

# What is a photon? Photon kinetic and electromagnetic structure simplified and explained and how one photon can go through two different holes at the same time.

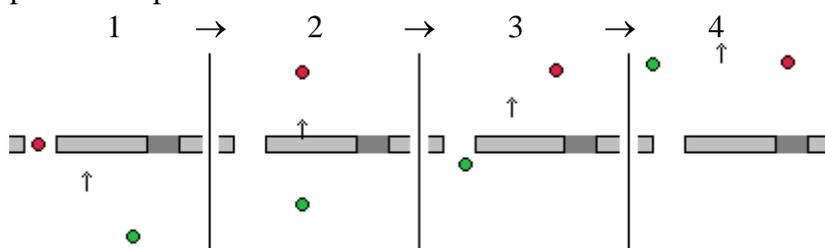
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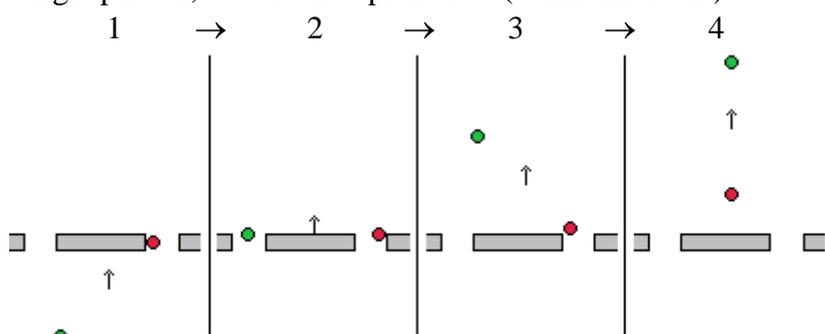
## Abstract.

To attempt to answer the question 'What is a photon?' we combine the kinetic and electromagnetic aspects of a photon and derive a straightforward picture of the photon that appears to readily explain a number of phenomena including some of the strange features of the double-slit experiment. By considering the kinetic properties of a photon first, we look at wave-particle duality from the point of view of a particle system behaving with wavelike properties as the kinetic complement of a wave-packet. We find that the photon is contained by the vacuum by a force that is more than 200 times stronger than electrostatic.

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Single photon, single slit - plan view (4 time intervals). Forward movement of the photon is upwards in each frame.



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Single photon, double slit - plan view (4 time intervals).



## Contents.

Kinetics. Photon containment 'strong force'.  
Electric and magnetic field fluctuations.  
Polarisation. Double-slit experiment. Pair production.  
Force between two electrons. Hydrogen atom.  
Fractional charges. References.

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## 1. Kinetics.

### 1.1 Photon translational and rotational/orbiting energy.

If we look at a photon as a kinetic system:  
with a rotational/orbiting component; and  
travelling at the speed of light  $c$  in a vacuum  
with relativistic mass  $m$ , equivalent to  $E/c^2$  where  $E$  is its total energy.  
then from:

$$E(\text{total}) = mc^2 \quad \text{----- (1.1a)}$$

$$E(\text{translational}) = \frac{1}{2} mc^2$$

$$E(\text{total}) = E(\text{translational}) + E(\text{rotational/orbiting})$$

we obtain:

$$E(\text{rotational/orbiting}) =$$

$$E(\text{total}) - E(\text{translational}) = mc^2 - \frac{1}{2} mc^2 = \frac{1}{2} mc^2 \quad \text{----- (1.1b)}$$

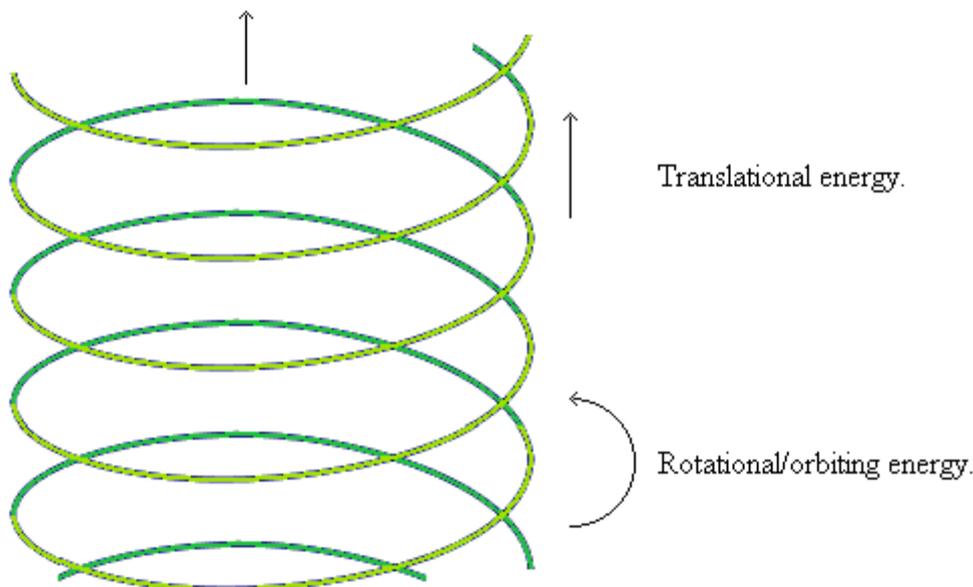


Figure 1.1a.

### 1.2 Photon translational and rotational/orbiting energy - speed, magnitude and location of rotating relativistic mass(es).

The photon as a system is travelling at the speed of light and therefore its components must all have zero rest mass. If some of the components are rotating/orbiting about a centre of mass they must also have zero rest mass and therefore their speed of rotation/orbiting must also be  $c$ , the speed of light in a vacuum.

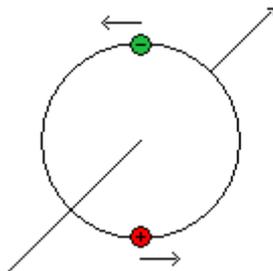
This is borne out by equation (1.1b), since the kinetic energy of a rotating/orbiting mass or masses is  $\frac{1}{2} \times \text{mass} \times (\text{speed of rotation/orbiting})^2$  and equation (1.1b) identifies the speed as  $c$ , the speed of light in a vacuum.

There can be a number of rotating relativistic masses since their individual masses

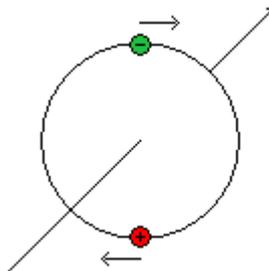
( $m_i$ ) can sum to the total mass ( $m$ ), that is  $m = \sum m_i$ .

However to comply with the electromagnetic properties of a photon the simplest solution is to have two equal masses carrying opposite charges.

Schematic diagrams of left-handed and right-handed photons showing only one of the infinite number of possible orientations of the orbit.



Left-handed photon.  
Figure 1.2a.



Right-handed photon.  
Figure 1.2b.

### 1.3 Photon translational and rotational/orbiting energy - assumptions on the rotating/orbiting relativistic mass(es).

In this paper it will be assumed that:

(a)  $m$  is composed of two identical relativistic masses, therefore  $m = \frac{1}{2}m + \frac{1}{2}m$ ; and

(b) the two masses rotate/orbit at the same average distance ( $r$ ) from the centre of revolution. Because of quantum uncertainty the distance  $r$  should not be regarded as rigidly fixed; and

(c) the two masses are at opposite ends of a diameter (diametrically opposite one another) in order to have the centre of mass of the system at the centre of revolution/orbit; and

(d) they possess equal and opposite charges of magnitude the same as the electron/proton; and

(e) neither one rotates/spins about its own axis.

The orientation of the plane of rotation does not have to be at right angles to the direction of propagation of the photon as indicated in Figures 1.2a and 1.2b. It can be in any one of an infinite number of possible orientations.

1.4 Check on the combined internal and external speeds of the photon and it's components to show that the speed of light is not exceeded.

Frames of reference.

'Stationary' observer E and travelling photon (E's frame of reference).

