Propositional Logic: An Extension Nary Anthropic

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Abstract[1]

The proposed extension of propositional logic appears to bridge gaps across areas as diverse as, inductive strength and deductive validity, morphisms and Russellian attempts at formal axiomatization, anthropic alternates, and generalized games—ultimately pointing to gradiency and orduality rationales.

Russell's Dream Come True (Almost)?

The conventional logic of propositions may not do any better job than set theory does, as they both stumble into the issue of whether sets can inherently be *related* (without invoking explicit group- or functional-theoretic mappings). Along similar lines, the Euler-Venn-Ballentine diagrams do not show exactly *how* the sets in question are related beyond naïve implication or conjunction and disjunction. The logic of predicates might improve upon the exposition while eking out on the semantic as well as pragmatic detail; but the very extent of interim detail, or Azimuthality (Shevenyonov, 2016a), might prove prohibitive when capturing the least informative utterances, let alone while connoting the finer shades of meaning.

For that matter, predicates appear to *emulate* the nature of functions, in which light their ability to *substantiate* it would have to be taken with a grain of salt. Higher-order predicates or logic (HOL) could further generalize so far as to capture functional-analytic cognates (e.g. 'functions of lines,' functional compositions as an extension of matrix product, and path integrals), but that would be just an unnecessarily involved and still 'cardinal' proxy for Orduality or relational premises (Shevenyonov, 2016c).

What could be the more parsimonious way of extending the simplest possible, propositional logic while providing bridges between deductive validity, inductive strength, functionality, and outright remote areas such as generalized game theory? In what ways could the latter equivalence be deployed in rationalizing the more arcane views of the 'anthropic' principle and the 'multiverse' approach to endogenizing world constants or parameters underpinning the key regularities?

[1] This tentative and incomplete effort is a dedication to Zoya Kosmodemyanskaya, Sandino, Patrice Lumumba, Che, and mayhap to Fidel alongside Leonardo Peltier and Joan of Arc. No deference to leftism per se, but rather reverence for the many hues of sanctity as a response to the perfidious and cherry-picking ways of mediocrity.

To begin with, consider a conventional implication of the form, $A \rightarrow B$, which equivalently can be rendered as $\bar{B} \rightarrow \bar{A}$. In predicate terms, this could build upon forms like

$$\forall x\, A(x) \rightarrow B(x)\ iff\ \ \forall x\, \bar{B}(x) \rightarrow \bar{A}(x)\ AND\ \ \exists x\, \bar{A}(x) \rightarrow \bar{B}(x)\ \ OR\ \ \exists x\, \bar{A}(x) \rightarrow B(x)$$

Thus far, there appears little that cannot be demonstrated with the magic of conventional diagrams—even though one would have difficulty to either rigorously distinguish between the universal specification (existence) operators or conceptualize the conjugate or dual predicate stretching forms. In other words, *induction* appears near meaningless in general, unless extra specifications and modes are phased in.

Needless to say, one would have liked for any such modifier operators to strike an optimum balance of completeness and simplicity (which invariably refers to Gradiency and Orduality). A good starting point might be to seek generalizations of the form,

$$1.1\ (pA \rightarrow qB) \equiv (\check{q}B \rightarrow \check{p}A)$$

This would simultaneously act to expand upon the previous while accounting for any interim operator or relational combinations. In the meantime, it could be instrumental to define a *general existence operator* as follows:

$$\exists_p x = p(x)$$

$$\lim_{p \rightarrow 0} \exists_p x = \emptyset {\sim} NOT {\sim} \bar{x}$$

$$\lim_{p \rightarrow 1} \exists_p x = (x) {\sim} \forall x$$

$$\lim_{p \rightarrow q \rightarrow 1} \exists_p \exists_q x\, (pA \rightarrow qB) = (A \rightarrow B)$$

$$\lim_{p \rightarrow q \rightarrow 0} \exists_p \exists_q x\, (pA \rightarrow qB) = (B \rightarrow A)$$

Incidentally, this is meant to be a prior illustration, so further extensions along the (1.1) will not hinge upon its validity or refined conventions. Moreover, the *p* and *q* operators could be used as a variety of predicates (to represent an *existential mode* as opposed to an *existential measure*, probabilistic or otherwise qualitative). In a sense, these very notions could be endogenized or ‘solved for’ as structural conventions, with operator ‘values’ referring to a structural status in a functional or relational space.

