The ABC Conjecture Does Not Hold Water

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Introduction: The ABC conjecture was proposed by Joseph Oesterle and David Masser in 1985. Yet it is still both unproved and un-negated a conjecture hitherto, although somebody claiming proved it on the internet.

AMS subject classification: 11A××, 11D88.

Abstract

The ABC conjecture seemingly is difficult to carry conviction in the face of satisfactory many primes and satisfactory many odd numbers of 6K±1 from operational results of computer programs. So we select and adopt a specific equality $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$. Then, proceed from the analysis of the limits of values of ε to find its certain particular values, thereby finally negate the ABC conjecture once and for all.

Keywords: ABC conjecture, illustrate with a counter-example, untenable

The Proof

The ABC conjecture states that if A, B and C are three co-prime positive integers satisfying A+B=C, then for any real number ε>0, there is merely at most a finite number of solutions to the inequality $C>Rad(A, B, C)^{1+\varepsilon}$, where Rad(A, B, C) denote the product of all distinct prime divisors of A, B and C.

As everyone knows, if anybody wants to prove the ABC conjecture, then that is surely a very difficult thing. But then, if we want to negate the ABC conjecture, so long as proved that there are infinitely many equalities of a certain form of A+B=C satisfying C>(Rad(A, B, C)) $^{1+\varepsilon}$ in which case regard ε as any real number > 0, that will suffice.

Since it is so, we try to let A or B be equal to 1, and another is equal to O^2-1, then C is equal to O^2 according to A+B=C, where O is a positive odd number, and the same below.

In this situation, equality A+B=C is turned into equality $1+(O^2-1)=O^2$. If want to negate the ABC conjecture, then merely need us to prove that there are infinitely many equalities like as $1+(O^2-1)=O^2$ satisfying O^2 >(Rad(1, O^2-1, O^2))$^{1+\varepsilon}$ i.e. satisfying O>(Rad(O^2-1))$^{1+\varepsilon}$ in which case regard ε as any real number > 0.

When O expresses from small to large each positive prime number or from small to large each positive odd number of 6K±1 with K≥1, please, see also appendices 1, 2 and 3 at the back of this article.

Perhaps we can obtain some enlightenment from those operational results of computer programs.

After you have seen three such appendices, whether you should not feel that there are merely finitely many equalities like as $1+(O^2-1)=O^2$ satisfying O>(Rad(O^2-1))$^{1+\varepsilon}$ in which case regard ε as any real number >0 ?

However, to my way of thinking, although the densities of satisfactory prime numbers and satisfactory odd numbers of 6K±1 are getting sparser and sparser along with which the values of O are getting greater and greater, but there are infinitely many prime numbers and infinitely many odd numbers of 6K±1 after all. To say nothing of this conjecture including better integers, namely regard O as each and every positive odd number up to each and every positive integer here.

Judging from this, inequalities like $O>(Rad(O^2-1))^{1+\varepsilon}$ should lasting forever.

Well then, thereinafter, let us cite a specific counter-example to negate the ABC conjecture.

From $O^2-1=(O+1)(O-1)$, we know that O+1 and O-1 are two even numbers, then, both of them have a common prime factor 2.

Such being the case, so let O+1 be equal to 2^N, then not only 2 is a common prime factor of O+1 and O-1, but also 2 is the unique prime factor of (O+1) further, where N is a natural number ≥ 2.

Thus, aforementioned satisfying $O>(Rad(O^2-1))^{1+\varepsilon}$, actually it is exactly satisfying $O>(Rad(O-1))^{1+\varepsilon}$ here.

After substitute 2^N for O+1 and do the relevant substitutions, equality $1+(O^2-1)=O^2$ satisfying $O>(Rad(O-1))^{1+\varepsilon}$ is transformed into equality $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$.

On the supposition of $2^N-1=(Rad(2^N-2))^{1+\varepsilon}$, it has $1+\varepsilon=\log_{rad(2^N-2)}(2^N-1)$, and $\varepsilon=[\log_{rad(2^N-2)}(2^N-1)]-1$.

Thus it can be seen, if $\varepsilon=[\log_{rad(2^N-2)}(2^N-1)]-1$, then $2^N-1=(Rad(2^N-2))^{1+\varepsilon}$;

If $0<\varepsilon<[\log_{rad(2^N-2)}(2^N-1)]-1$, then $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$, and that there are infinitely many real numbers of ε between 0 and $[\log_{rad(2^N-2)}(2^N-1)]-1$;

If $\varepsilon>[\log_{rad(2^N-2)}(2^N-1)]-1$, then $2^N-1<(Rad(2^N-2))^{1+\varepsilon}$, of course, there are infinitely many real numbers of ε in which case $\varepsilon>[\log_{rad(2^N-2)}(2^N-1)]-1$ too.