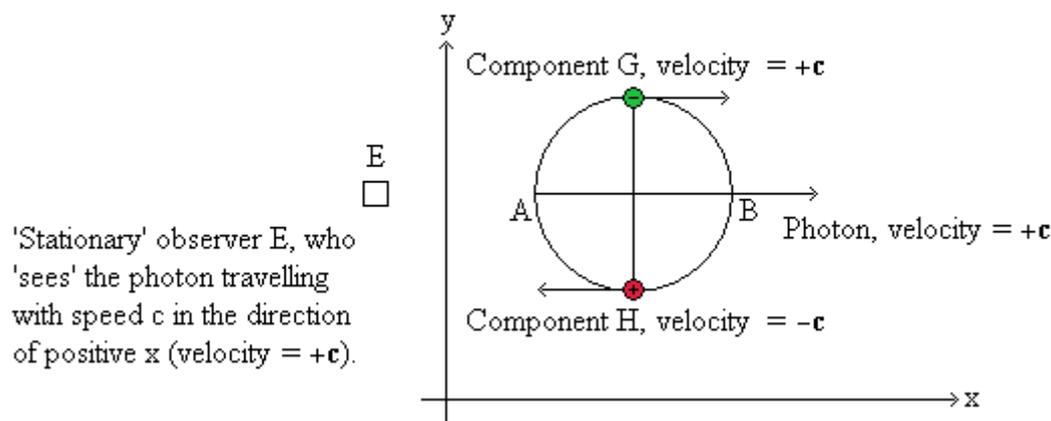


Figure 1.4a.

Consider, for example, when a diameter (AB in Figure 1.4a) of the plane of the orbit of the photon's components is parallel to the direction of the photon's propagation and the components are at right angles to the centre of that diameter.

All velocities are shown with reference to the positive x direction, so a velocity of  $-c$  is actually a speed of  $c$  in the negative x direction.

Photon's frame of reference.

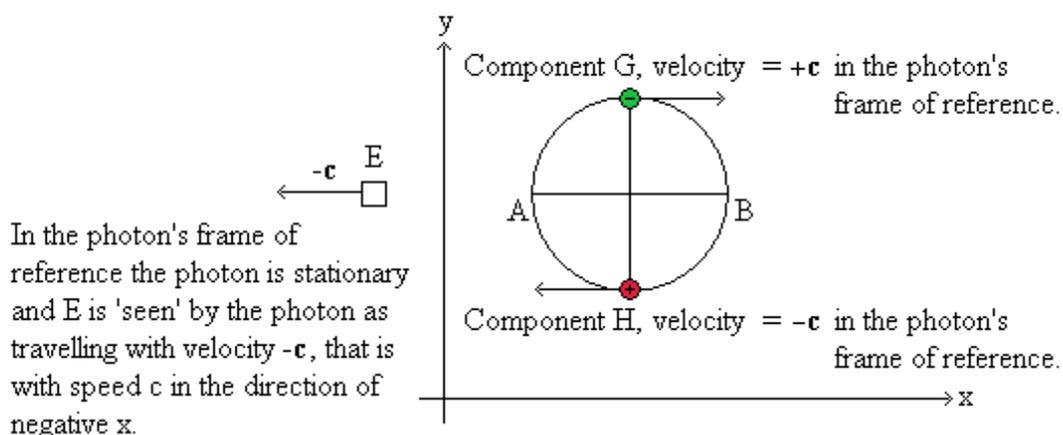


Figure 1.4b.

### Calculation.

What is the velocity of the photon component G when viewed by the 'stationary' observer E, not travelling with the photon? This is shown as  $V_x'$  in the calculation below.

The following calculation [*Reference 1*] shows that the speed of light ( $c$ ) is not exceeded.

Velocity of Component G in the photon's frame of reference =  $V_x$  ( $= +c$ )

Velocity of Component G in E's frame of reference =  $V_x'$

Velocity of E's frame of reference with respect to the photon's frame of reference =  $v$  ( $= -c$ )

The formula to obtain  $V_x'$  from  $V_x$  and  $v$  is:

$$V_x' = (V_x - v)/(1 - (v/c^2)V_x)$$

$$V_x' = (+c - (-c))/(1 - ((-c)/c^2)(+c))$$

$$V_x' = (c + c)/(1 - ((-c^2)/c^2))$$

$$V_x' = (2c)/(1 - (-1))$$

$$V_x' = (2c)/(1 + 1)$$

$$V_x' = (2c)/2 = c$$

So the component G, travelling with velocity  $+c$  in the photon's frame of reference and is part of the photon which is also travelling with velocity  $+c$  with respect to the 'stationary' observer E has a velocity  $V_x' = c$  as seen by E and therefore the speed of light is not exceeded.

### 1.5 Frequency of the photon's components.

With the frequency  $\nu$ , of rotation of an orbiting object; and  $\tau$ , the period of revolution, the time taken to complete one orbit is:

$$\nu = 1/\tau$$

If one of the photon's masses completes a full orbit in a time  $\tau$ , then from:  
circular orbit distance = speed x circular orbit time:

$$2\pi r = c\tau$$

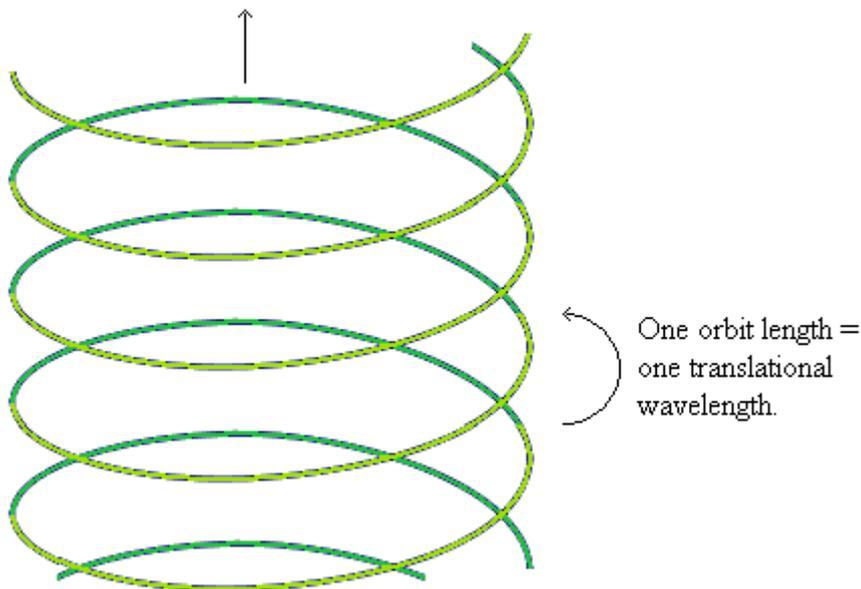
$$\Rightarrow \tau = 2\pi r/c$$

$$\nu = (1/\tau) = 1/(2\pi r/c) = c/2\pi r$$

$$\Rightarrow \nu \times 2\pi r = c \quad \text{----- (1.5a)}$$

For light: frequency x wavelength = speed, or in symbols:  $\nu\lambda = c$

From equation (1.5a) the wavelength  $\lambda = 2\pi r$ , the distance travelled in one revolution.  
So one wavelength of the photon matches exactly one revolution of the rotating masses. Translational wavelength,  $\lambda =$  rotational wavelength,  $\lambda = 2\pi r$



Only one component is shown in this figure. For two orbiting components the figure would show a double helix.

Figure 1.5a.

### 1.6 Planck's constant and photon angular momentum.

The total energy of a photon  $E(\text{total})$  is proportional to the photon's frequency  $\nu$ . The constant of proportionality is  $h$ , Planck's constant (also called the Planck constant):  
 $E(\text{total}) = h\nu$  ----- (1.6a)

Combining this with equation (1.1a)

$E(\text{total}) = mc^2$  gives:

$$E(\text{total}) = mc^2 = h\nu$$
 ----- (1.6b)

If we consider an orbiting object which does not rotate/spin about its own axis (condition 1.3 (e)):

$m$  = mass of orbiting (and non-spinning) object;

$v$  = speed of orbiting object;

$r$  = distance of object from its centre of orbit

then:

angular momentum =  $mvr = mcr$  if the speed is  $c$ , the speed of light in a vacuum.

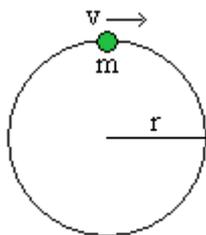


Figure 1.6a.

If we assume that we can model a photon as having orbiting relativistic mass or masses then we can put the energy of a photon into a form containing the frequency,  $\nu$ .