For now, the convention will be so simple as to overlook any particular (while capturing all) narrowings along the aforementioned lines. [Somewhat mnemonically, or as a matter of meta-formalization or automorphism, that could imply:

$$\forall_p \exists_p \leftrightarrow\ \forall_p \exists_{\hat{p}} (\hat{p} \rightarrow p)$$

Again, this has little to do with the way the meta-induction proceeds from now on.]

To begin with, one may refer to the standard conventions for set operations (Appendix) to arrive at the following:

$$1.2\ (pA \to qB) = (1 + pA)(qB) = \widetilde{(pA)}(qB)$$

$$1.3\ (\check{q}B \to \check{p}A) = (1 + \check{q}B)(\check{p}A) = \widetilde{(\check{q}B)}(\check{p}A)$$

Based on the (1.1) identity or definition, it obtains that:

$$1.4\ \widetilde{(pA)}(qB) \equiv \widetilde{(\check{q}B)}(\check{p}A)$$

$$1.5\ (1 + pA)(qB) = (1 + \check{q}B)(\check{p}A)$$

The dual operators $(\check{q}, \check{p})$ can be solved for based on the knowledge of the direct ones (p, q) subject to the particular nature of the objects making up the basis(A, B). Rather than providing closed-form solutions of the $A = A(B)$ sort (which might be less than meaningful outside intended functional maps) based on the weighted-average type reductions as per the special case of $p = q$,

$$-\frac{\check{p}A - pB}{\check{p} - p} = AB(p + \check{p})$$

it should be enough as a starting point to further restrict this to the $A = B$ instance, to arrive at

$$-B = B^2(p + \check{p})$$

One should be able to appreciate that even the very special cases could prove fairly involved while acting to generalize the convention of $B + B^2 \equiv \emptyset$ (as in the Appendix). Even more importantly, in the event the $A = B$ restriction is applied alone, it follows that

$$1.6\ \check{q} = p, \check{p} = q$$

Additional special cases may have to be studied prior to reverting to the cross-disciplinary implications. For instance, the syllogisms of the kind $\emptyset \to X$ (valid for some X) versus $X \to \emptyset$ (invalid for all X) could be studied in line with restricting either p or q to a zero value (irrespective of how a minimal existential status is defined for practical purposes). In the former case, it follows that, $qB = (1 + \check{q}B)(\check{p}A)$. In the latter, either $\check{p} = 0$ or $A \sim \emptyset$ or $\check{q}B = 1$. In other words, even the degenerate cases can be formalized rigorously as well as consistently.

By contrast, should either one be set to unity (which captures induction without collapsing to the naïve $A \to B$ instance), it would obtain that,

either $B\widetilde{(pA)} = \check{p}^2 A(\widetilde{\frac{\check{q}}{\check{p}} * B})$

or, $(qB)\check{A} = (\check{p}A)\widetilde{(\check{q}B)}$

Once both these restrictions apply, equivalence obtains, which identically reduces to the naïve case. This can be deployed to infer some extra relationships:

$$1.7\ B\check{A} = (\check{p}A)\widetilde{(\check{q}B)} = \check{p}^2 A\widetilde{(\frac{\check{q}}{\check{p}} * B)}$$

