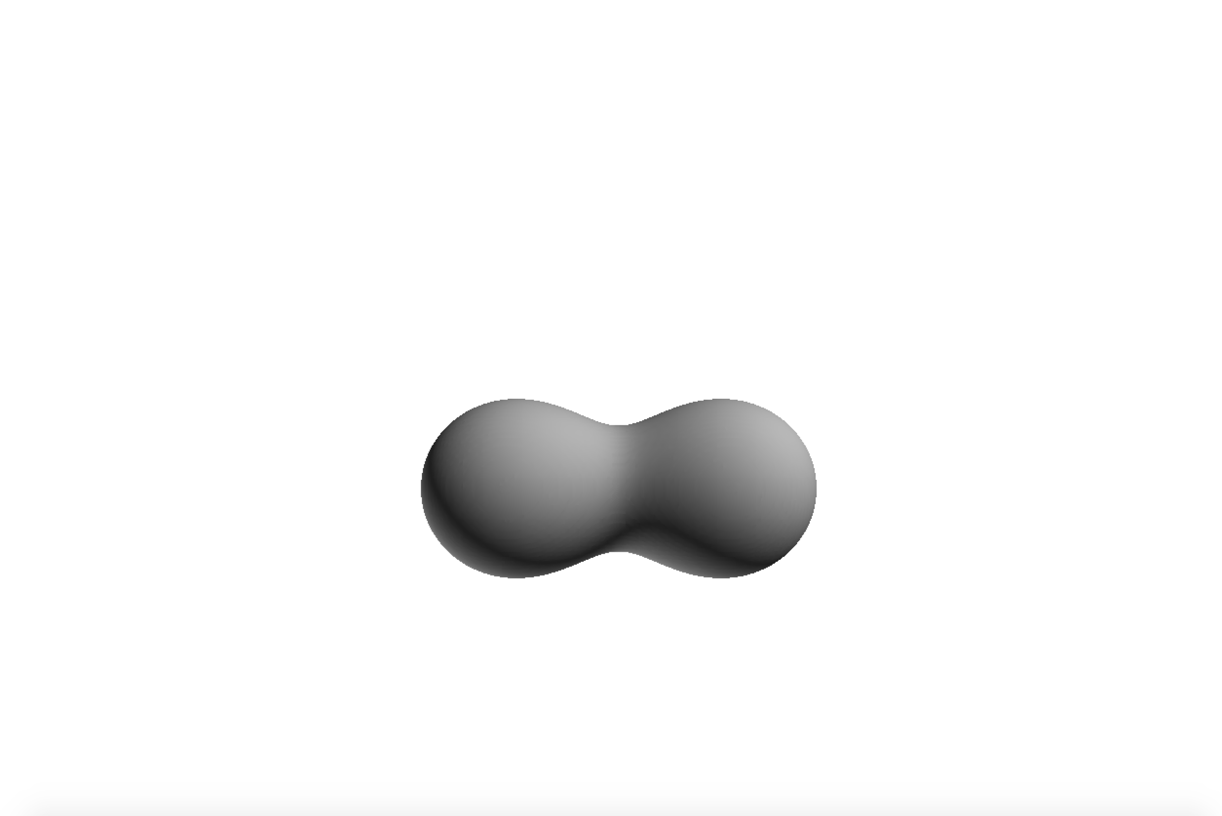

Figure 5: Two black holes of unit mass each, at a distance of 7.

