Coherent Detection of Signals below Noise Level

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Abstract: The search for a phase coherent signal near 50.93 µHz in the data set of the superconducting gravimeter H1 indicates a weakly damped signal 14 dB below noise. It might belong to the long-sought Slichter triplet.

Introduction

Very strong earthquakes can probably move the inner, solid core, which is surrounded by the liquid outer core. This should lead to a harmonic oscillation around its resting position and measurable fluctuations of gravitation here on the surface. The existence of the ${}_1S_1$ natural mode was postulated 55 years ago by Slichter[1], but could never be confirmed by measurements despite intensive search.

The main problem is that no theory provides reliable predictions, but only very vague assumptions. With the known data of the Earth, the expected period of oscillation is likely to be about five hours (± two hours). The frequency difference of the three desired spectral lines is unknown, as is the damping. If the putative oscillation deep inside the Earth is triggered by changes on the surface, the search should focus on the period after a strong earthquake. This investigation focuses on the strongest event on 2004-12-26 since the invention of the Superconducting Gravimeter.

The many unsuccessful attempts have shown one thing: the periodic change of gravitation is extremely small and disappears in the noise level caused by the frequent earthquakes. Moreover, in almost all previous searches, simple and best practices to reduce the noise have been omitted [2]. On the contrary, extremely wide-band methods such as Fourier analysis were used and narrow-band filters were generally dispensed with. For this reason, a selective detection system has been developed which is capable of detecting signal whose amplitude is only about 10% of the average noise level.

An Integrating Coherent Detector for Noisy Signals

Resonance is the preferred method in communications engineering to identify extremely weak signals. The example of a swing shows that many tiny impulses arriving in the right rhythm, lead to a substantial whole amplitude. From the amplitude increase over time, the supplied energy can be calculated. If the pulses are randomly distributed (noise), the amplitude will fluctuate around a small average value. A small frequency deviation produces a beat, because the excitation is periodically antiphase.

A particular property of the new method allows to determine the kind of the excitation: when the excitation (A) occurs with a *constant* amplitude and exactly on the resonant frequency, the amplitude (B) of the oscillating circuit increases in proportion to time. The method is robust, since spurious spikes, deviating frequencies or noise can only slightly disturb the expected linear amplitude increase.

If the stimulating amplitude (A) is not constant, the time dependency of the envelope of the integrated amplitude (B) of the oscillating circuit can be either calculated with the relationship

$A = const \cdot \frac{d}{dt} B$ or by reconstructing the excitation (trial-and-error).

In order to investigate the gravitational data, the resonant and loss-free oscillating circuit is not implemented mechanically or by electronic components, it is simulated by software. The oscillation is triggered by the serial data, which were measured by superconducting gravimeters at regular time

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intervals Δt. The amplitude of the resonant circuit is calculated in the same rhythm. This can be done very simply by extending the known trigonometric identities.

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \quad \text{and} \quad \cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

We take two variables x and y. At time t = 0, we start with the initial values x = 0 and y = 0. We proceed from on time step to the next

$$x(t+\Delta t)=P\cdot x(t)+Q\cdot y(t) \quad \text{and} \quad y(t+\Delta t)=P\cdot y(t)-Q\cdot x(t)+S(t)$$

The resonant frequency f_0 and sampling period Δt define the constant values

$$P=\cos(2\pi\,\Delta t\, f_0) \quad \text{and} \quad Q=\sin(2\pi\,\Delta t\, f_0)$$

If the signal *S* remains zero, the resonant circuit never starts to oscillate. If the signal is noise, the circuit responds with an oscillation of variable amplitude. If the signal contains a small component of the frequency f_0, it forces the circuit to oscillate with approximately the same phase. An in-phase, long lasting signal component increases the integrated amplitude remarkably. Noise results in a wavy envelope.

