Every even integer greater than six can be expressed as the sum of two co-prime odd integers at least one of which is a prime

Prashanth R. Rao

Abstract: In this paper we prove a simple theorem that is distantly related to the Even Goldbach conjecture and is weaker than Chen’s theorem regarding the expression of any even integer as the sum of a prime number and a semiprime number. We show that any even integer greater than six can be written as the sum of two odd integers coprime to one another and at least one of them is a prime.

Proof:

Consider any even integer 2n > 6.
Let it be 2n = 2p, 1p, 2p, 3p, ……p, k where p, 1, p, 2, p, 3, ……p, k are all the remaining prime factors.

Bertrand’s postulate suggests that there must be at least one positive prime integer between n = p, 1p, 2p, 3p, ……p, k and 2n = 2p, 1p, 2p, 3p, ……p, k

Let us call this prime as p and this will be an odd prime integer. This prime integer p will be co-prime to all the factors of 2n.

Therefore there must exist another odd integer x such that:

\[ p + x = 2n \]

Since p is coprime to all the prime factors of 2n, x must also be coprime to all the prime factors of 2n and similarly x must be co-prime to p (otherwise common factors would arise for the three integers x, p, 2n).

Consistently n < p < 2n and
x is an odd integer such that: \[ 3 \leq x < n. \]
Also note that p, x, n are unequal.