Hereinafter, we will divide the procedure relating to the negative proof into four aspects in accordance with disparate evaluations of ε to explain the relation inter se until negate the conjecture finally.

Firstly, when ε=0, there are infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$. Evidently, this has nothing to do with the conjecture because ε=0 is inconformity to the requirement of the conjecture.

Secondly, when $0<\varepsilon<[\log_{rad(2^N-2)}(2^N-1)]-1$, there are infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$. Then, both there are infinitely many real numbers within the limits which satisfy ε of each and every inequality, and there are infinitely many natural numbers N ≥2 under these circumstances.

Thirdly, when $\varepsilon=[\log_{rad(2^N-2)}(2^N-1)]-1$, there is equality $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $2^N-1=(Rad(2^N-2))^{1+\varepsilon}$. This has nothing to do with the conjecture too because $2^N-1=(Rad(2^N-2))^{1+\varepsilon}$ is inconformity to the requirement of the conjecture.

Fourthly, when $\varepsilon>[\log_{rad(2^N-2)}(2^N-1)]-1$, there are infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ with N≥2 satisfying $2^N-1<(Rad(2^N-2))^{1+\varepsilon}$. Likewise, this has nothing to do with the conjecture because $2^N-1<(Rad(2^N-2))^{1+\varepsilon}$ is inconformity to the requirement of the conjecture.

By this token, if we want to negate the ABC conjecture, then can only from secondly aspect i.e. the case when $0<\varepsilon<[\log_{rad(2^N-2)}(2^N-1)]-1$ to consider it.

By now, let us list N and $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$ after evaluations of headmost N as follows, where $0<\varepsilon<[\log_{rad(2^N-2)}(2^N-1)]-1$, but real numbers which satisfy ε of each inequality are incomplete alike.

N,	2^N,	$2^N(2^N-2)$,	$2^N-1>(Rad(2^N-2))^{1+\varepsilon}$,	$1+2^N(2^N-2)=(2^N-1)^2$
2,	4,	8,	$3>2^{1+\varepsilon}$,	1+8=9
3,	8,	48,	$7>(2*3)^{1+\varepsilon}=6^{1+\varepsilon}$,	1+48=49
4,	16,	224,	$15>(2*7)^{1+\varepsilon}=14^{1+\varepsilon}$,	1+224=225
5,	32,	960,	$31>(2*3*5)^{1+\varepsilon}=30^{1+\varepsilon}$,	1+960=961
6,	64,	3968,	$63>(2*31)^{1+\varepsilon}=62^{1+\varepsilon}$,	1+3968=3969
7,	128,	16128,	$127>(2*3*7)^{1+\varepsilon}=42^{1+\varepsilon}$,	1+16128=16129
8,	256,	65024,	$255>(2*127)^{1+\varepsilon}=254^{1+\varepsilon}$,	1+65024=65025
9,	512,	261120,	$511>(2*3*5*17)^{1+\varepsilon}=510^{1+\varepsilon}$,	1+261120=261121
10,	1024,	1046528,	$1023>(2*7*73)^{1+\varepsilon}=1022^{1+\varepsilon}$,	1+1046528=1046529
11,	2048,	4190208,	$2047>(2*3*11*31)^{1+\varepsilon}=2046^{1+\varepsilon}$,	1+4190208=4190209
12,	4096,	16769024,	$4095>(2*23*89)^{1+\varepsilon}=4094^{1+\varepsilon}$,	1+16769024=16769025
13,	8192,	67092480,	$8191>(2*3*5*7*13)^{1+\varepsilon}=2730^{1+\varepsilon}$,	1+67092480=67092481
14,	16384,	268402688,	$16383>(2*8191)^{1+\varepsilon}=16382^{1+\varepsilon}$,	1+268402688 268402689
15,	32768,	1073676288,	$32767>(2*3*43*127)^{1+\varepsilon}=32766^{1+\varepsilon}$,	1+1073676288=1073676289
…	…	…	…	…

As listed above and their extensions, we are not difficult to make out that smallest difference of 2^N-1 minus $Rad(2^N-2)$ is 1, and that values of ε are getting smaller and smaller up to infinitesimal along with which values of N are getting greater and greater up to infinite great.

