

New Lower Bounds for Van der Waerden Numbers

Alexey V. Komkov

February 2, 2017

Abstract

This work contains certificates numbers Van der Waerden, was found using SAT Solver. These certificates establish the best currently known lower bounds of the numbers Van der Waerden $W(7, 3)$, $W(8, 3)$, $W(9, 3)$, $W(10, 3)$, $W(11, 3)$.

1 Introduction

Van der Waerden's theorem states that for any positive integers r and k there exists a positive integer N such that if the integers $\{1, 2, \dots, N\}$ are colored, each with one of r different colors, then there are at least k integers in arithmetic progression all of the same color. The smallest such N is the van der Waerden number $W(r, k)$. Van der Waerden Numbers are quite difficult to calculate, at the time of writing the article, the exact values are known only for 7 van der Waerden numbers, and for the rest of the numbers only bounds are known. To prove the lower bound of a van der Waerden number it is sufficient to get a certificate of this number, i. e., a sequence of numbers $\{1, 2, \dots, N\}$ of r colors, and not having same-coloured arithmetic progressions of length k . Then $N+1$ will be the lower bound of $W(r, k)$ number.

2 Certificates

Certificate $W(7, 3) > 336$

00113434103225034635411511224545214336145046522622335656325440256150633033446060436551
36026104414455010154066240130215525566121265100351241326636600232306211462352430040011
34341032250346354115112245452143361450465226223356563254402561506330334460604365513602
610441445501015406624013021552556612126510035124132663660023230621146235243004

Certificate $W(8, 3) > 449$

12140440452561661536300300732420300701415255217573373367404373363202141142616776775603
07677574342022045256656615373656616070434430141551552176751552536307007320211211426561
21141757363367434224220451524220261675775607044044301214044345256166153630030073242030
07014152552175733733674043733632021411426167767756030767757434202204525665661537365661
60704344301413515521767515525363070073202112114265612114175736336743422422045152422026
1675775607044044305

Certificate $W(9, 3) > 559$

```
24403861826453756867557736282433551407203756486707866884730354466251831486750781807700
58414655773620425078618020188116052576688473153618072013120022716368770058426472018312
42311338274708811605375831204235342244038581002271648604231534645335514060211338275071
53426457564466251713224403861826453756867557736282433551407203756486707866884730354466
25183148675078180770058414655773620425078618020188116052576688473153618072013120022716
36877005842647201831242311338274708811605375831204235342244038581002271648604231534645
3355140602113382750715342645756446625171320
```

Certificate $W(10, 3) > 701$

```
78804022787297879915122151228989915899151338983089800262332623390900269002624490941909
11373443734401011370113735501052010224845548455121224812248466121631213359566595662323
35923359577232742324460677606773434460344606883438534355717887178845455714557179945496
45466828998289956566825668280056507565779390093900676779367793911676186768804011040117
87880478804022787297879915122151228989915899151338983089800262332623390900269002624490
94190911373443734401011370113735501052010224845548455121224812248466121631213359566595
66232335923359577232742324460677606773434460344606883438534355717887178845455714557179
94549645466823998239956566825668280056507565779390093900676779367793911676186768804011
0401178788040
```

Certificate $W(11, 3) > 816$

```
3568726047804797AA45469066939253A179746798371589558A8005657A977A4A364028A8578A948269A2
69091167680A88050475139096890A5937A077A1A227879109916158624A1A79A106A48018802033898A21
AA272697350208A02170591299131449A9032003837A84613190132816A23AA24255A0A143114948095724
2A12439270340035366010254225A591A6835302354A38145114647712136533606A207946413465049256
2257588232476447170318A5752457615A367336869934358755828142906868568726047844797AA45469
866939253A179746798371589558A8005657A977A4A364028A8578A948269A669091167680A88050475139
096890A5937A077A1A227879109916158624A1A79A106A48018802033898A21AA272697350208A02170591
299131449A9032003837A84613190132816A23AA24255A0A1431149480957242A124392703400353660102
54225A591A6835302354A38145114647712136533606A2079464134650492562257588232476447170318A
5752457615A3673368699343587578281429068609
```

3 Results

Number $W(r, k)$	Old Lower Bounds	New Lower Bounds
$W(7, 3)$	> 273 [1]	> 336
$W(8, 3)$	> 354 [1]	> 449
$W(9, 3)$	> 454 [1]	> 559
$W(10, 3)$	> 592 [1]	> 701
$W(11, 3)$	> 731 [1]	> 816

References

- Alexey V. Komkov, "New Lower Bounds for Van Der Waerden Numbers Using Genetic Algorithm", 2016