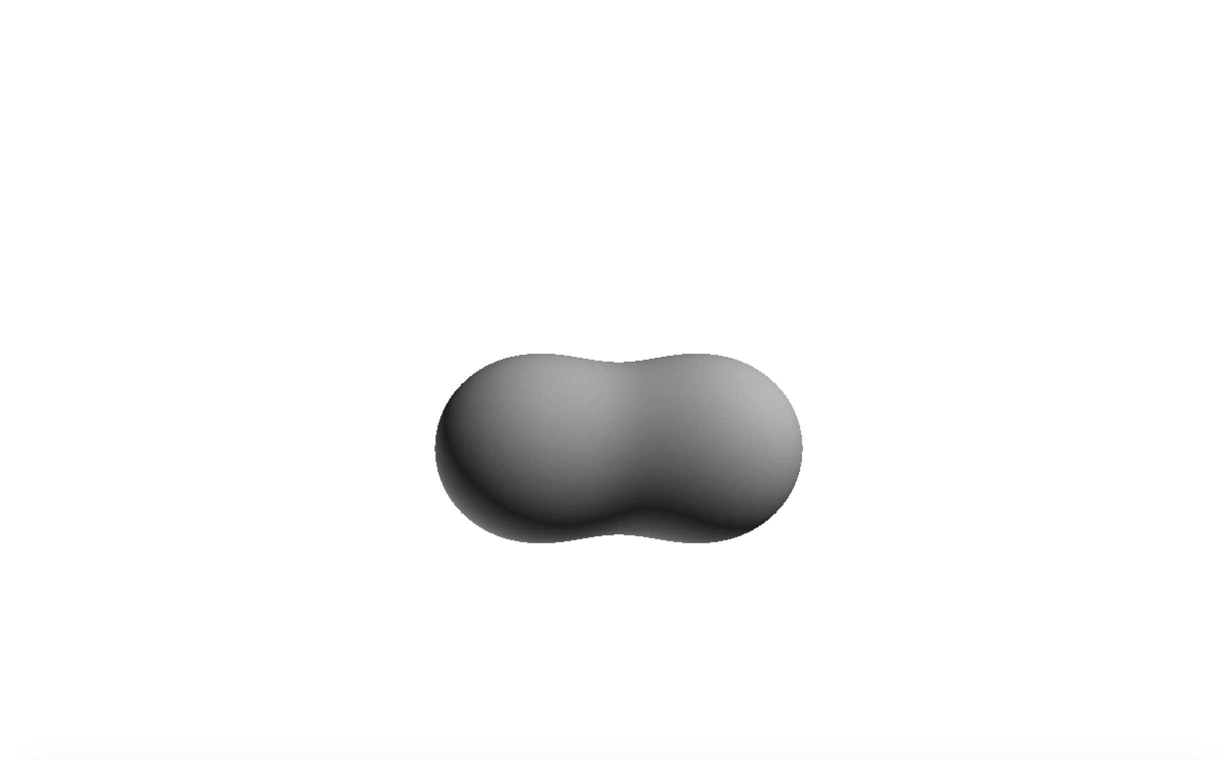

Figure 6: Two black holes of unit mass each, at a distance of 6.

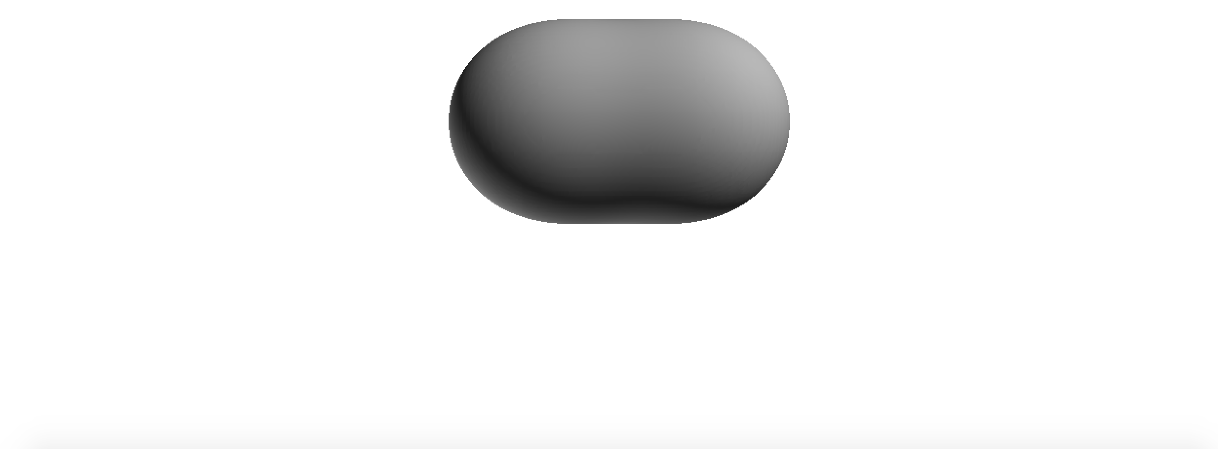

Figure 7: Two black holes of unit mass each, at a distance of 5.

