Conjecture on 2-Poulet numbers of the form (4h+1)(4k+1)

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Abstract. In this paper I conjecture that for any 2-Poulet number (Fermat pseudoprime to base 2 with two prime factors, see the sequence A214305 in OEIS) of the form (4*h + 1)*(4*k + 1) is true that h and k can not be relatively primes (in fact, for sixteen from the first twenty 2-Poulet numbers of this form is true that k is a multiple of h and this is also the case for four from a randlomly chosen set of five consecutive, much larger, such numbers).

Conjecture:

For any 2-Poulet number (Fermat pseudoprime to base 2 with two prime factors, see the sequence A214305 in OEIS) of the form (4*h + 1)*(4*k + 1) is true that h and k can not be relatively primes (in fact, for sixteen from the first twenty 2-Poulet numbers of this form is true that k is a multiple of h and this is also the case for four from a randlomly chosen set of five consecutive, much larger, such numbers).

Note: In the case of the 2-Poulet numbers of the form (4*h + 1)*(4*k + 3), i.e. 1387, 2047, 13747, 14491, 19951 (...) the values of (h, k) are (18, 4), (22, 5), (58, 14), (84, 14), (70, 17) (...), also in the case of the 2-Poulet numbers of the form (4*h + 3)*(4*k + 3), i.e. 341, 4681, 5461, 10261, 15709 (...) the values of (h, k) are (2, 7), (7, 37), (10, 31), (7, 82), (5, 170) (...) so h and k are sometimes relatively primes and sometimes they share factors.

Verifying the conjecture:
(for the first twenty 2-Poulet numbers of this form)

: 2701 = 37*73 = (4*9 + 1)*(4*18 + 1) and 18 = 2*9;
: 3277 = 29*113 = (4*7 + 1)*(4*28 + 1) and 28 = 4*7;
: 4033 = 37*109 = (4*9 + 1)*(4*27 + 1) and 27 = 3*9;
: 4369 = 17*257 = (4*4 + 1)*(4*64 + 1) and 64 = 16*4;
: 7957 = 73*109 = (4*18 + 1)*(4*27 + 1) and 27 and 18 share the factor 9;
: 8321 = 53*157 = (4*13 + 1)*(4*39 + 1) and 39 = 3*13;

: 18721 = 97*193 = (4*24 + 1)*(4*48 + 1) and 48 = 2*24;

: 23377 = 97*241 = (4*24 + 1)*(4*60 + 1) and 60 and 24 share the factor 12;

: 31417 = 89*353 = (4*22 + 1)*(4*88 + 1) and 88 = 4*22;

: 31609 = 37*433 = (4*18 + 1)*(4*108 + 1) and 108 = 6*18;

: 35333 = 89*397 = (4*22 + 1)*(4*99 + 1) and 99 and 22 share the factor 11;

: 49141 = 157*313 = (4*39 + 1)*(4*78 + 1) and 78 = 2*39;

: 60701 = 101*601 = (4*25 + 1)*(4*150 + 1) and 150 = 6*25;

: 65281 = 97*673 = (4*24 + 1)*(4*168 + 1) and 168 = 7*24;

: 80581 = 61*1321 = (4*15 + 1)*(4*330 + 1) and 330 = 22*15;

: 85489 = 53*1613 = (4*13 + 1)*(4*403 + 1) and 403 = 31*13;

: 88357 = 149*593 = (4*37 + 1)*(4*148 + 1) and 148 = 4*37;

: 104653 = 229*457 = (4*57 + 1)*(4*114 + 1) and 114 = 2*57;

: 129889 = 193*673 = (4*48 + 1)*(4*168 + 1) and 168 and 48 share the factor 24.

Verifying the conjecture:
(for five consecutive larger 2-Poulet numbers of this form)

: 27686175193 = 74413*372061 = (4*18603 + 1)*(4*93015 + 1) and 93015 = 5*18603;

: 27702689701 = 83221*332881 = (4*20805 + 1)*(4*83320 + 1) and 83320 = 4*20805;

: 27708447397 = 135913*203869 = (4*33978 + 1)*(4*50967 + 1) and 50967 and 33978 share the factor 16989;

: 27712970209 = 74449*372241 = (4*18612 + 1)*(4*93060 + 1) and 93060 = 5*18612;

: 27716297941 = 41621*665921 = (4*10405 + 1)*(4*166480 + 1) and 166480 = 16*10405.