We need:

angular speed  $\omega = 2\pi \times \text{frequency}$  ( $\omega = 2\pi\nu$ )

speed  $v = \text{radius} \times \text{angular speed}$  ( $v = r\omega$ ); and

so with  $v = c$  we obtain:  $c = r\omega = r2\pi\nu$

and therefore  $mc^2 = mcc = mcr2\pi\nu$

Comparing this with equation (1.6b):  $mc^2 = h\nu$

$$mcr2\pi\nu = h\nu$$

$$\Rightarrow mcr2\pi = h$$

$$\Rightarrow mcr = h/2\pi$$
 ----- (1.6c)

which is the established spin (angular momentum) of a photon.

Planck's constant,  $h$  is referred to as a quantum of action in quantum mechanics and has the same dimensions as angular momentum for a rotating object, so  $h/2\pi$  also has the dimensions of angular momentum.

1.7 Photon angular momentum divided between two equal relativistic masses.

For two equal relativistic masses each of mass  $\frac{1}{2}m$ , equation (1.6c) needs to be modified to show the individual angular momentum of each of the two equal masses.

Total angular momentum:  $mcr = h/2\pi$

Angular momentum of one mass:  $(\frac{1}{2}m)cr = \frac{1}{2}(mcr) = \frac{1}{2}(h/2\pi) = h/4\pi$  ----- (1.7a)

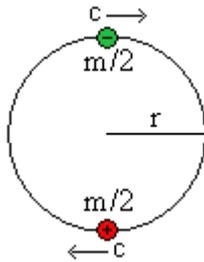


Figure 1.7a.

## 2. Photon angular momentum and whether components rotate/spin.

### 2.1 Photon angular momentum and whether components rotate/spin - introduction.

Equation 1.6c shows the angular momentum of the photon components as:

$$mcr = h/2\pi$$

A fuller formula for the total angular momentum  $L$ , of a mass  $m$ , travelling at speed  $v$ , a distance  $r$  from the point about which it orbits also includes a term for the rotation of the mass.

This term  $I\omega$  depends on the moment of inertia  $I$ , of the object and the angular speed  $\omega$ , of its rotation.

So the total angular momentum of the object is

$$L = mvr + I\omega \quad \text{-----}(2.1a)$$

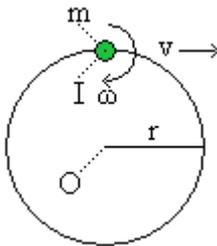


Figure 2.1a.

$I\omega$  is just an alternative way of writing mass x speed x radius for the object spinning about its own axis. We can use  $MVR$  for the angular momentum of the object due to it spinning to avoid confusion with  $mvr$  for the object orbiting about the point O in figure 2.1a.

$$MVR = M \times R\omega \times R = MR^2\omega$$

since speed,  $V = \text{radius} \times \text{angular speed} = R\omega$

There is not a single radius  $R$  for a composite object unless it is, for example, a ring of infinitely small width.

So we need to add up all the individual masses x their radii from the centre of spin to get the moment of inertia  $I$ .  $\omega$  is the same for all the parts of the spinning object if it is rigid.

$$I = MR^2 = \sum M_i R_i^2$$

$$\text{So } MVR = MR^2\omega = \sum M_i R_i^2 \omega = I\omega$$

2.2 Photon angular momentum and whether components rotate/spin - comparing full angular momentum formula with actual angular momentum.

If we compare equation 2.1a (with  $v = c$  for the photon components) with equation 1.6c the established angular momentum of a photon:

$$L = mcr + I\omega \quad \text{-----(2.1a)}$$

$$L = mcr = h/2\pi \quad \text{-----(1.6c)}$$

we get

$$mcr + I\omega = mcr$$

therefore  $I\omega = 0$  and either  $I = 0$  or  $\omega = 0$  or both  $I = 0$  and  $\omega = 0$

Check to see if  $I = 0$ :

Each photon component has mass  $m/2$  where  $m =$  the photon's mass equivalent. If  $m$  is not equal to zero, then all of the mass parts  $M_i$  in the formula  $I = \sum M_i R_i^2$  cannot also be zero. The only way left for  $I$  to be zero would be for the component to have no physical size and therefore all  $R_i = 0$ . If we rule out the possibility of a physical particle having no size then we are left with  $\omega = 0$

For no angular speed we need each component to point in the same direction as the radius from the centre of its orbit so that it is not spinning on its own axis as it moves in its circular orbit. In this way an observer travelling with the particle 'sees' it as not spinning.

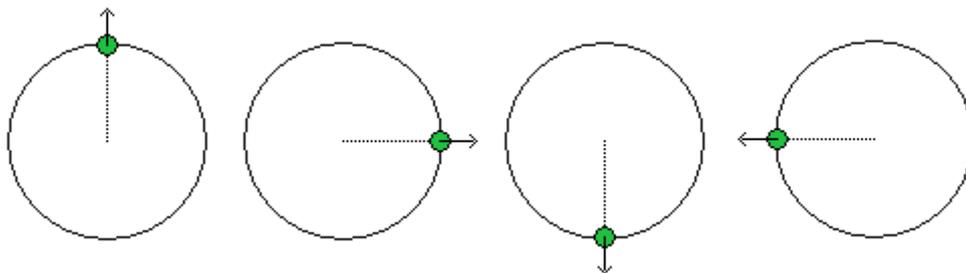


Figure 2.2a.

Figure 2.2b.

Figure 2.2c.

Figure 2.2d.

So each component has a constant fixed direction parallel to the radius from the centre of its orbit to its present position. If each component is asymmetric, does this mean that one part of the component is key to interacting with the vacuum in the process of containment?

As an example of  $\omega$  not equal to 0 ( $\omega \neq 0$ ), a component with a fixed direction pointing upwards in the above diagrams it would actually appear to an observer travelling with it to spin once on its own axis in one complete revolution around its orbit.

### 3. Force containing one of the photon masses.

#### 3.1 Photon component outward force.

The outward (centrifugal) force of a particle of mass  $\frac{1}{2}m$  travelling at speed  $c$  in an orbit of radius  $r$  is:

$$((\frac{1}{2}m)c^2)/r$$

From equation (1.7a):

$$(\frac{1}{2}m)cr = h/4\pi$$

$$((\frac{1}{2}m)cr)/r^2 = (h/4\pi)/r^2$$

$$((\frac{1}{2}m)c)/r = h/(4\pi r^2)$$

So the outward force of one component is:

$$((\frac{1}{2}m)c^2)/r = (hc)/(4\pi r^2) \quad \text{----- (3.1a)}$$

The total centrifugal force (of both particles) is:

$$(mc^2)/r = (hc)/(2\pi r^2)$$

### 4. Force between photon components allowing for light speed force transmission time.

#### 4.1 Force between photon components allowing for light speed force transmission time - geometry.

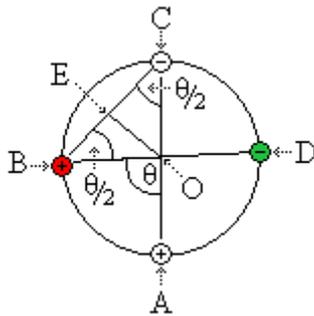


Figure 4.1a.

In Figure 4.1a the positive and negative components are at A and C at some time. In the time that the positively charged photon component has travelled a curved distance from A to B at the speed of light, a force can have travelled at the speed of light from C to B.

$$\text{radius} = r = OA = OB$$

$$\angle AOB = \theta$$

curved distance  $AB = r\theta$  where  $\theta$  is in radians

$$\angle BOC = \pi - \theta$$

$$\angle BEO = \angle CEO = \pi/2 \quad (\text{by construction})$$

$$\angle BOE = \angle COE = (\angle BOC)/2 = (\pi - \theta)/2$$

$$\angle OBE = \angle OCE = (\pi - \angle BEO - \angle BOE) = (\pi - \pi/2 - ((\pi - \theta)/2)) = (\pi/2 - \pi/2 + \theta/2)$$

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$$\angle OBE = \angle OCE = \theta/2$$

$$\begin{aligned} BE/OB &= BE/r = \cos(\angle OBE) = \cos(\theta/2) \\ BE &= r \cos(\theta/2) \end{aligned}$$

Similarly

$$\begin{aligned} CE/OC &= CE/r = \cos(\angle OCE) = \cos(\theta/2) \\ CE &= r \cos(\theta/2) \end{aligned}$$

$$CB = CE + EB = 2r \cos(\theta/2)$$

*4.2 Force between photon components allowing for light speed force transmission time - calculating  $\theta$ .*

CB = curved length AB when:

$$2r \cos(\theta/2) = r\theta$$

$$\Rightarrow \cos(\theta/2) = (\theta/2)$$

by tabulating  $\cos(\theta/2)$  and  $(\theta/2)$  we see that they are approximately equal when:

$$\cos(\theta/2) = (\theta/2) \approx 0.739$$

$$(\theta/2) \approx 0.739 \approx 42.35^\circ$$

$$\text{and therefore: } \theta \approx 1.478 \approx 84.69^\circ$$

## 5. Force containing one of the photon masses compared with the electrostatic force between two static charges.

### 5.1 Force containing one of the photon masses compared with the electrostatic force between two static charges - instantaneous transmission.