Hereto to negate the ABC conjecture, this means it be necessary to prove that for any real number ε>0, there are infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon}$.

In this case, we suppose that smallest positive real number which 0 and ε border on each other is ε_0. That is to say, there is not a real number between 0 and ε_0. Then, there are still infinitely many real numbers between ε_0 and $\log_{rad(2^N-2)}(2^N-1)-1$. Therefore, no matter how great a natural number N, such that $1+2^N(2^N-2)=(2^N-1)^2$, it is able to satisfy $2^N-1>(Rad(2^N-2))^{1+\varepsilon_0}$.

In other words, when $\varepsilon=\varepsilon_0$, there are infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon_0}$.

Since natural numbers of $N\geq 2$ both have infinitely many and are up to infinitely great, thus although values of ε are getting smaller and smaller along with which values of N are getting greater and greater, but ε forever cannot reach smallest positive real number ε_0 due to N forever cannot reach ideal greatest natural number because there is an unbridgeable line of demarcation between finite field and infinite field forever.

To move forward single step to speaking, start from ε_0, we suppose and list orderly-increasing neighboring real numbers ε_0, ε_1, $\varepsilon_2 \ldots \varepsilon_y$, one by one, where corner mark y is a concrete natural number which consists of Arabic numerals. Without doubt, for any real number ε_y, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ with $N\geq 2$ satisfying $2^N-1>(Rad(2^N-2))^{1+\varepsilon_y}$, because ε forever cannot reach ε_y either.

In addition, three terms 1, $2^N(2^N-2)$ and $(2^N-1)^2$ in the equality are co-prime positive integers assuredly.

It is obvious that all qualifications about aforementioned equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ are completely in conformity with the requirement of the conjecture.

Only this and nothing more, for the ABC conjecture asserted the argument that if A, B and C are three co-prime positive integers and satisfying A+B=C, for any real number $\varepsilon>0$, there is merely at most a finite number of solutions to the inequality $C>Rad(A, B, C)^{1+\varepsilon}$, has to be negated by infinitely many equalities like as $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>(rad(1, 2^N(2^N-2), (2^N-1)^2))^{1+\varepsilon}$, i.e. satisfying $2^N-1>(rad(2^N-2))^{1+\varepsilon}$, where $N\geq2$, $0 < \varepsilon = \varepsilon_0, \varepsilon_1, \varepsilon_2\ldots\varepsilon_y$, and y is a concrete natural number.

The proof was thus brought to a close. As a consequence, the ABC conjecture does not hold water.

Appendix 1: Prime number P and equality $1+ (P^2-1)=P^2$ satisfying $P >(Rad(P^2-1))^{1+\varepsilon}$ after the evaluations of headmost P are listed as follows, but real numbers which satisfy ε of each inequality are incomplete alike.

P,	P^2-1,	Rad (P^2-1)
7,	48,	2*3=6
17,	288,	2*3=6
31,	960,	2*3*5=30
97,	9408,	2*3*7=42
127,	16128,	2*3*7=42
251,	63000,	2*3*5*7=210
449,	201600,	2*3*5*7=210
487,	237168,	2*3*61=366
577,	332928,	2*3*17=102
1151,	1324800,	2*3*5*23=690
1249,	1560000,	2*3*5*13=390
1567,	2455488,	2*3*7*29=1218
1999,	3996000,	2*3*5*37=1110
2663,	7091568,	2*3*11*37=2442