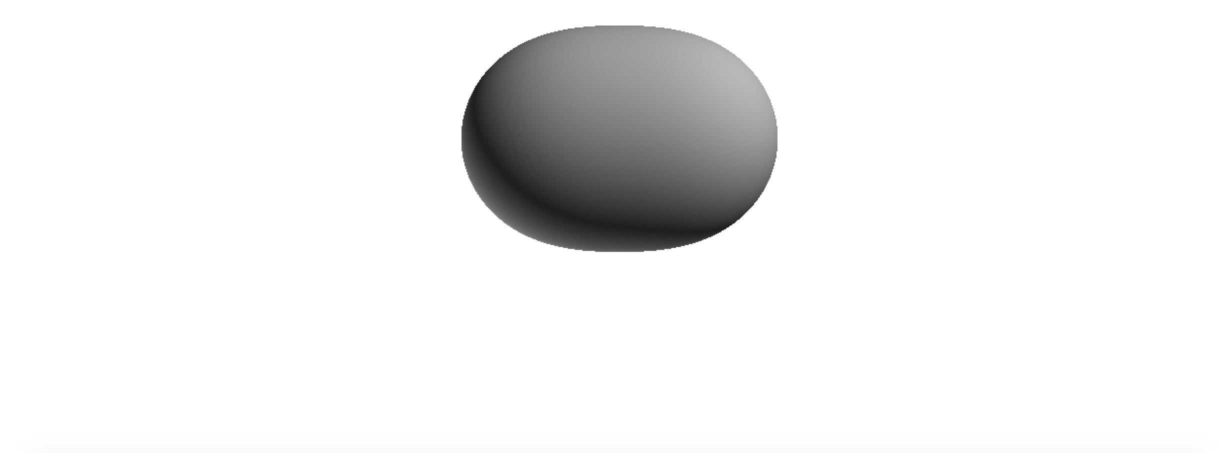

Figure 8: Two black holes of unit mass each, at a distance of 4.