(In section 5.2 we perform an alternative calculation - allowing for light speed force transmission time.)

Coulomb's law describes the electrostatic force  $F$ , between two electrically charged particles. Using  $+e$  and  $-e$ , (where  $e$  is the magnitude of the electron's charge) on two particles a distance  $2r$ , apart in vacuum and where  $\epsilon_0$  is the permittivity of free space, Coulomb's law becomes:

$$F = -e^2/(4\pi\epsilon_0(2r)^2) \quad \text{----- (5.1a)}$$

The minus sign in front of the expression shows us that the force is attractive.

The outward (centrifugal) force of one photon component is equation (3.1a):  $((1/2m)c^2)/r = (hc)/(4\pi r^2)$  This is balanced by an equal and opposite inward (centripetal) force and we can compare the inward force with the electrostatic force between the two components with electrostatic charges  $+e$  and  $-e$  a distance  $2r$  apart as in equation (5.1a)

$$F = -e^2/(4\pi\epsilon_0(2r)^2) \text{ as if the photon components were stationary.}$$

The force is fictional because in reality the components are not static and there isn't the time for a signal to be communicated at or below the speed of light between the charges when they are a distance  $2r$  apart.

Special relativity theory assumes that no signal can travel faster than the speed of light in a vacuum. A signal from one photon component cannot reach the other photon component instantaneously but if we compare the fictitious force with the actual outward force on one photon component we obtain a straightforward equation.

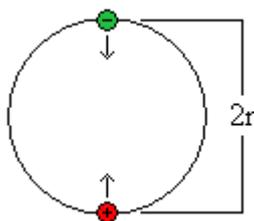


Figure 5.1a.

To compare the outward force  $(hc)/(4\pi r^2)$  with the magnitude of the electrostatic force  $e^2/(4\pi\epsilon_0(2r)^2)$  we manipulate the  $(hc)/(4\pi r^2)$  as follows:

$$\begin{aligned} (hc)/(4\pi r^2) &= [(4\epsilon_0 e^2)(hc)]/[(4\epsilon_0 e^2)(4\pi r^2)] = [2(2\epsilon_0 hc/e^2)] \times [e^2/(4\pi\epsilon_0(2r)^2)] \\ (hc)/(4\pi r^2) &= (2/\alpha) \times [e^2/(4\pi\epsilon_0(2r)^2)] \quad \text{----- (5.1b)} \end{aligned}$$

where  $\alpha$  is the fine structure constant.

$$\alpha = e^2/(2\epsilon_0 hc) \approx 1/(137.036) \quad 1/\alpha = (2\epsilon_0 hc/e^2) \approx 137.036$$

$\alpha$  is also called Sommerfeld's constant and it defines the strength of the

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electromagnetic interaction.

In summary:

$$(hc)/(4\pi r^2) = (2/\alpha) \times [e^2/(4\pi\epsilon_0(2r)^2)] \approx (2 \times 137.036) \times (e^2)/(4\pi\epsilon_0(2r)^2)$$

So a photon component is subject to an inward force of approximately 2x137 times stronger than electrostatic.

It is interesting to note the relative strengths of the four fundamental interactions of nature from strongest to weakest: Strong nuclear : Electromagnetic : Weak nuclear : Gravity.

5.2 Force containing one of the photon masses compared with the electrostatic force between two static charges - allowing for light speed force transmission time.

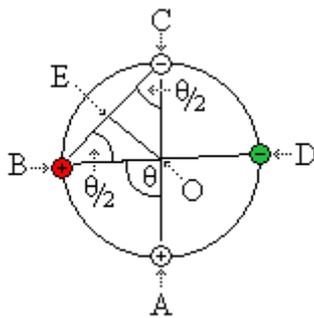


Figure 4.1a.

When the positive photon component is at B in Figure 4.1a it can receive the 'electrostatic' force that has come from the negative photon component when it was at C. From section 4.1:  $CB = CE + EB = 2r \cos(\theta/2)$  and  $\cos(\theta/2) \approx 0.739$

'Electrostatic' force in the direction BC:

$$F = -e^2/(4\pi\epsilon_0(CB)^2) = -e^2/(4\pi\epsilon_0(2r \cos(\theta/2))^2) = -e^2/(4\pi\epsilon_0(2r)^2 \times (\cos(\theta/2))^2)$$

'Electrostatic' force from point B towards the centre of orbit:

$$F = -e^2/(4\pi\epsilon_0(CB)^2) \times \cos(\angle CBO) = -e^2/(4\pi\epsilon_0(CB)^2) \times \cos(\theta/2)$$

$$F = [-e^2/(4\pi\epsilon_0(2r)^2 \times (\cos(\theta/2))^2)] \times \cos(\theta/2)$$

$$F = [-e^2/(4\pi\epsilon_0(2r)^2)] / \cos(\theta/2)$$

$$F \approx [-e^2/(4\pi\epsilon_0(2r)^2)] / 0.739$$

$$F \approx (1/0.739) (-e^2/(4\pi\epsilon_0(2r)^2))$$

$$F \approx 1.353 (-e^2/(4\pi\epsilon_0(2r)^2)) \quad \text{----- (5.2a)}$$

Using Equation 5.1b:

$$(hc)/(4\pi r^2) = (2/\alpha) \times [e^2/(4\pi\epsilon_0(2r)^2)] = (2/\alpha) \times [e^2/(4\pi\epsilon_0(2r)^2)] \times (1.353/1.353)$$

$$(hc)/(4\pi r^2) \approx (1/1.353) \times (2/\alpha) \times [1.353 \times e^2/(4\pi\epsilon_0(2r)^2)]$$

So a photon component is subject to an inward force of approximately  $(1/1.353) \times (2/\alpha) \approx (2/1.353) \times 137 \approx$  1.478 x 137 times stronger than the electrostatic force assuming force transmission speed to comply with special relativity theory.

## 6. Force containing one of the photon masses compared with the magnetic force between two moving charges.

### 6.1 Force containing one of the photon masses compared with the magnetic force between two moving charges - magnetic field of a moving charge.

Please note that symbols in this and other sections written in **bold** type denote vectors.

**B** = magnetic field vector (magnetic flux density vector) at a point a distance  $r_1$  and direction vector  $\mathbf{r}_1$  away from a charge  $q_1$  moving with velocity  $\mathbf{v}_1$  and  $\mu_0$  is the permeability of free space.

$$\mathbf{B} = (\mu_0/4\pi) \times (q_1 \mathbf{v}_1 \times \mathbf{r}_1 / r_1^3) \quad \text{----- (6.1a)}$$

In SI units **B** is measured in  $\text{Wb m}^{-2} = (\text{kg m}^2 \text{C}^{-1} \text{s}^{-1}) \text{m}^{-2} = \text{kg C}^{-1} \text{s}^{-1} = \text{T}$  where C = Coulomb, T = Tesla, Wb = Weber,  $\mathbf{x}$  in bold type denotes the vector cross product.

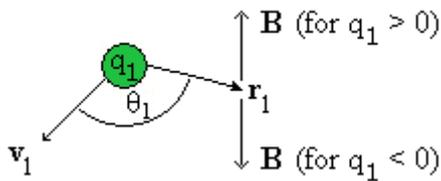


Figure 6.1a.

$\mathbf{v}_1 \times \mathbf{r}_1$  is the vector cross product which takes account of the magnitude and direction of both  $\mathbf{v}_1$  and  $\mathbf{r}_1$  and the angle  $\theta_1$  between them. It also defines a vector at right angles (in a right handed sense) to both  $\mathbf{v}_1$  and  $\mathbf{r}_1$  to show the direction of **B**.  $r_1$  is the magnitude of the vector  $\mathbf{r}_1$  and denotes the distance, but not the direction of the point where **B** is to be evaluated, from the charge  $q_1$ .