4801,	23049600,	2*3*5*7=210
4999,	24990000,	2*3*5*7*17=3570
7937,	62995968,	2*3*7*31=1302
8191,	67092480,	2*3*5*7*13=2730
12799,	163814400,	2*3*5*79=2370
13121,	172160640,	2*3*5*41=1230
13183,	173791488,	2*3*13*103=8034
15551,	241833600,	2*3*5*311=9330
31249,	976500000,	2*3*5*7*31=6510
31751,	1008126000,	2*3*5*7*127=26670
32257,	1040514048,	2*3*7*127=5334
33857,	1146296448,	2*3*11*19*23=28842
35153,	1235733408,	2*3*7*13*31=16926
39367,	1549760688,	2*3*7*19*37=29526
65537,	4295098368,	2*3*11*331=21846
79201,	6272798400,	2*3*5*11*199=65670
81919,	6710722560,	2*3*5*37*41=45510
85751,	7353234000,	2*3*5*7*397=83370
115249,	13282332000,	2*3*5*7*461=96810
117127,	13718734128,	2*3*11*241=15906
124001,	15376248000,	2*3*5*31*83=77190
126001,	15876252000,	2*3*5*7*251=52710
131071,	17179607040,	2*3*5*17*257=131070
153089,	23436241920,	2*3*5*7*13*23=62790
160001,	25600320000,	2*3*5*2963=88890
161839,	26191861920,	2*3*5*7*17*37=132090
165887,	27518496768,	2*3*7*17*41=29274
196831,	38742442560,	2*3*5*6151=184530
215297,	46352798208,	2*3*29*443=77082
281249,	79101000000,	2*3*5*11*17*47=263670
442367,	195688562688,	2*3*29*263=45762
474337,	224995589568,	2*3*61*487=178242
511757,	261895227048,	2*3*7*13*373=203658
524287,	274876858368,	2*3*7*19*73=58254
538001,	289445076000,	2*3*5*41*269=330870
665857,	443365544448,	2*3*17*577=58854
715823,	512402567328,	2*3*71*1657=705882
902501,	814508055000,	2*3*5*19*619=352830
911249,	830374740000,	2*3*5*13*337=131430
988417,	976968165888,	2*3*11*13*19*37=603174
1039681,	1080936581760,	2*3*5*7*19*103=410970
1062881,	1129716020160,	2*3*5*7*13*73=199290
1102249,	1214952858000,	2*3*5*7*4409=925890
1179649,	1391571763200,	2*3*5*23593=707790

1229311,	1511205534720,	2*3*5*7*29*157=956130
1246589,	1553984134920,	2*3*5*7*19*211=841890
1272833,	1620103845888,	2*3*11*97*113=723426
...	...	...

Appendix 2: Odd number 6K-1and equality $1+((6K-1)^2-1)=(6K-1)^2$ satisfying $6K-1>(\mathrm{Rad}((6K-1)^2-1))^{1+\varepsilon}$ after the evaluations of headmost 6K-1 are listed as follows, but real numbers which satisfy ε of each inequality are incomplete alike.

6K-1	$(6K-1)^2-1$,	$\mathrm{Rad}((6K-1)^2-1)$
17,	288,	2*3=6
161,	25920,	2*3*5=30
251,	63000,	2*3*5*7=210
449,	201600,	2*3*5*7=210
485,	235224,	2*3*11=66
1025,	1050624,	2*3*19=114
1151,	1324800,	2*3*5*23=690
1457,	2122848,	2*3*7*13=546
2177,	4739328,	2*3*11*17=1122
2663,	7091568,	2*3*11*37=2442
4607,	21224448,	2*3*7*47=1974
5291,	27994680,	2*3*5*7*23=4830
7775,	60450624,	2*3*13*23=1794
7937,	62995968,	2*3*7*31=1302
9827,	96569928,	2*3*7*13*17=9282
10751,	115584000,	2*3*5*7*43=9030
11663,	136025568,	2*3*7*17=714
13121,	172160640,	2*3*5*41=1230
14849,	220492800,	2*3*5*11*29=9570
15551,	241833600,	2*3*5*311=9330
19601,	384199200,	2*3*5*7*11=2310
24335,	592192224,	2*3*13*23=1794
25001,	625050000,	2*3*5*463=13890
28673,	822140928,	2*3*7*59=2478
31751,	1008126000,	2*3*5*7*127=26670
33281,	1107624960,	2*3*5*13*43=16770