It would appear that the *linear*-operator or *homogeneity* case could be one solution, or operator concept (albeit not necessarily applicable outside the corner conditions):

$$1.8\ B\check{A} = \check{p}\check{q}A\check{B}$$

This appears to be a *non-commutative* generalization of the naïve-case equivalence (Appendix):

$$B(1+A) \neq A(1+B)\ OR\ \ (A \to B) \neq (B \to A)$$

Alternatively, (1.7) could more neutrally be reduced to,

$$1.9\ B\check{A} = (\check{p}A)\widetilde{(\check{q}B)}$$

By directly juxtaposing the terms pairwise, it can be obtained that, $\check{A} = \check{p}A$ if and only if $B = \widetilde{(\check{q}B)}$. This appears to be valid insofar as it refers to the identity definitions as implied from the outset. Rather than making any use of the homogeneity assumption, it embarks on the symmetry of [double] negation, albeit without assuming any general operator, i.e. $\check{p} \neq \check{q}$.

The more 'rigorous' approach would be to embark on the initial conventions (Appendix):

$$(1+A)B = (1+\check{q}B)(\check{p}A)\ OR\ \ (A \to B) = (\check{q}B \to \check{p}A)$$

Since this merely refers to the original equivalence (1.1) yet to be reduced, this checks the correctness of (1.7) through (1.9) without providing any further reduced or closed forms. This might reiterate the multitude of meta-solutions or operator concepts as per the initial conjecture, with the linearity or homogeneity case (1.8) still yielding a tentative yet parsimonious non-commutativity of the form:

$$1.10\ \check{p}\check{q} = \left(A\check{B}\right)^{-1}(B\check{A})$$

In passing, please note that an alternate juxtaposition as per (1.9) could be:

$$\check{A} = \check{q}B\ iff\ B = \check{p}A$$

Combined, this might suggest the reduced forms:

$$1.11\ \check{A} = \check{q}(\check{p}A), B = \check{p}\widetilde{(\check{q}B)}$$

Should linearity or homogeneity be maintained in this case, the composite or product $\check{p}\check{q}$ could indeed amount to the *universal* negation (inversion) operator or acting as commutator consistent with (1.8) and (1.10). One should bear in mind that this pertains to the special case still offering some nontrivial implications.

Haunting Overlaps: Generalized Games, Anthropic Principle & Relational Premises

In actuality, the dual operators as in (1.8) delineate the existence domain or specification modality for the applicable subset of X. Better yet, as long as the primal operator couple secures the actual direct mapping, the conjugate counterparts point to the inverse mapping, thus providing for a bridge in between propositional logic and functional or relational representations:

$$\exists_{(p,q)}(A,B)\colon (A \xrightarrow{(p,q)} B) \equiv (pA \to qB) = 1 \sim \{f\colon A \to B\}$$

$$\exists_{(\check{p},\check{q})}(A,B)\colon (B \xrightarrow{(\check{p},\check{q})} A) \equiv (\check{q}B \to \check{p}A) = 1 \sim \{f^{-1}\colon B \to A\}$$

Among other things, the above may refer to the $A \xrightarrow{p<1} B$ Gradiency rationale as in Shevenyonov (2016a) or the Orduality premises $(A,B)^{\rho}$ as in Shevenyonov (2016c).

Moreover, if one were to expand (1.5) directly as[2],

$$2.1\ (qB + pqAB) = (\check{p}A + \check{p}\check{q}AB)$$

a striking equivalence is revealed between this *general propositional logic* (GPL) and the *generalized game* (GG) featuring a consistent treatment of cooperative and non-cooperative games alike (in fact, an entire continuum of interior cases) as in Shevenyonov (2016o):

$$2.2\ (\ )_1 s\bar{s} + (\ )_2 s = \overline{(\ )_1} s\bar{s} + \overline{(\ )_2}\bar{s}$$

The parenthetical terms (or their ratios) refer to the implied *relationships*, notably taking on the rho values that may capture cases as diverse as, strategic complementarity (collusions or control), substitutability (rivalry or costly production), and neutrality or independence (which may also border on stochasticity unless invariance can be ensured). In fact, largely the same holds for the respective operator composition terms, which again refers to Shevenyonov (2016c) for the original ordinalcy foundations.