Without using vectors the above equation becomes:

$$B = (\mu_0/4\pi) \times ((q_1 v_1 \sin\theta_1) / r_1^2) \quad \text{----- (6.1b)}$$

where B is now the magnitude of **B**.

6.2 Force containing one of the photon masses compared with the magnetic force between two moving charges - force on a charged particle moving in a magnetic field.

If a charge  $q_2$  is moving with velocity  $\mathbf{v}_2$  in a magnetic flux density of strength and direction  $\mathbf{B}$ , then the force  $\mathbf{F}$  on  $q_2$  is given by:

$$\mathbf{F} = q_2 (\mathbf{v}_2 \times \mathbf{B}) \quad \text{----- (6.2a)}$$

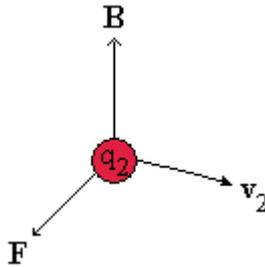


Figure 6.2a.

6.3 Force containing one of the photon masses compared with the magnetic force between two moving charges - force on one charged photon component moving in the magnetic field due to the other photon component - instantaneous transmission.

(In section 6.5 we perform an alternative calculation - allowing for light speed force transmission time).

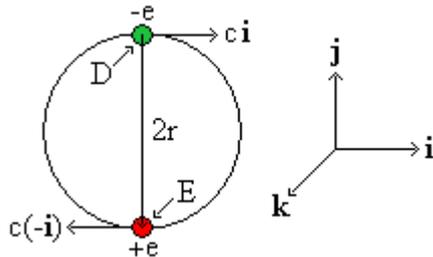


Figure 6.3a.

In Figure 6.3a  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  are unit (magnitude = 1) direction vectors at right angles to each other.

To find the magnetic flux density vector  $\mathbf{B}$  at the place (point E) where  $+e$  is situated due to the charge  $-e$  (at point D at the top of Figure 6.3a.) we need:

$$q_1 = -e, \mathbf{v}_1 = c\mathbf{i}, \mathbf{r}_1 = 2r(-\mathbf{j}), \quad r_1 = 2r$$

So:

$$\mathbf{B} = (\mu_0/4\pi) \times (q_1 \mathbf{v}_1 \times \mathbf{r}_1 / r_1^3) \quad \text{----- (6.1a)}$$

$$\mathbf{B} = (\mu_0/4\pi) \times (-e (c\mathbf{i} \times 2r(-\mathbf{j})) / (2r)^3)$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-ec)/(2r)^2)(\mathbf{i} \times (-\mathbf{j}))$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-ec)/(2r)^2)(-\mathbf{k})$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((ec)/(2r)^2)(\mathbf{k}) \quad \text{----- (6.3a)}$$

To find the force  $\mathbf{F}$ , on the charge  $+e$  at point E in Figure 6.3a we need:

$$q_2 = +e, \mathbf{v}_2 = c(-\mathbf{i})$$

So:

$$\mathbf{F} = q_2 (\mathbf{v}_2 \times \mathbf{B}) \quad \text{----- (6.2a)}$$

$$\mathbf{F} = e (c(-\mathbf{i}) \times (\mu_0/4\pi) \times ((ec)/(2r^2))(\mathbf{k}))$$

$$\mathbf{F} = (\mu_0/4\pi) ((ec)^2/(2r^2)) ((-\mathbf{i}) \times (\mathbf{k}))$$

$$\mathbf{F} = (\mu_0/4\pi) ((ec)^2/(2r^2)) (-\mathbf{i} \times \mathbf{k})$$

$$\mathbf{F} = (\mu_0/4\pi) ((ec)^2/(2r^2)) (-(-\mathbf{j}))$$

$$\mathbf{F} = (\mu_0/4\pi) ((ec)^2/(2r^2))\mathbf{j} \quad \text{----- (6.3b)}$$

So there is a fictitious attractive force in the direction of positive  $\mathbf{j}$  caused by the charge  $-e$  on the charge  $+e$ . It is fictitious because the signal between the charges cannot travel fast enough. (In section 6.5 we perform an alternative calculation - allowing for light speed force transmission time). As with the comparison with the electrostatic charge discussed in section 5, special relativity theory assumes that no signal can travel faster than the speed of light in a vacuum. A signal from point D cannot reach point E instantaneously but if we compare the fictitious force with the actual outward force on one photon component we obtain a straightforward equation.

*6.4 Force containing one of the photon masses compared with the magnetic force between two moving charges - magnetic force compared with outward (centrifugal) force.*

The outward (centrifugal) force on one photon component is equation (3.1a):

$$((1/2m)c^2)/r = (hc)/(4\pi r^2)$$

This is balanced by an equal and opposite inward (centripetal) force which we can compare with a fictional magnetic force between the two components with electric charges  $-e$  and  $+e$  a distance  $2r$  apart as in equation (6.3b):  $\mathbf{F} = (\mu_0/4\pi) ((ec)^2/(2r^2))\mathbf{j}$

The magnitude of the force in equation 6.3b can be rearranged to:

$$F = (\mu_0/4\pi) ((ec)^2/(2r^2))$$

$$F = (e^2 (\mu_0 c^2))/(4\pi(2r^2))$$

$$F = (e^2 (\mu_0(1/(\mu_0 \epsilon_0))))/(4\pi(2r^2)) \quad [\text{using } c^2 = 1/(\mu_0 \epsilon_0)]$$

$$F = e^2/(4\pi \epsilon_0 (2r)^2)$$

This is the same magnitude as the fictitious electrostatic force derived in section 5 and shown in Equation 5.1a, so the outward force on a photon component is balanced by an inward force 2x137 times stronger than magnetic:

We can also show that the outward force on a photon component is  $(2/\alpha \approx 2 \times 137)$  times stronger than the magnitude of Equation 6.3b as follows:

$$(hc)/(4\pi r^2) = [(4hc)/(4\pi(2r)^2)] \times [e^2/e^2] \times [(c^2/c^2)]$$

$$(hc)/(4\pi r^2) = [2 \times (2hc/e^2)] \times [1/(4\pi(2r)^2)] \times [e^2] \times [(\mu_0 \epsilon_0)(c^2)] \quad [\text{using } c^2 = 1/(\mu_0 \epsilon_0)]$$

$$(hc)/(4\pi r^2) = [2 \times (2\epsilon_0 hc/e^2)] \times (\mu_0/4\pi) \times ((e^2 c^2)/(2r)^2)$$

$$(hc)/(4\pi r^2) = (2/\alpha) \times (\mu_0/4\pi) \times ((e^2 c^2)/(2r)^2) \quad [\text{using } 1/\alpha = (2\epsilon_0 hc/e^2) \approx 137.036]$$

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6.5 Force containing one of the photon masses compared with the magnetic force between two moving charges - allowing for light speed force transmission time - geometry.

We start by referring back to section 4 (Force between photon components allowing for light speed force transmission time) with this modified diagram to show additional lines:

BG perpendicular to AC; and

GB extended to point I; and

the tangent BH, perpendicular to the radius OB, showing the direction of the velocity vector of the positive photon component at the point B; and

BJ perpendicular to IBG.