33857,	1146296448,	2*3*11*19*23=28842
35153,	1235733408,	2*3*7*13*31=16926
36449,	1328529600,	2*3*5*17*67=34170
48599,	2361862800,	2*3*5*11*47=15510
49151,	2415820800,	2*3*5*983=29490
52001,	2704104000,	2*3*5*13*107=41730
53249,	2835456000,	2*3*5*13*71=27690
58751,	3451680000,	2*3*5*17*47=23970
65537,	4295098368,	2*3*11*331=21846
67229,	4519738440,	2*3*5*7*83=17430
73001,	5329146000,	2*3*5*23*73=50370
83105,	6906441024,	2*3*7*19*53=42294
85751,	7353234000,	2*3*5*7*397=83370
95831,	9183580560,	2*3*5*7*11*37=85470
98495,	9701265024,	2*3*11*19*37=46398
101249,	10251360000,	2*3*5*7*113=23730
118097,	13946901408,	2*3*11*61=4026
124001,	15376248000,	2*3*5*31*83=77190
130049,	16912742400,	2*3*5*17*127=64770
145001,	21025290000,	2*3*5*11*13*29=124410
153089,	23436241920,	2*3*5*7*13*23=62790
160001,	25600320000,	2*3*5*2963=88890
165887,	27518496768,	2*3*7*17*41=29274
171395,	29376246024,	2*3*17*23*71=166566
194399,	37790971200,	2*3*5*37*71=78810
207647,	43117276608,	2*3*7*47*103=203322
209951,	44079422400,	2*3*5*13*17*19=125970
215297,	46352798208,	2*3*29*443=77082
246401,	60713452800,	2*3*5*7*11*13=30030
258065,	66597544224,	2*3*59*127=44958
259199,	67184121600,	2*3*5*19*359=204630
275561,	75933864720,	2*3*5*7*83=17430
281249,	79101000000,	2*3*5*11*17*47=263670
297755,	88658040024,	2*3*53*919=292242
360449,	129923481600,	2*3*5*11*89=29370
415151,	172350352800,	2*3*5*19*23*31=406410
433025,	187510650624,	2*3*11*17*199=223278
439001,	192721878000,	2*3*5*29*439=381930

442367,	195688562688,	2*3*29*263=45762
456191,	208110228480,	2*3*5*7*11*19=43890
511757,	261895227048,	2*3*7*13*373=203658
526337,	277030637568,	2*3*19*257=29298
538001,	289445076000,	2*3*5*41*269=330870
595349,	354440431800,	2*3*5*7*13*107=292110
628865,	395471188224,	2*3*7*17*23*31=509082
663551,	440299929600,	2*3*5*23*577=398130
672281,	451961742960,	2*3*5*7*13*17=46410
692225,	479175450624,	2*3*13*4273=333294
715823,	512402567328,	2*3*71*1657=705882
778751,	606453120000,	2*3*5*7*13*89=242970
780449,	609100641600,	2*3*5*11*29*43=411510
795905,	633464769024,	2*3*17*3109=317118
802817,	644515135488,	2*3*7*14867=624414
816641,	666902522880,	2*3*5*11*29*71=679470
830465,	689672116224,	2*3*7*13*811=442806
845153,	714283593408,	2*3*7*11*37*47=803418
902501,	814508055000,	2*3*5*19*619=352830
907925,	824327805624,	2*3*61*389=142374
911249,	830374740000,	2*3*5*13*337=131430
943937,	891017059968,	2*3*7*43*229=413574
964895,	931022361024,	2*3*7*19*23*41=752514
983039,	966365675520,	2*3*5*7*1433=300930
1024001,	1048578048000,	2*3*5*7*43=9030
1062881,	1129716020160,	2*3*5*7*13*73=199290
1098305,	1206273873024,	2*3*11*43*131=371778
1226177,	1503510035328,	2*3*7*17*23*29=476238
1240577,	1539031292928,	2*3*41*2423=596058
1246589,	1553984134920,	2*3*5*7*19*211=841890
1272833,	1620103845888,	2*3*11*97*113=723426
1283201,	1646604806400,	2*3*5*89*401=1070670
1336337,	1785796577568,	2*3*17*73*113=841398
1349633,	1821509234688,	2*3*11*13*659=565422
1354751,	1835350272000,	2*3*5*7*5419=1137990
1376255,	1894077825024,	2*3*7*11*47=21714
1431431,	2048994707760,	2*3*5*7*11*13*47=1411410
1524095,	2322865569024,	2*3*7*11*13*73=438438