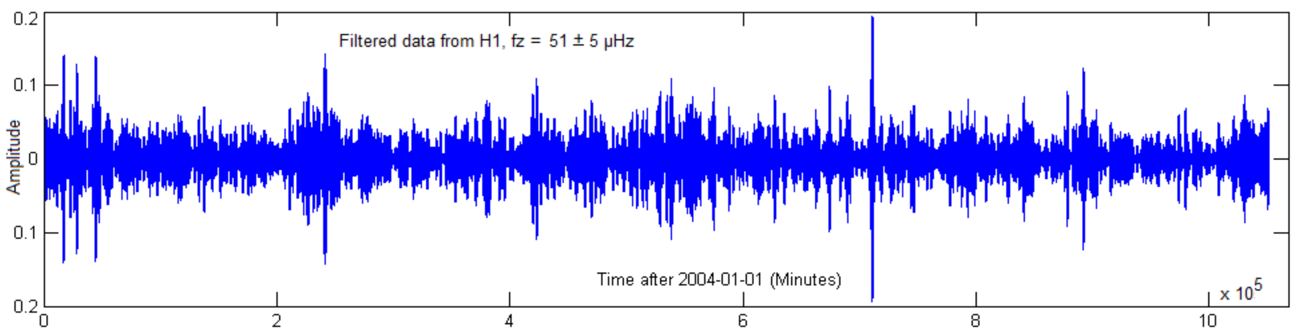
Using MATLAB, a few lines of code will do the job:

```
j=2e-6*pi*Ts*f0; P=cos(j); Q=sin(j);
L=length(S); x=zeros(L,1); y=x;
  for j=2:L
    x(j)=x(j-1)*P+y(j-1)*Q; %Add-Theorem
    y(j)=y(j-1)*P-x(j-1)*Q+S(j);
  end
plot(x), title('Coherent Demodulation')
```

The only freely selectable parameters in the formulas above are the desired resonance frequency f_0 and the start time of the integration. Experiments have shown that (at least near 50 µHz with time steps Δt = 600 seconds) no feedback is necessary to reduce the bandwidth. This avoids all problems that accompany IIR filters. Likewise, no complex control circuit is required to bring the phases of the signal and the resonant circuit into coincidence.

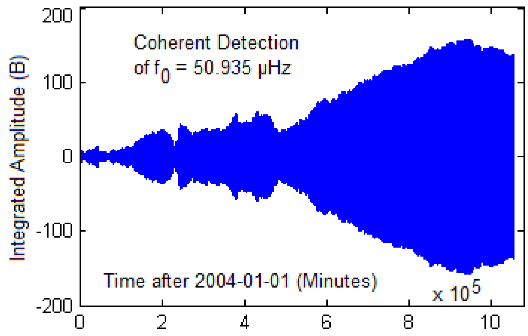
Analyzing 50.9 µHz

Previous investigations[3][4] have shown that near 50.9 µHz, there is a weak signal which is not listed in the catalog HW95[5]. The signal-to-noise ratio is very poor, which is why the signal can only be detected in the data of a few stations. After preparing the 2004/2005 CORMIN records in the usual way, the two-year data chain recorded by the station H1 looks like this:



There are no abnormalities or longer data gaps and the very strong earthquake on 2004-12-26 (time = 518459 minutes after 2004-01-01) produces no significant signal in the specified frequency range. It is advisable to clean up the data series of unbalanced (unipolar) spikes, which are very common in some stations and are probably caused by faulty electronics. Disturbances generated by earth-quakes are always symmetrical to the zero line.

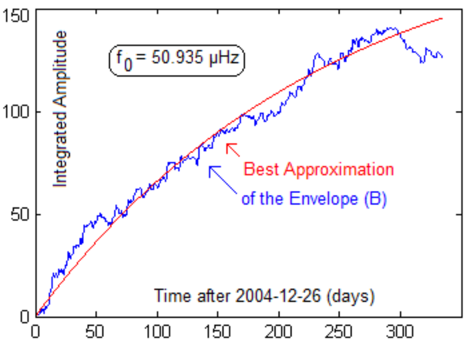
The following figure shows the output signal (B) of the integrating detector with the resonance frequency 50.935 µHz. The surprisingly large increase of the amplitudes can be explained by the extraordinarily high phase consistency of the exciting signal. Because of the very long integration time, the process is very sensitive to tiny frequency deviations. A change of only ± 0.003 µHz reduces the integrated amplitude by about 10%.



It can be seen that the mysterious vibration near 50.935 µHz has obviously been fueled twice: in March 2004 by a weak earthquake and in December 2004 by a much stronger one. Because of the better SNR, only the latter is examined.

