Conjecture on 3-Carmichael numbers of the form (4h+1)(4j+1)(4k+1)

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Abstract. In this paper I conjecture that for any 3-Carmichael number (absolute Fermat pseudoprime with three prime factors, see the sequence A087788 in OEIS) of the form (4*h + 1)*(4*j + 1)*(4*k + 1) is true that h, j and k must share a common factor (in fact, for seven from a randomly chosen set of ten consecutive, reasonably large, such numbers it is true that both j and k are multiples of h). The conjecture is probably true even for the larger set of 3-Poulet numbers (Fermat pseudoprimes to base 2 with three prime factors, see the sequence 215672 in OEIS).

Conjecture:

For any 3-Carmichael number (absolute Fermat pseudoprime with three prime factors, see the sequence A087788 in OEIS) of the form (4*h + 1)*(4*j + 1)*(4*k + 1) is true that h, j and k must share a common factor (in fact, for seven from a randomly chosen set of ten consecutive, reasonably large, such numbers it is true that both j and k are multiples of h). The conjecture is probably true even for the larger set of 3-Poulet numbers (Fermat pseudoprimes to base 2 with three prime factors, see the sequence 215672 in OEIS).

Note: The number of the 3-Carmichael numbers of the form (4*h + 1)*(4*j + 1)*(4*k + 1) respectively (4*h + 1)*(4*j + 3)*(4*k + 3) seems to be prevalent in front of those of the form (4*h + 1)*(4*j + 1)*(4*k + 3) or (4*h + 3)*(4*j + 3)*(4*k + 3).

Verifying the conjecture:

(for ten consecutive 3-Carmichael numbers of this form)

: 7166415855504133 = 6829*279949*3748573 = (4*1707 + 1)*(4*69987 + 1)*(4*937143 + 1) and 69987 = 41*1707, also 937143 = 549*1707);

: 7176175908880001 = 80513*201281*442817 = (4*20128 + 1)*(4*50320 + 1)*(4*110704 + 1) and 20128, 50320 and 110704 share the factor 10064;

: 7181222478490321 = 9781*185821*3951121 = (4*2445 + 1)*(4*46455 + 1)*(4*987780 + 1) and 46455 = 19*2445, also 987780 = 404*2445);

: 7182633224049097 = 7517*22549*42375209 = (4*1879 + 1)*(4*5637 + 1)*(4*10593802 + 1) and 5637 = 3*1879, also 10593802 = 5638*1879);

: 7197847038184129 = 13633*34649*15237737 = (4*3408 + 1)*(4*8662 + 1)*(4*3809434 + 1) and 3408, 8662 and 3809434 share the factor 142;

: 7203600125162641 = 49333*147997*986641 = (4*12333 + 1)*(4*36999 + 1)*(4*246660 + 1) and 36999 = 3*12333, also 246660 = 20*12333);

: 7205074807056961 = 86837*102161*812173 = (4*21709 + 1)*(4*25540 + 1)*(4*203043 + 1) and 21709, 25540 and 203043 share the factor 1277;

: 7206253022807569 = 106297*212593*318889 = (4*26574 + 1)*(4*53148 + 1)*(4*79722 + 1) and 53148 = 2*26574, also 79722 = 3*26574);

: 7210574407615489 = 50497*353473*403969 = (4*12624 + 1)*(4*88368 + 1)*(4*100992 + 1) and 88368 = 7*12624, also 100992 = 8*12624);

: 7214334239197441 = 10433*93889*7364993 = (4*2608 + 1)*(4*23472 + 1)*(4*1841248 + 1) and 23472 = 9*2608, also 1841248 = 706*2608).

Note: Every case when j and k are multiples of h give us a third degree polynomial which might generate an entire set of 3-Carmichael numbers, e.g. the polynomial (4*n + 1)*(8*n + 1)*(12*n + 1), suggested by the number 7206253022807569, or even a fourth degree polynomial, e.g. the polynomial (4*n + 1)*(12*n + 1)*(12*n^2 + 4*n + 1), suggested by the number 7182633224049097.

Note: In the case of the 3-Carmichael numbers of the form (4*h + 3)*(4*j + 3)*(4*k + 3) I couldn’t find a similar pattern; for instance, in the case of the Carmichael number 7203119040117571 = 5791*35899*34648519, we have h = 1447 prime, j = 8974 = 2*7*641 and k = 8662129 = 7*17*83*877.