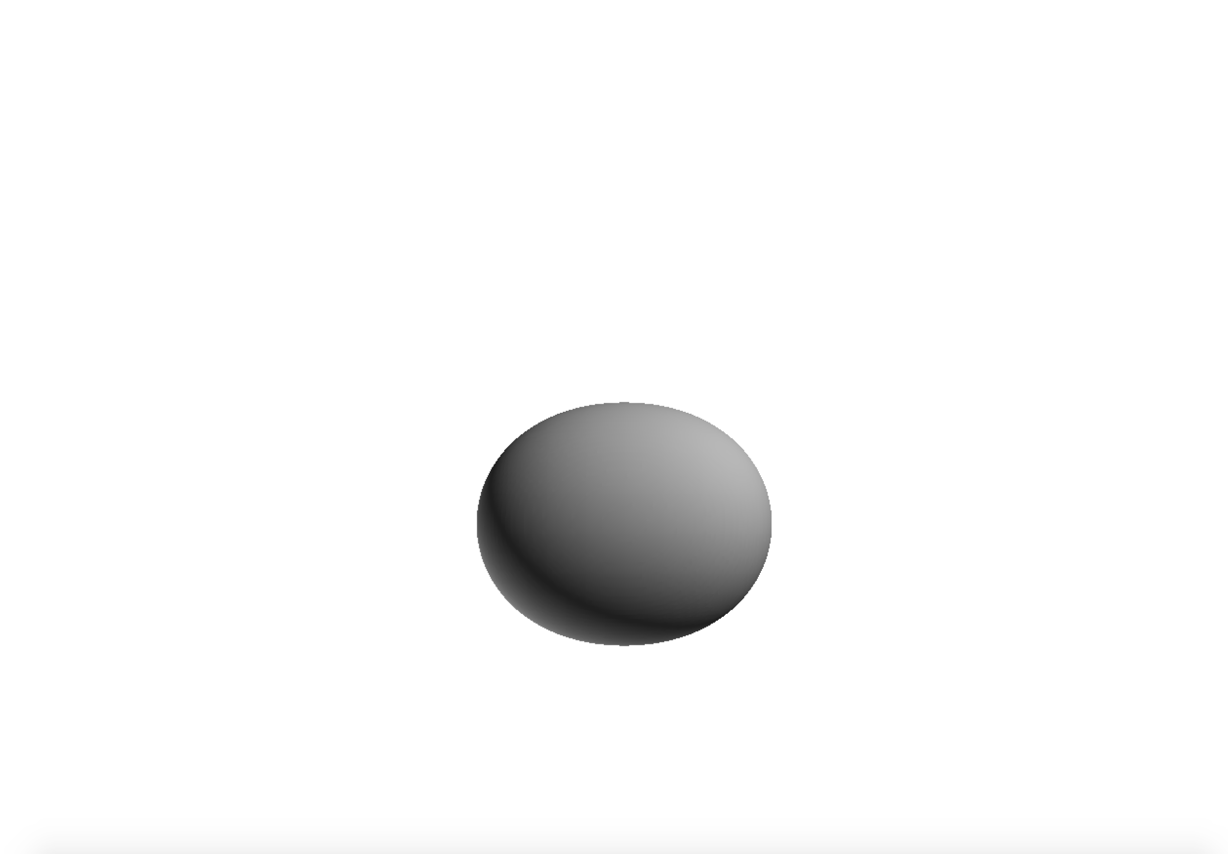

Figure 9: Two black holes of unit mass each, at a distance of 3.

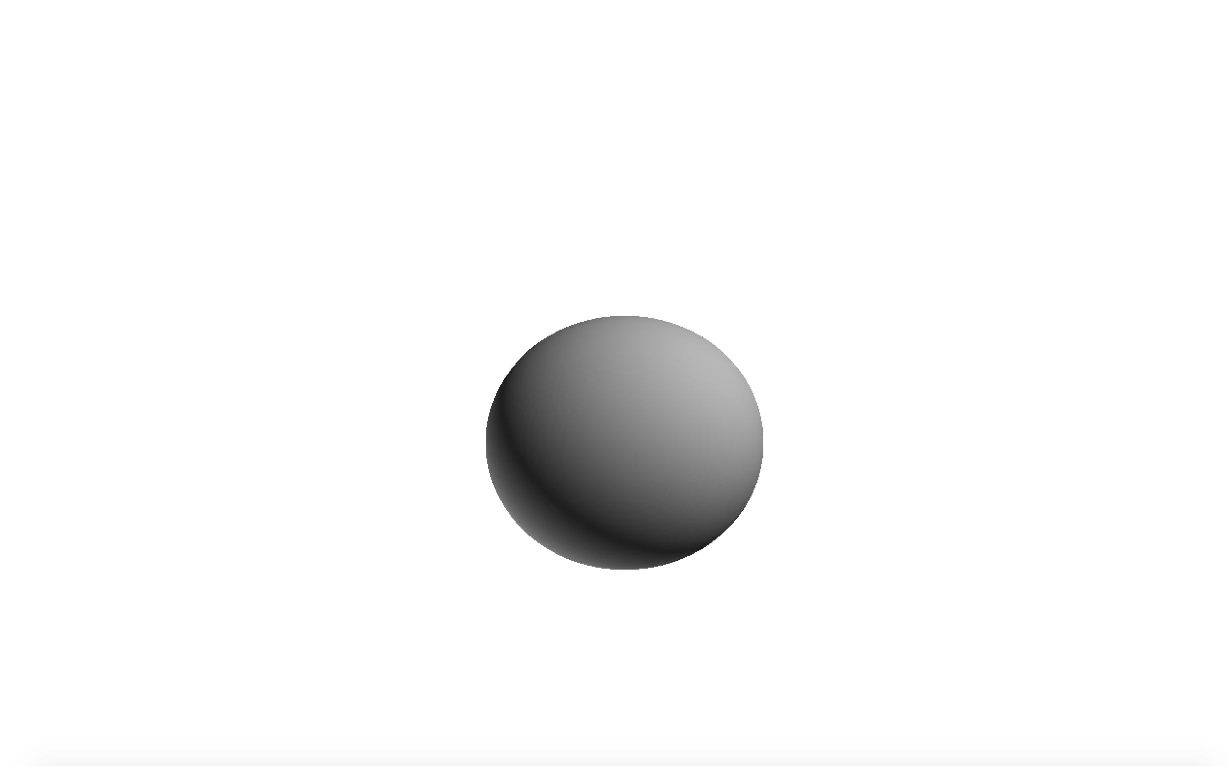

Figure 10: Two black holes of unit mass each, at a distance of 2.