1712501,	2932659675000,	2*3*5*11*31*137=1401510
1714751,	2940370992000,	2*3*5*13*19*229=1696890
1721249,	2962698120000,	2*3*5*17*19*149=1443810
1781249,	3172848000000,	2*3*5*7*19*71=283290
1843199,	3397382553600,	2*3*5*7*31*137=891870
1850201,	3423243740400,	2*3*5*11*29*47=449790
1882385,	3543373288224,	2*3*7*11*3169=1464078
1996001,	3984019992000,	2*3*5*37*499=553890
2024999,	4100620950000,	2*3*5*59*131=231870
2093057,	4380887605248,	2*3*7*11*31*73=1045506
2218751,	4922856000000,	2*3*5*71*107=227910
2261249,	5113247040000,	2*3*5*11*67*73=1614030
2371841,	5625629729280,	2*3*5*11*17*109=611490
2426111,	5886014584320,	2*3*5*13*19*113=837330
2433401,	5921440426800,	2*3*5*23*1669=1151610
2450087,	6002926307568,	2*3*19*107*199=2427402
2550251,	6503780163000,	2*3*5*101*461=1396830
2618999,	6859155762000,	2*3*5*19*41*97=2266890
2725001,	7425630450000,	2*3*5*7*89*109=2037210
2834351,	8033545591200,	2*3*5*56687=1700610
2862251,	8192480787000,	2*3*5*43*107=138030
2882465,	8308604476224,	2*3*13*41*659=2107482
2952449,	8716955097600,	2*3*5*19*607=345990
3014657,	9088156827648,	2*3*23*6203=856014
3130001,	9796906260000,	2*3*5*139*313=1305210
3429215,	11759515516224,	2*3*7*47*191=377034
3512321,	12336398807040,	2*3*5*7*11*73=168630
3548447,	12591476111808,	2*3*11*31*37*43=3255186
3694085,	13646263987224,	2*3*11*31*691=1413786
3792257,	14381213154048,	2*3*13*17*43*53=3021954
3906251,	15258796875000,	2*3*5*7*5167=1085070
4000751,	16006008564000,	2*3*5*7*13*1231=3360630
4046849,	16376986828800,	2*3*5*13*17*19*23=2897310
4194305,	17592194433024,	2*3*43*5419=1398102
4687499,	21972646875000,	2*3*5*47*1061=1496010
4691555,	22010688318024,	2*3*7*19*977=779646
4899851,	24008539822200,	2*3*5*43*53*71=4854270
5458751,	29797962480000,	2*3*5*11*13*397=1703130

5544449,	30740914713600,	2*3*5*7*13*17*37=1717170
5771249,	33307315020000,	2*3*5*7*19*227=905730
5786801,	33487065813600,	2*3*5*7*17*23*37=3038070
5848415,	34203958012224,	2*3*7*11*13*967=5807802
…	…	…

Appendix 3: Odd number 6K+1 and equality 1+ $((6K+1)^2 -1) = (6K+1)^2$ satisfying $6K+1 > (\text{Rad}\ ((6K+1)^2-1))^{1+\varepsilon}$ after the evaluations of headmost 6K+1 are listed as follows, but real numbers which satisfy ε of each inequality are incomplete alike.

6K+1	$(6K+1)^2$ -1,	$\text{Rad}((6K+1)^2-1)$
7,	48,	2*3=6
31,	960,	2*3*5=30
49,	2400,	2*3*5=30
55,	3024,	2*3*7=42
97,	9408,	2*3*7=42
127,	16128,	2*3*7=42
487,	237168,	2*3*61=366
511,	261120,	2*3*5*17=510
577,	332928,	2*3*17=102
649,	421200,	2*3*5*13=390
721,	519840,	2*3*5*19=570
1249,	1560000,	2*3*5*13=390
1351,	1825200,	2*3*5*13=390
1567,	2455488,	2*3*7*29=1218
1921,	3690240,	2*3*5*31=930
1999,	3996000,	2*3*5*37=1110
2047,	4190208,	2*3*11*31=2046
2431,	5909760,	2*3*5*19=570
4375,	19140624,	2*3*547=3282
4801,	23049600,	2*3*5*7=210
4999,	24990000,	2*3*5*7*17=3570
5617,	31550688,	2*3*13*53=4134
6049,	36590400,	2*3*5*7*11=2310
6751,	45576000,	2*3*5*211=6330

