

Conjecture on the pairs of consecutive primes having the same number of digits involving concatenation

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Abstract. In this paper I make the following conjecture: For any pair of consecutive primes $[p_1, p_2]$, $p_2 > p_1 > 43$, p_1 and p_2 having the same number of digits, there exist a prime q , $5 < q < p_1$, such that the number n obtained concatenating (from the left to the right) q with p_2 , then with p_1 , then again with q is prime. Example: for $[p_1, p_2] = [961748941, 961748947]$ there exist $q = 19$ such that $n = 1996174894796174894119$ is prime. Note that the least values of q that satisfy this conjecture for twenty consecutive pairs of consecutive primes with 9 digits are 19, 17, 107, 23, 131, 47, 83, 79, 61, 277, 163, 7, 41, 13, 181, 19, 7, 37, 29 and 23 (all twenty primes lower than 300!), the corresponding primes n obtained having 20 to 24 digits! This method appears to be a good way to obtain big primes with a high degree of ease and certainty.

Conjecture:

For any pair of consecutive primes $[p_1, p_2]$, $p_2 > p_1 > 43$, p_1 and p_2 having the same number of digits, there exist a prime q , $5 < q < p_1$, such that the number n obtained concatenating (from the left to the right) q with p_2 , then with p_1 , then again with q is prime.

Example: for $[p_1, p_2] = [961748941, 961748947]$ there exist $q = 19$ such that $n = 1996174894796174894119$ is prime.

The least such primes q for the first twenty pairs of consecutive primes (such that $p_2 > p_1 > 43$):

- : $q = 23$ for $[p_1, p_2] = [47, 53]$, because $n = 23534723$ is prime;
- : $q = 41$ for $[p_1, p_2] = [53, 59]$, because $n = 41595341$ is prime;
- : $q = 7$ for $[p_1, p_2] = [59, 61]$, because $n = 761597$ is prime (note that 11615911, 37615937 and 53615953 are also primes);

: $q = 7$ for $[p_1, p_2] = [61, 67]$, because $n = 767617$ is prime;

: $q = 13$ for $[p_1, p_2] = [67, 71]$, because $n = 13716713$ is prime (note that 23716723, 37716737, 59716759 and 61716761 are also primes);

: $q = 23$ for $[p_1, p_2] = [71, 73]$, because $n = 23737123$ is prime (note that 29737129, 31737131, 43737143, 47737147 and 67737167 are also primes);

: $q = 61$ for $[p_1, p_2] = [73, 79]$, because $n = 61797361$ is prime;

: $q = 11$ for $[p_1, p_2] = [79, 83]$, because $n = 11837911$ is prime (note that 13837913, 47837947 and 59837959 are also primes);

: $q = 17$ for $[p_1, p_2] = [83, 89]$, because $n = 17898317$ is prime (note that 29898329, 53898353, 59898359 and 71898371 are also primes);

: $q = 7$ for $[p_1, p_2] = [89, 97]$, because $n = 797897$ is prime (note that 19978919, 43978943 and 73978973 are also primes);

: $q = 7$ for $[p_1, p_2] = [101, 103]$, because $n = 71031017$ is prime (note that 1710310117, 1910310119, 4710310147 and 5310310153 are also primes);

: $q = 11$ for $[p_1, p_2] = [103, 107]$, because $n = 1110710311$ is prime (note that 1710710317, 2310710323 and 5910710359 are also primes);

: $q = 43$ for $[p_1, p_2] = [107, 109]$, because $n = 4310910743$ is prime (note that 4710910747, 5310910753 and 7110910771 are also primes);

: $q = 17$ for $[p_1, p_2] = [109, 113]$, because $n = 1711310917$ is prime (note that 4311310943 is also prime);

: $q = 11$ for $[p_1, p_2] = [113, 127]$, because $n = 1112711311$ is prime (note that 1312711313, 3112711331, 4112711341, 4712711347 and 7912711379 are also primes);

: $q = 11$ for $[p_1, p_2] = [127, 131]$, because $n = 1113112711$ is prime (note that 2313112723, 6713112767, 7113112771, 8313112783 and 101131127101 are also primes);