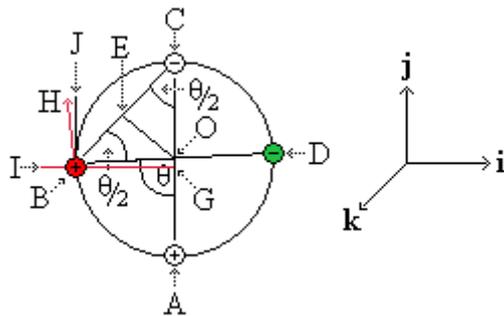


Figure 6.5a.

radius =  $r = OA = OB$

From section 4.1:  $CB = CE + EB = 2r \cos(\theta/2)$

$\angle AOB = \theta$

$\angle BGO = \pi/2$  by construction of BG perpendicular to AC

$\angle GBO + \angle AOB + \angle BGO = \pi$  (sum of angles in a triangle)

$\angle GBO + \theta + \pi/2 = \pi$

$\angle GBO = \pi - \theta - \pi/2 = \pi/2 - \theta$

$\angle IBJ = \pi/2$

$\angle GBJ = \pi/2$

$\angle GBC = \angle GBO + \angle OBC = (\pi/2 - \theta) + \theta/2 = \pi/2 - \theta/2$

$\angle CBJ = \angle GBJ - \angle GBC = \pi/2 - (\pi/2 - \theta/2) = \pi/2 - \pi/2 + \theta/2 = \theta/2$

$\angle GBJ = \pi/2$

$\angle OBJ = \angle GBJ - \angle GBO = \pi/2 - (\pi/2 - \theta) = \pi/2 - \pi/2 + \theta = \theta$

$\angle HBO = \pi/2$

$\angle HBJ = \angle HBO - \angle OBJ = \pi/2 - \theta$

$\angle IBH = \angle IBJ - \angle HBJ = \pi/2 - (\pi/2 - \theta) = \pi/2 - \pi/2 + \theta = \theta$

The vector **BH** points in the direction:

$-\mathbf{i} \cos(\angle IBH) + \mathbf{j} \cos(\angle HBJ) = -\mathbf{i} \cos(\theta) + \mathbf{j} \cos(\pi/2 - \theta) = -\mathbf{i} \cos(\theta) + \mathbf{j} \sin(\theta)$

The velocity vector of the positive photon component at B is:  
 $c \times \mathbf{BH} = c(-\mathbf{i} \cos(\theta) + \mathbf{j} \sin(\theta))$

The vector  $\mathbf{CB} = \mathbf{CG} + \mathbf{GB}$

$$\mathbf{CB} = -\mathbf{j} \text{ CG} - \mathbf{i} \text{ GB} = -\mathbf{j} (\text{CO} + \text{OG}) - \mathbf{i} \text{ GB} = -\mathbf{j} (r + r \cos(\angle \text{AOB})) - \mathbf{i} r \cos(\angle \text{GBO})$$

$$\mathbf{CB} = -\mathbf{j} (r + r \cos(\theta)) - \mathbf{i} r \cos(\pi/2 - \theta) = -\mathbf{i} r \sin(\theta) - \mathbf{j} (r + r \cos(\theta))$$

6.6 Force containing one of the photon masses compared with the magnetic force between two moving charges - allowing for light speed force transmission time - magnetic flux density vector calculation.

$\mathbf{B}$  = magnetic field vector (magnetic flux density vector) at a point a distance  $r_1$  and direction vector  $\mathbf{r}_1$  away from a charge  $q_1$  moving with velocity  $\mathbf{v}_1$  and  $\mu_0$  is the permeability of free space.

$$\mathbf{B} = (\mu_0/4\pi) \times (q_1 \mathbf{v}_1 \times \mathbf{r}_1 / r_1^3) \quad \text{----- (6.1a)}$$

For the set up in this section (with  $q_1$  being the negative photon component when it was at C):

$$q_1 = -e$$

$$\mathbf{v}_1 = \mathbf{i} c$$

$$\mathbf{r}_1 = \mathbf{CB} = -\mathbf{i} r \sin(\theta) - \mathbf{j} (r + r \cos(\theta))$$

$$r_1 = \text{CB} = 2r \cos(\theta/2)$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-e) \times (\mathbf{i} c) \times [-\mathbf{i} r \sin(\theta) - \mathbf{j} (r + r \cos(\theta))]/[2r \cos(\theta/2)]^3)$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-ec) \times [-\mathbf{i} \times \mathbf{i} r \sin(\theta) - \mathbf{i} \times \mathbf{j} r(1 + \cos(\theta))]/[2r \cos(\theta/2)]^3)$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-ec) \times [-\mathbf{k} \times r(1 + \cos(\theta))]/[2r \cos(\theta/2)]^3) \text{ as } \mathbf{i} \times \mathbf{i} = \mathbf{0} \text{ and } \mathbf{i} \times \mathbf{j} = \mathbf{k}$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((-ec) \times [-\mathbf{k} \times r(1 + \cos(\theta))]/[2r \times (2r)^2 \times \cos^3(\theta/2)])$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((ec) \times [\mathbf{k} \times (1 + \cos(\theta))]/[2 \times (2r)^2 \times \cos^3(\theta/2)])$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((ec)/(2r)^2) \times [(1 + \cos(\theta))]/[2\cos^3(\theta/2)] \mathbf{k} \quad \text{----- (6.6a)}$$

6.7 Force containing one of the photon masses compared with the magnetic force between two moving charges - allowing for light speed force transmission time - magnetic force calculation.

Magnetic force:

$$\mathbf{F} = q_2 (\mathbf{v}_2 \times \mathbf{B}) \quad \text{----- (6.2a)}$$

For the set up in this section:

$$q_2 = +e$$

$$\mathbf{v}_2 = c(-\mathbf{i} \cos(\theta) + \mathbf{j} \sin(\theta)) \quad \text{from section 6.5}$$

$$\mathbf{B} = (\mu_0/4\pi) \times ((ec)/(2r)^2) \times [(1 + \cos(\theta))]/[2\cos^3(\theta/2)] \mathbf{k} \quad \text{----- (6.6a)}$$

$$\mathbf{F} = q_2 (\mathbf{v}_2 \times \mathbf{B})$$

$$\mathbf{F} = +e (c(-\mathbf{i} \cos(\theta) + \mathbf{j} \sin(\theta))) \times [(\mu_0/4\pi) \times ((ec)/(2r)^2) \times [(1 + \cos(\theta))]/[2\cos^3(\theta/2)] \mathbf{k}]$$

$$\mathbf{F} = (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [1 + \cos(\theta)]/[2\cos^3(\theta/2)] \times (-\mathbf{i} \cos(\theta) + \mathbf{j} \sin(\theta)) \times \mathbf{k}$$

$$\mathbf{F} = (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [1 + \cos(\theta)]/[2\cos^3(\theta/2)] \times (-\mathbf{i} \times \mathbf{k}) \cos(\theta) + (\mathbf{j} \times \mathbf{k}) \sin(\theta)$$

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$$\mathbf{F} = (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [1 + \cos(\theta)]/[2\cos^3(\theta/2)] \times [\mathbf{j} \cos(\theta) + \mathbf{i} \sin(\theta)]$$

$$\mathbf{F} = (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [1 + \cos(\theta)]/[2\cos^3(\theta/2)] \times [\mathbf{i} \sin(\theta) + \mathbf{j} \cos(\theta)]$$

$$\mathbf{F} \approx 1.353 \times (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [\mathbf{i} \sin\theta + \mathbf{j} \cos\theta] \quad \text{----- (6.7a)}$$

The factor  $[(1 + \cos\theta)/(2(\cos^3(\theta/2)))]$  is evaluated with  $\theta \approx 1.478$  radians  $\approx 84.69^\circ$  as follows:

$$[(1 + \cos\theta)/(2(\cos^3(\theta/2)))] \approx [(1 + 0.093)/(2 \times (0.739)^3)] \approx [1.093/(2 \times 0.404)]$$

$$\approx [1.093/0.808] \approx 1.353$$

The factor  $[\mathbf{i} \sin\theta + \mathbf{j} \cos\theta]$  is the unit vector pointing towards the centre of orbit of the photon components. As it is a unit vector it has a magnitude of 1 and shows the direction of the force without changing the size of the force.

The magnitude of a vector is found by taking the square root of the sum of the squares of the components of the vector. The components here are  $\sin\theta$  and  $\cos\theta$ , so the magnitude of the vector  $[\mathbf{i} \sin\theta + \mathbf{j} \cos\theta]$  is  $\sqrt{(\sin^2\theta + \cos^2\theta)} = \sqrt{1} = 1$  as  $(\sin^2\theta + \cos^2\theta)$  is the trigonometric identity which is equal to 1 for all  $\theta$ .

Equation 6.7a has the same magnitude as equation 5.2a - the 'electrostatic' force from point B towards the centre of orbit:

$$\mathbf{F} \approx 1.353 \times (\mu_0/4\pi) \times (ec)^2/(2r)^2 \times [\mathbf{i} \sin\theta + \mathbf{j} \cos\theta] \quad \text{----- (6.7a)}$$

$$\mathbf{F} \approx 1.353 \times (e^2 (\mu_0 c^2))/(4\pi(2r)^2) \times [\mathbf{i} \sin\theta + \mathbf{j} \cos\theta]$$

$$\mathbf{F} \approx 1.353 \times (e^2 (\mu_0(1/(\mu_0\epsilon_0))))/(4\pi(2r)^2) \times [\mathbf{i} \sin\theta + \mathbf{j} \cos\theta] \quad [\text{using } c^2 = 1/(\mu_0\epsilon_0)]$$

$$\mathbf{F} \approx 1.353 \times e^2/(4\pi\epsilon_0(2r)^2) \times [\mathbf{i} \sin\theta + \mathbf{j} \cos\theta]$$

The magnitude of the 'electrostatic' force from point B towards the centre of orbit:

$$F \approx 1.353 (-e^2/(4\pi\epsilon_0(2r)^2)) \quad \text{----- (5.2a)}$$

## 7. Force containing one of the photon masses compared with the magnetic force between two parallel electric currents.

### 7.1 Force containing one of the photon masses compared with the magnetic force between two parallel electric currents - introduction.