8191,	67092480,	2*3*5*7*13=2730
8449,	71385600,	2*3*5*11*13=4290
8749,	76545000,	2*3*5*7=210
12151,	147646800,	2*3*5*7*31=6510
12799,	163814400,	2*3*5*79=2370
13183,	173791488,	2*3*13*103=8034
18751,	351600000,	2*3*5*293=8790
18817,	354079488,	2*3*7*97=4074
21295,	453477024,	2*3*7*11*13=6006
27379,	749609640,	2*3*5*13*37=14430
27649,	764467200,	2*3*5*7*79=16590
29281,	857376960,	2*3*5*11*61=20130
31249,	976500000,	2*3*5*7*31=6510
32257,	1040514048,	2*3*7*127=5334
32767,	1073676288,	2*3*43*127=32766
33535,	1124596224,	2*3*23*131=18078
39367,	1549760688,	2*3*7*19*37=29526
43903,	1927473408,	2*3*7*271=11382
51841,	2687489280,	2*3*5*7*23=4830
53137,	2823540768,	2*3*41*163=40098
56251,	3164175000,	2*3*5*7*41=8610
57121,	3262808640,	2*3*5*7*13*17=46410
62425,	3896880624,	2*3*7*13*17=9282
74359,	5529260880,	2*3*5*11*13*17=72930
79201,	6272798400,	2*3*5*11*199=65670
81919,	6710722560,	2*3*5*37*41=45510
100351,	10070323200,	2*3*5*7*223=46830
110593,	12230811648,	2*3*11*457=30162
115249,	13282332000,	2*3*5*7*461=96810
116161,	13493377920,	2*3*5*11*241=79530
117127,	13718734128,	2*3*11*241=15906
118099,	13947373800,	2*3*5*1181=35430
119071,	14177903040,	2*3*5*7*61=12810
126001,	15876252000,	2*3*5*7*251=52710
131071,	17179607040,	2*3*5*17*257=131070
132097,	17449617408,	2*3*43*257=66306
137215,	18827956224,	2*3*7*11*67=30954
143749,	20663775000,	2*3*5*11*23=7590

146881,	21574028160,	2*3*5*17*271=138210
161839,	26191861920,	2*3*5*7*17*37=132090
167041,	27902695680,	2*3*5*17 *29=14790
181249,	32851200000,	2*3*5*29*59=51330
189001,	35721378000,	2*3*5*7*11*71=164010
196831,	38742442560,	2*3*5*6151=184530
202501,	41006655000,	2*3*5*19*73=41610
211249,	44626140000,	2*3*5*13*163=63570
220159,	48469985280,	2*3*5*43*151=194790
221185,	48922804224,	2*3*7*37*61=94794
227137,	51591216768,	2*3*7*13*337=184002
235297,	55364678208,	2*3*7*19*43=34314
236671,	56013162240,	2*3*5*7*23*43=207690
244903,	59977479408,	2*3*7*11*17*23=180642
260641,	67933730880,	2*3*5*19*181=103170
262087,	68689595568,	2*3*11*19*181=226974
285769,	81663921360,	2*3*5*7*17*41=146370
302527,	91522585728,	2*3*7*29*163=198534
312499,	97655625000,	2*3*5*643=19290
320761,	102887619120,	2*3*5*11*13*73=313170
330751,	109396224000,	2*3*5*7*17*19=67830
337501,	113906925000,	2*3*5*11*23*29=220110
354295,	125524947024,	2*3*67*661=265722
373249,	139314816000,	2*3*5*1493=44790
403201,	162571046400,	2*3*5*7*449=94290
406783,	165472409088,	2*3*7*31*227=295554
470449,	221322261600,	2*3*5*11*97=32010
474337,	224995589568,	2*3*61*487=178242
500095,	250095009024,	2*3*7*3907=164094
522241,	272735662080,	2*3*5*7*17*73=260610
524287,	274876858368,	2*3*7*19*73=58254
546751,	298936656000,	2*3*5*8543=256290
559681,	313242821760,	2*3*5*11*23*53=402270
559873,	313457776128,	2*3*7*29*197=239946
583201,	340123406400,	2*3*5*17*1009=514590
661249,	437250240000,	2*3*5*7*23*41=198030
665335,	442670662224,	2*3*7*37*109=169386
665857,	443365544448,	2*3*17*577=58854