- : $q = 89$ for $[p_1, p_2] = [131, 137]$, because $n = 8913713189$ is prime (note that 107137131107 and 113137131113 are also primes);
- : $q = 7$ for $[p_1, p_2] = [137, 139]$, because $n = 71391377$ is prime (note that 1113913711, 4313913743 and 6113913761 are also primes);
- : $q = 11$ for $[p_1, p_2] = [139, 149]$, because $n = 1114913911$ is prime (note that 2914913929, 4314913943, 4714913947, 6714913967, 8314913983, 101149139101, 127149139127 and 137149139137 are also primes);
- : $q = 17$ for $[p_1, p_2] = [149, 151]$, because $n = 1715114917$ is prime (note that 2915114929, 5315114953, 103151149103, 113151149113 and 131151149131 are also primes).

The least such primes q for twenty larger consecutive pairs of consecutive primes:

- : $q = 19$ for $[p_1, p_2] = [961748941, 961748947]$, because $n = 1996174894796174894119$ is prime;
- : $q = 17$ for $[p_1, p_2] = [961748947, 961748951]$, because $n = 1796174895196174894717$ is prime;
- : $q = 107$ for $[p_1, p_2] = [961748951, 961748969]$, because $n = 107961748969961748951107$ is prime;
- : $q = 23$ for $[p_1, p_2] = [961748987, 961748969]$, because $n = 2396174898796174896923$ is prime;
- : $q = 131$ for $[p_1, p_2] = [961748987, 961748993]$, because $n = 131961748993961748987131$ is prime;
- : $q = 47$ for $[p_1, p_2] = [961748993, 961749023]$, because $n = 4796174902396174899347$ is prime;
- : $q = 83$ for $[p_1, p_2] = [961749023, 961749037]$, because $n = 83961749037961749023$ is prime;
- : $q = 79$ for $[p_1, p_2] = [961749037, 961749043]$, because $n = 7996174904396174903779$ is prime;
- : $q = 61$ for $[p_1, p_2] = [961749043, 961749067]$, because $n = 6196174906796174904361$ is prime;

- : $q = 277$ for $[p_1, p_2] = [961749067, 961749079]$,
because $n = 277961749079961749067277$ is prime;
- : $q = 163$ for $[p_1, p_2] = [961749079, 961749091]$,
because $n = 163961749091961749079163$ is prime;
- : $q = 7$ for $[p_1, p_2] = [961749091, 961749097]$, because
 $n = 79617490979617490917$ is prime;
- : $q = 41$ for $[p_1, p_2] = [961749097, 961749101]$,
because $n = 4196174910196174909741$ is prime;
- : $q = 13$ for $[p_1, p_2] = [961749101, 961749121]$,
because $n = 1396174912196174910113$ is prime;
- : $q = 181$ for $[p_1, p_2] = [961749121, 961749157]$,
because $n = 181961749157961749121181$ is prime;
- : $q = 19$ for $[p_1, p_2] = [961749157, 961749167]$,
because $n = 1996174916796174915719$ is prime;
- : $q = 7$ for $[p_1, p_2] = [961749167, 961749193]$, because
 $n = 79617491939617491677$ is prime;
- : $q = 37$ for $[p_1, p_2] = [961749193, 961749199]$,
because $n = 3796174919996174919337$ is prime;
- : $q = 29$ for $[p_1, p_2] = [961749199, 961749221]$,
because $n = 2996174922196174919929$ is prime;
- : $q = 23$ for $[p_1, p_2] = [961749221, 961749227]$,
because $n = 2396174922796174922123$ is prime.

Note that the least values of q that satisfy this conjecture for twenty consecutive pairs of consecutive primes with 9 digits are 19, 17, 107, 23, 131, 47, 83, 79, 61, 277, 163, 7, 41, 13, 181, 19, 7, 37, 29 and 23 (all twenty primes lower than 300!), the corresponding primes n obtained having 20 to 24 digits! This method appears to be a good way to obtain big primes with a high degree of ease and certainty.