An electric current is caused by moving electric charges. As the supposed photon components are charged and move we can portray them as electric currents.

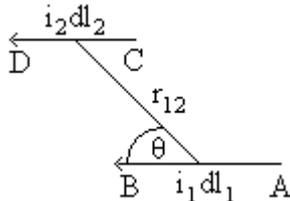


Figure 7.1a.

The magnetic force  $dF$  (on each current element) between two parallel current elements  $i_1dl_1$  and  $i_2dl_2$  separated by a distance  $r_{12}$  at an angle  $\theta$  between the transversal and the parallel current elements [*Reference 2*] is:

$$dF = (\mu_0/4\pi) (1/r_{12}^2) i_1dl_1 i_2dl_2 \sin\theta \quad \text{----- (7.1a)}$$

where:

$\mu_0$  = permeability of free space.

$r_{12}$  = distance between the two current elements.

$i_1$  = current (charge per unit time) flowing along the path AB.

$dl_1$  = length of the current element AB.

$i_2$  = current (charge per unit time) flowing along the path CD.

$dl_2$  = length of the current element CD.

$\theta$  = angle between the two current elements.

Parallel electric currents where both currents point in the same direction are attractive.

### 7.2 Force containing one of the photon masses compared with the magnetic force between two parallel electric currents - photon 'electric currents'.

The direction of an electric current is the direction of movement of positive charge. With a flow of negative charge the direction of the electric current is in the opposite direction to the flow of the negatively charged particles.

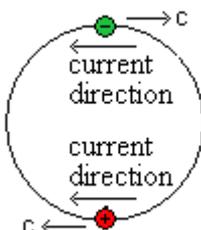


Figure 7.2a.

Treating the charged components in our picture of the photon as currents we see in Figure 7.2a that the positive charge at the bottom of the figure is travelling to the left and therefore the current direction is also to the left. The negative charge at the top of the figure is travelling to the right but because the charge is negative this current is also pointing to the left.

As a result the currents are:  
parallel; and  
pointing in the same direction; and  
the force between them is attractive.

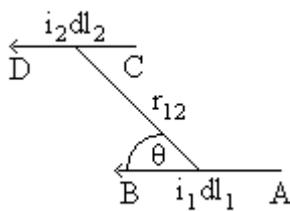


Figure 7.1a.

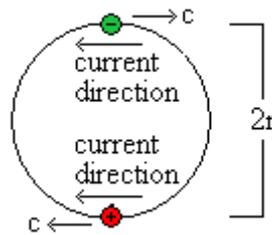


Figure 7.2a.

If we compare Figure 7.2a with Figure 7.1a we see that:

$$r_{12} = 2r$$

$$i_1 = e \text{ Coulombs}/(\text{time in seconds taken by the charge to move along a short distance AB})$$

$$= e/(AB/c) = (ec)/AB$$

since (time taken by the charge to move along a short distance AB) = (distance AB)/(speed of travel of the charge) = AB/c

$$dl_1 = AB$$

$$i_2 = e \text{ Coulombs}/(\text{time in seconds taken by the charge to move along a short distance CD})$$

$$= e/(CD/c) = (ec)/CD$$

since (time taken by the charge to move along a short distance CD) = (distance CD)/(speed of travel of the charge) = CD/c

$$dl_2 = CD$$

$$\theta = \pi/2 = 90^\circ \text{ and } \sin(\pi/2) = 1$$

In equation 7.1a:

$$dF = (\mu_0/4\pi) (1/r_{12}^2) i_1 dl_1 i_2 dl_2 \sin\theta$$

$$dF = (\mu_0/4\pi) (1/(2r)^2) ((ec)/AB) \times AB ((ec)/CD) \times CD \sin(\pi/2)$$

$$dF = (\mu_0/4\pi) (1/(2r)^2) (ec) \times (ec)$$

$$dF = (\mu_0/4\pi(2r)^2) (ec)^2 \text{ ----- (7.2a)}$$

Equation 7.2a shows the attractive force on each photon mass between the moving charged photon masses treated as electric currents.

7.3 Force containing one of the photon masses compared with the magnetic force between two parallel electric currents - calculation.

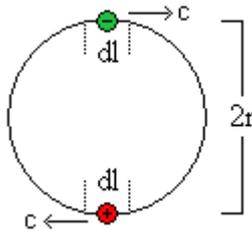


Figure 7.3a.

The force required to contain one photon component is:

$$((1/2)mc^2)/r = (hc)/(4\pi r^2) \quad \text{----- (3.1a)}$$

$$\begin{aligned} (hc)/(4\pi r^2) &= (4hc)/(4\pi(2r)^2) \times (\mu_0 \epsilon_0 e^2)/(\mu_0 \epsilon_0 e^2) \\ (hc)/(4\pi r^2) &= [2(2\epsilon_0 hc/e^2)] \times [\mu_0/(4\pi(2r)^2)] \times [e^2/\mu_0 \epsilon_0] \\ (hc)/(4\pi r^2) &= [2(1/\alpha)] \times [\mu_0/(4\pi(2r)^2)] \times (e^2 c^2) \quad \text{[using } c^2 = 1/(\mu_0 \epsilon_0)] \\ &\quad \text{and [using } 1/\alpha = (2\epsilon_0 hc/e^2) \approx 137.036] \\ (hc)/(4\pi r^2) &= (2/\alpha) \times [\mu_0/(4\pi(2r)^2)] \times (ec)^2 \end{aligned}$$

This is  $(2/\alpha \approx 2 \times 137)$  times greater than equation 7.2a (magnetic force from electric currents calculation).

So the force required to contain one photon component is  $(2/\alpha \approx 2 \times 137)$  times stronger than the force produced by the charged components treated as electric currents.

In fact it has the same value compared with the electrostatic force derived in section 5.

$$\begin{aligned} (hc)/(4\pi r^2) &= (2/\alpha) \times [\mu_0/(4\pi(2r)^2)] \times (e^2 c^2) \\ (hc)/(4\pi r^2) &= (2/\alpha) \times [\mu_0/(4\pi(2r)^2)] \times e^2 \times (1/\mu_0 \epsilon_0) \\ (hc)/(4\pi r^2) &= (2/\alpha) \times [e^2/(4\pi \epsilon_0 (2r)^2)] \\ (hc)/(4\pi r^2) &= (2/\alpha) \times [e^2/(4\pi \epsilon_0 (2r)^2)] \quad \text{----- (5.1b)} \end{aligned}$$

This is the same as equation 5.1b from the electrostatic charge calculation.

This calculation assumes instant transmission of the magnetic signal between the moving charges. Calculations allowing for light speed transmission are shown in sub-sections 5.2 and 6.5, 6.6 and 6.7.

## 8. Containing the photon masses - vacuum pressure.

### 8.1 Vacuum pressure - introduction.

To identify the force between the two photon masses we can look at containment from the outside.

The outward (centrifugal) force of a component held in an approximately circular orbit has to be balanced by an inward (centripetal) force and this force may be provided by the vacuum. The outward force redirects vacuum energy outwards and the vacuum attempts to balance the energy deficit by transferring energy inwards.

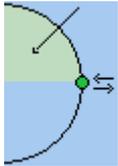


Figure 8.1a.

Two components having the same mass  $m_1$  will travel in an approximately circular path at the same radius  $r$  because that is where the force from the vacuum pressure matches the centrifugal force  $m_1c^2/r$ .