702463,	493454266368,	2*3*7*47*53=104622
781249,	610350000000,	2*3*5*13*313=122070
818749,	670349925000,	2*3*5*7*19*131=522690
826687,	683411395968,	2*3*7*12917=542514
842401,	709639444800,	2*3*5*11*13*59=253110
907741,	823993723080,	2*3*5*11*31*41=419430
913951,	835306430400,	2*3*5*13*677=264030
919999,	846398160000,	2*3*5*23*631=435390
938449,	880686525600,	2*3*5*7*19*137=546630
966655,	934421889024,	2*3*13*17*59=78234
988417,	976968165888,	2*3*11*13*19*37=603174
1039681,	1080936581760,	2*3*5*7*19*103=410970
1059967,	1123530041088,	2*3*7*13*727=396942
1102249,	1214952858000,	2*3*5*7*4409=925890
1102735,	1216024480224,	2*3*41*2269=558174
1128001,	1272386256000,	2*3*5*47*751=1058910
1179649,	1391571763200,	2*3*5*23593=707790
1202851,	1446850528200,	2*3*5*7*11*17*19=746130
1229311,	1511205534720,	2*3*5*7*29*157=956130
1370929,	1879446323040,	2*3*5*11*13*103=441870
1387777,	1925925001728,	2*3*7*13*17*139=1290198
1417177,	2008390649328,	2*3*7*14461=607362
1434817,	2058699823488,	2*3*7*11*47*53=1150842
1518751,	2306604600000,	2*3*5*31*1531=1423830
1555849,	2420666110800,	2*3*5*7*29*37=225330
1581229,	2500285150440,	2*3*5*7*11*461=1064910
1653751,	2734892370000,	2*3*5*7*37*151=1173270
1685503,	2840920363008,	2*3*7*13*823=449358
1823509,	3325185073080,	2*3*5*13*37*83=1197690
1831249,	3353472900000,	2*3*5*157*293=1380030
1847041,	3411560455680,	2*3*5*13*31*37=447330
1915999,	3671052168000,	2*3*5*7*19*479=1911210
1999999,	3999996000000,	2*3*5*7*11*13*37=1111110
2086399,	4353060787200,	2*3*5*53*163=259170
2097151,	4398042316800,	2*3*5*11*31*41=419430
2101249,	4415247360000,	2*3*5*19*41=23370
2234497,	4992976843008,	2*3*7*11*23*151=1604526
2281249,	5204097000000,	2*3*5*73*89=194910

2367487,	5604994695168,	2*3*11*17*1087=1219614
2400001,	5760004800000,	2*3*5*11*43*59=837210
2456245,	6033139500024,	2*3*7*13*19*43=446082
2649601,	7020385459200,	2*3*5*23*1151=794190
2655505,	7051706805024,	2*3*7*79*683=2266194
2739199,	7503211161600,	2*3*5*7*11*107=247170
2898919,	8403731368560,	2*3*5*11*23*137=1039830
2957311,	8745688350720,	2*3*5*19*1217=693690
2965951,	8796865334400,	2*3*5*11*13*383=1643070
2970343,	8822937537648,	2*3*13*17*571=757146
3001249,	9007495560000,	2*3*5*7*17*613=2188410
3114751,	9701673792000,	2*3*5*23*4153=2865570
3120001,	9734406240000,	2*3*5*13*1249=487110
3188647,	10167469690608,	2*3*398581=2391486
3271681,	10703896565760,	2*3*5*71*1279=2724270
3483649,	12135810355200,	2*3*5*7*19*193=770070
3529471,	12457165539840,	2*3*5*7*17*811=2895270
3543121,	12553706420640,	2*3*5*7*11*19*37=1623930
3650401,	13325427460800,	2*3*5*7*13*193=526890
3684751,	13577389932000,	2*3*5*17*41*137=2864670
3704401,	13722586768800,	2*3*5*7*13*17*29=1345890
3748321,	14049910319040,	2*3*5*19*37*137=2889330
3781249,	14297844000000,	2*3*5*11*43*229=3249510
3786751,	14339483136000,	2*3*5*11*17*43=241230
3909631,	15285214556160,	2*3*5*19*23*83=1088130
4176049,	17439385250400,	2*3*5*17*19*241=2335290
4218751,	17797860000000,	2*3*5*23*1433=988770
4245697,	18025943015808,	2*3*7*13*31*47=795522
4257361,	18125122684320,	2*3*5*73*1459=3195210
4605823,	21213605507328,	2*3*13*35983=2806674
4620799,	21351783398400,	2*3*5*7*13*19*31=1607970
4910977,	24117695094528,	2*3*7*29*1567=1908606
5030911,	25310065489920,	2*3*5*17*6211=3167610
5038849,	25389999244800,	2*3*5*179*563=3023310
5126401,	26279987212800,	2*3*5*89*1601=4274670
5196799,	27006719846400,	2*3*5*7*17*29*37=3830610
5353777,	28662928165728,	2*3*17*59*769=4627842
5540833,	30700830333888,	2*3*11*13*53*97=4410978

5651521,	31939689613440,	2*3*5*7*29*41=249690
5658247,	32015759113008,	2*3*11*17*29*41=1334058
5918719,	35031234600960,	2*3*5*13*17*449=2976870
...	...	...

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