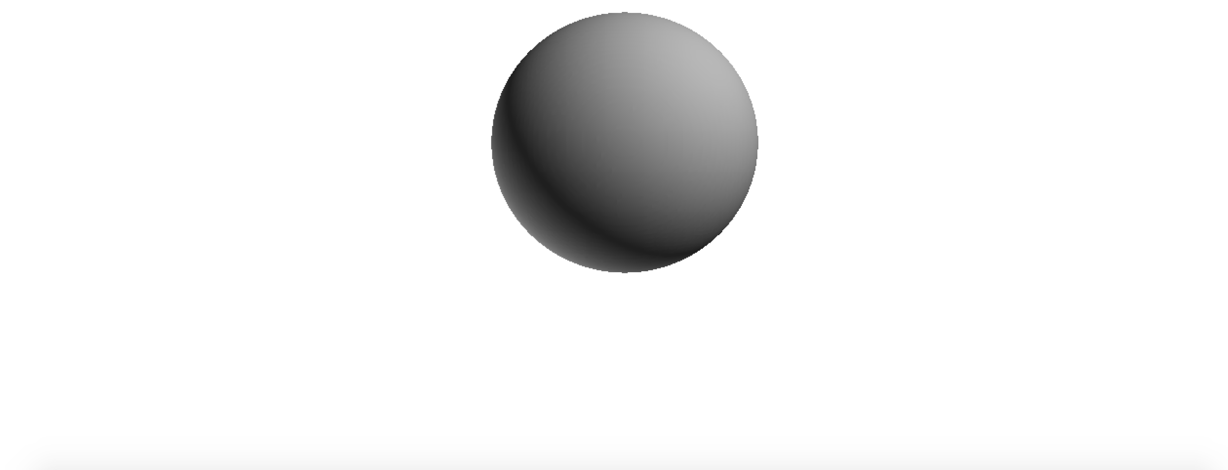

Figure 11: Two black holes of unit mass each, at a distance of 1.

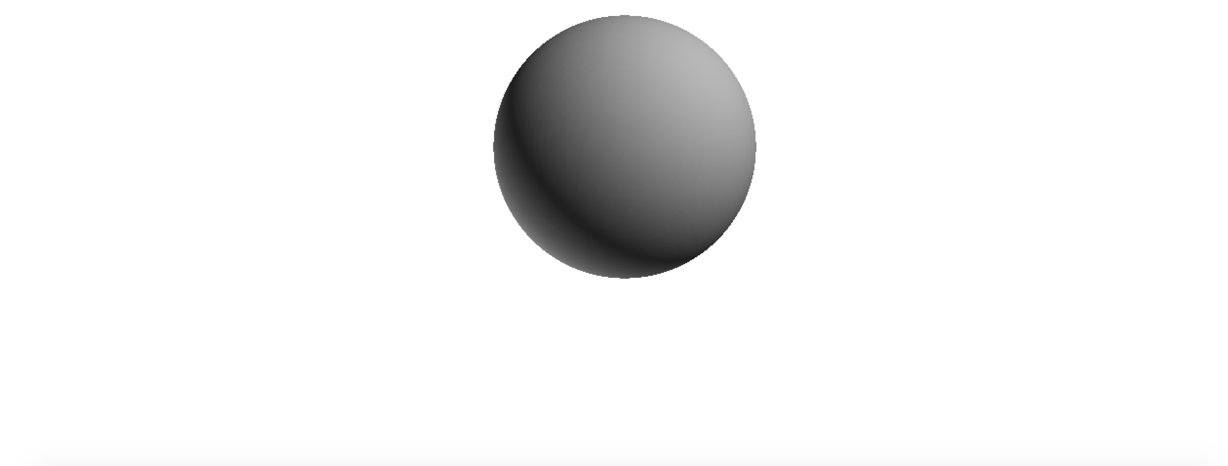

Figure 12: *One* black hole of mass = 2.