Now, this need not suggest that the A and B sets be restricted to the *strategic space* (referring to a game with the opportunity frontier, internal production system and external market included, acting formally as a 'chance' player). In fact, both the left- and right-hand sides of (2.2) refer to an excess or net gain $\Delta G \equiv G - \bar{G}$ which could dually be seen as a potential opportunity shift (indeed an *XX-shift*) yet to be attained as a matter of appropriating

[2] While at it, one might be tempted to spot utmost simplicity as well as lowest risk to tapping into the complete or general case juxtaposition, with $pq = \check{p}\check{q}$ being an option that is hart to resist or discard. Simultaneously, though, this hinges on the validity of $qB = \check{p}A$, which does appear to expand upon the naïve case of $qB = \check{p}A$. As a safe bet, this could be of interest as an illustration distinguishing between a *complete*- versus *partial-operator* action—with largely the same holding for the object.

the chance player's payoff otherwise seen as a cost or outlay. In fact, it can be shown that the closed-form optimized values for the strategy resembles the (1.8) while clearly constituting a duality:

$$\Delta G(s) = (\,)_2[\frac{1}{s} - \frac{(\,)_1}{\overline{(\,)_1}s + \overline{(\,)_2}}]^{-1} \underset{s}{\rightarrow} max$$

$$s = \overline{(\,)_2}[1 \mp \sqrt{\frac{\overline{(\,)_1}}{(\,)_1}}]^{-1}$$

The corner cases (unity chance-strategy case being the ultimately coveted one) ensure weak symmetry of the payoffs making room for strategy-invariance, or irrelevance:

$$\bar{s} = 0: (\,)_2 s = 0$$

Zero value parentheses suggests symmetry in the upside quadrant. In contrast, the upside corner $\bar{s} = 1$ yields a generalization over the conventional strategy:

$$s = \frac{\overline{(\,)_2}}{(\,)_2 + (\,)_1 - \overline{(\,)_1}} = \frac{\pi_{12} - \pi_{22}}{(\pi_{11} - \pi_{21}) - (\overline{\pi_{11}} - \overline{\pi_{12}} - \overline{\pi_{21}} + \overline{\pi_{22}})}$$

Consider negation as, $\check{A} \equiv (\gamma + \delta A)$. Akin to the above,

$$A = \frac{\gamma - 1}{1 - \delta}$$

Much of this in turn resembles some of the results as well as rationale as in Shevenyonov (2016n).

Afterthoughts

The literature in the social and natural sciences alike has been replete with the quasi-anthropic or semi-kabbalistic *"as if" equivalence* principle, whereby markets as manned by bounded-rational individuals work *as if* they complied with the 'representative person' or 'rational expectations' premises; bears browse around as if to feel for the soil's chemical makeup; and even the laws and universal constants or parameters such as the speed of light have been dust-settled on as-if competitive premises or otherwise as if to best accommodate the observer that is most likely to emerge wherever they are optimal. The same pseudo-Darwinian rationale might be posited for how the organs and tissues are coordinated within the body, or the past generations implicitly negotiate with the future as per the odds of having been [made] part of a computer-aided simulation.

It is to be hoped that the increasingly popular frameworks that verge on absurdity could still be brought to terms by the grand equivalence as proposed for [implied] GG versus their likely GPL underpinning.

References

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Appendix

The following conventions can be looked up as early as in Stoll (1960), or discerned directly from a handful of basic identities:

$$(A - B) \equiv A \cap \bar{B}$$

$$A + B \equiv (A - B) \cup (B - A)$$

$$A + B = B + A$$

$$A + A = \emptyset$$

$$1 + A = \bar{A}, \qquad 1 + 1 = \emptyset$$

$$A \cup B = AB, \;\; A \cap B = A + B + AB$$

$$(A \to B) = (1 + A)B$$

$$(A \to A) = (1 + A)A = A + A = \emptyset$$

$$(A \leftrightarrow B) = A + B$$