Analysis of the period after 2004-12-26

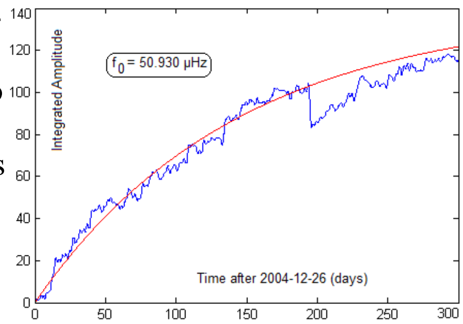
The resonant data integration starts 518459 minutes after the 2004-01-01 and yields the illustrated blue envelope of the integrated amplitude (B).



The red curve is the best approximation during the first 300 days after the earthquake, followed by disturbances because of the bad SNR. The integrated amplitude is described by:

$$y(B)=194\cdot(1-\mathrm{e}^{-t/T_0}) \quad \text{with} \quad T_0=242\,days$$

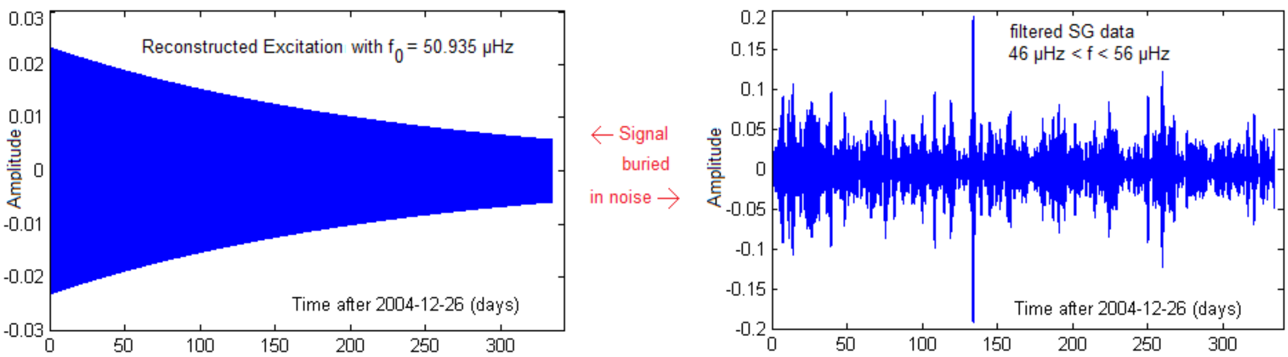
The curved envelope (B) shows that the amplitude of the exciting oscillation (A) is not constant but decreases with time. When the signal frequency deviates from the resonance frequency of the coherent detector, phase jumps of the signal can be strongly emphasized, as shown in the lower figure. In contrast to the previous picture, the earthquake on 2005-03-28 is clearly visible here. The tiny frequency deviation of only 98 ppm causes a slight drop of the integrated amplitude.



Now, for comparison and using the same program, the original data series of station H1 is replaced by a synthetic attenuated oscillation (f_0 = 50.935 µHz). The initial amplitude and the damping are varied until exactly the same output signal of the integrating detector is obtained. This synthetic oscillation is described by

$$y_{Replace}=0.0232\cdot\exp\left(\frac{-t}{T_0}\right)\cdot\sin(2\pi f_0\cdot t) \quad \text{with} \quad T_0=242\,days$$

and is shown in the left figure below. Comparing with the data, one realizes that the signal strength is well below the average noise level of the SG data, corresponding to an SNR of about -14 dB.



The damping of this mysterious oscillation on f_0 = 50.935 μHz may be characterized by the very high Q-factor $Q=\pi f T_0=3346$. Perhaps it is one of the long-sought Slichter triplet lines.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[6].

[1] Slichter, L., 1961. The fundamental free mode of the Earth’s inner core. Proc. Natl. Acad. Sci. U.S.A. 47, 186–190.
[2] https://en.wikipedia.org/wiki/Signal-to-noise_ratio#Improving_SNR_in_practice
[3] H. Weidner, Candidates for the Slichter Triplet, http://vixra.org/abs/1607.0115
[4] H. Weidner, Candidates for the Slichter Triplet II, http://vixra.org/abs/1608.0176
[5] T. Hartmann, H. Wenzel, The HW95 tidal potential catalogue, 1995
[6] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html