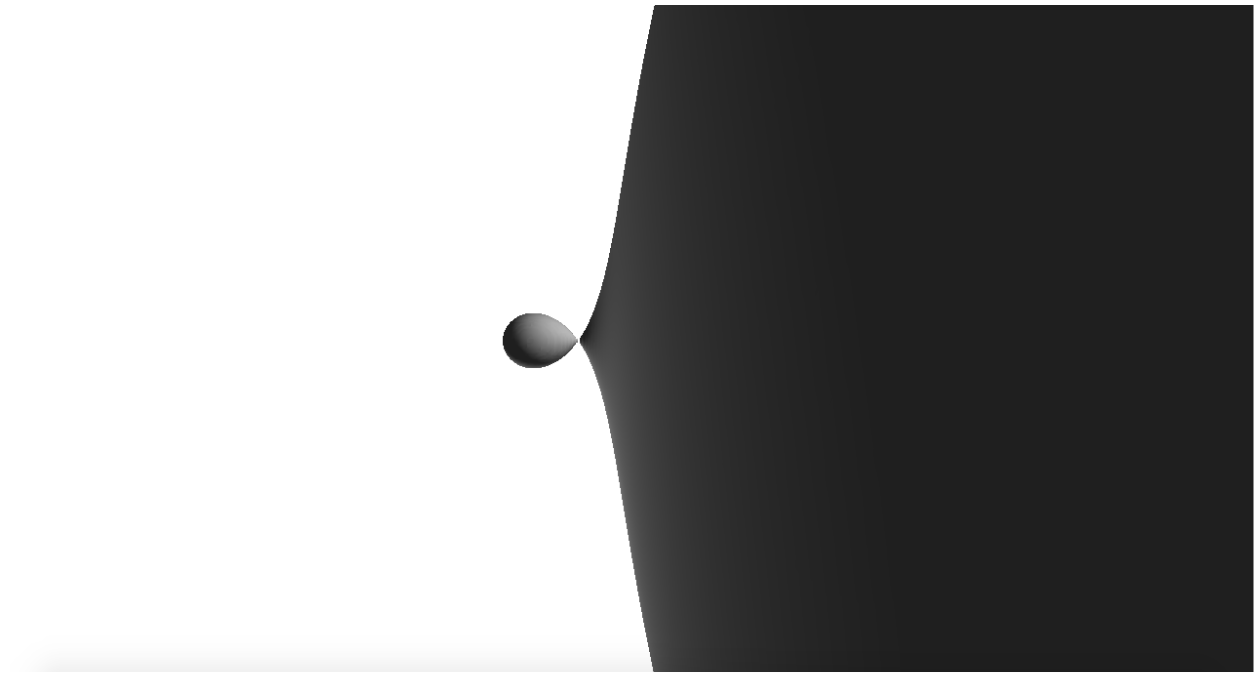

Figure 13: A unit mass black hole merging with a black hole of mass = 1000. Note that the event horizon resembles an elliptic curve [4].

References

[1] Blinn J. *A generalization of algebraic surface drawing* ACM Transactions on Graphics, Vol. 1, No. 3 (1982)

[2] Lorensen W, Cline H. *Marching Cubes: A high resolution 3D surface construction algorithm* Computer Graphics, Vol. 21, No. 4 (1987)

[3] Halayka, S. *C++ code* (2017) https://github.com/sjhalayka/bhmerger

[4] Emparan R, Martinez M. *Exact event horizon of a black hole merger* (2016) arXiv:1603.00712 [gr-qc]

[5] http://physics.stackexchange.com/questions/213603/why-do-people-put-exponentials-there

[6] http://physics.stackexchange.com/questions/18769/black-hole-collision-and-the-event-horizon

[7] http://physics.stackexchange.com/questions/127334/differences-between-wavefunction-probability-and-probability-density