Equation 3.1a shows the force needed to contain a single photon component of mass  $\frac{1}{2}m$  travelling at speed  $c$  in a circular path of radius  $r$  as  $(hc)/(4\pi r^2)$ :

$$((\frac{1}{2}m)c^2)/r = (hc)/(4\pi r^2) \quad \text{----- (3.1a)}$$

If we re-label  $\frac{1}{2}m$  as  $m_1$  then we obtain:

$$(m_1c^2)/r = (hc)/(4\pi r^2) \quad \text{----- (8.1a)}$$

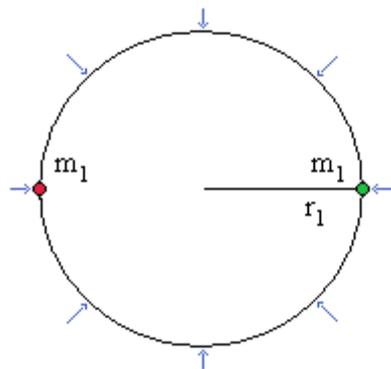


Figure 8.1b.

### 8.2 Vacuum pressure - varying the radius - specific example.

We can see from equation 8.1a that an inward vacuum force at  $r_1$  of  $F_1 = (hc)/(4\pi r_1^2)$  spread over a spherical area  $4\pi r_1^2$  gives a vacuum pressure of  $F_1/(4\pi r_1^2) = (hc)/(4\pi r_1^2)^2$

If the force  $F_1$  present at  $r_1$  continues inwards without loss of strength to a spherical region at half the radius  $r_2 = 1/2 r_1$  then it give rise to a pressure 4 times as great because the area reduces to  $1/4 (4\pi r_1^2)$

The pressure at  $r_2$  is  $F_1/(4\pi r_2^2) = F_1/(4\pi (1/2 r_1)^2) = F_1/(1/4(4\pi r_1^2)) = 4F_1/(4\pi r_1^2)$

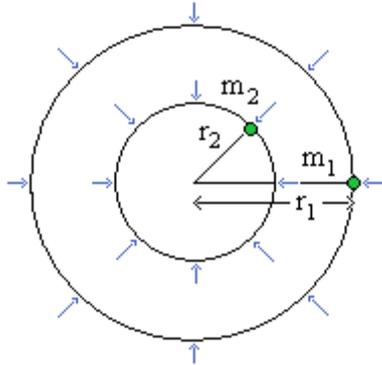


Figure 8.2a.

### 8.3 Vacuum pressure - mass dependent on radius.

The pressure needs to be 4 times as much because with the radius being half as much it means that the mass becomes twice as much from  $mr = \text{constant}$  as shown below.

Equation 1.6c states:

$$mcr = h/2\pi \quad \text{----- (1.6c)}$$

so for a single photon component of mass  $m_1 = 1/2 m$  at radius  $r_1$ :

$$m_1 r_1 = 1/2 mcr_1 = 1/2 h/2\pi = h/4\pi$$

$$m_1 r_1 = h/(4\pi c) = \text{constant} \quad (\text{since } h, \pi \text{ and } c \text{ are constants}).$$

Similarly:

$$m_2 r_2 = h/(4\pi c) = \text{constant} = m_1 r_1$$

$$m_2 r_2 = m_1 r_1$$

$$m_2 = (m_1 r_1)/r_2$$

$$\text{with } r_2 = 1/2 r_1$$

$$m_2 = (m_1 r_1)/(1/2 r_1)$$

$$m_2 = 2m_1$$

The force has to match  $(m_2 c^2)/r_2 = (2m_1 c^2)/(1/2 r_1) = 4(m_1 c^2)/r_1 = 4$  times the force needed to contain a component at  $r_1$

#### 8.4 Vacuum pressure - varying the radius - general formula.

For a change of radius  $r_2 = (1/b)r_1$  the spherical area at  $r_2$  is  $(1/b^2)(4\pi r_1^2)$

If the force  $F_1$  present at  $r_1$  continues inwards without loss of strength to a spherical region of radius  $r_2 = (1/b)r_1$  then it give rise to a pressure  $b^2$  times as great as the pressure at  $r_1$  because the area reduces to  $(1/b^2)(4\pi r_1^2)$  as shown here:

The pressure at  $r_2$  is:

$$F_1/(4\pi r_2^2) = F_1/(4\pi((1/b)r_1)^2) = F_1/((1/b^2)(4\pi r_1^2)) = b^2 F_1/(4\pi r_1^2) = b^2 \times (\text{pressure at } r_1)$$

## 9. Electric field of a photon.

*Purpose of this section:* to calculate  $E_y$ , the component of the electric field strength at right angles to the direction of propagation of a photon.

### 9.1 Electric field of a photon - introduction.

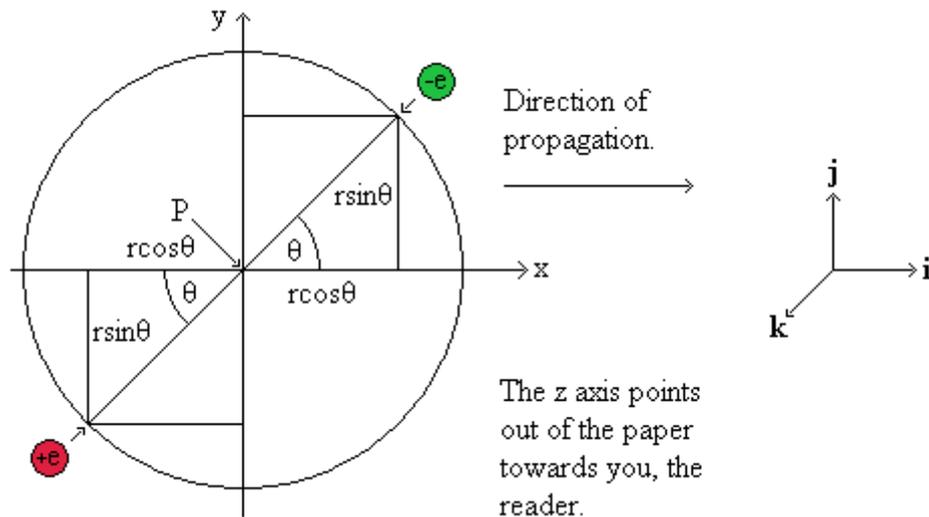


Figure 9.1a.

Figure 9.1a shows a photon travelling at light speed in the direction of increasing positive  $x$ . At the same time the negative and positive charges are moving anticlockwise at light speed a fixed distance  $r$  from the centre of rotation  $P$ , so that the angle  $\theta$  increases uniformly with time.

For diagram simplicity the orbit of rotation of the photon components has been chosen to be in the  $x$ - $y$  plane. The photon could equally well be orbiting in the  $y$ - $z$  plane or any other plane as long as we define the  $y$  axis to be at right angles to the direction of propagation.

For tilted planes of orbit we use a diameter of orbit as an intermediate axis and then project the component of the electric field along the intermediate axis onto the  $y$  axis. The electric field still fluctuates only with  $\theta$  as the  $y$  axis is at a fixed angle to the intermediate axis.

## 9.2 Electric field of a photon - electric field calculation.

The electric field  $\mathbf{E}$  at a point P is a vector quantity which gives the force acting on a unit positive charge at the point P.

$$\mathbf{E} = (1/(4\pi\epsilon_0))\{(q_1/r_1^2) \mathbf{r}_1 + (q_2/r_2^2) \mathbf{r}_2 + \dots \}$$

A particle with a charge of magnitude  $q_1$  is at a distance  $r_1$  from P.  $\mathbf{r}_1$  is the unit vector pointing from  $q_1$  to P.

Similarly for any further charged particles with properties  $q_2, r_2, \mathbf{r}_2 \dots$

Referring to Figure 9.1a with

$r$  = radius of orbit; and

$\mathbf{i}$  = unit vector in the direction of increasing  $x$ : and

$\mathbf{j}$  = unit vector in the direction of increasing  $y$ :

$$\mathbf{E} = (1/(4\pi\epsilon_0))\{(-e/r^2)([-r\cos\theta\mathbf{i} - r\sin\theta\mathbf{j}]/r) + (e/r^2)([r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}]/r)\}$$

$$\mathbf{E} = (1/(4\pi\epsilon_0))\{(-e/r^2)(-\cos\theta\mathbf{i} - \sin\theta\mathbf{j}) + (e/r^2)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\}$$

$$\mathbf{E} = (1/(4\pi\epsilon_0))(e/r^2)\{\cos\theta\mathbf{i} + \sin\theta\mathbf{j} + \cos\theta\mathbf{i} + \sin\theta\mathbf{j}\}$$

$$\mathbf{E} = (1/(4\pi\epsilon_0))(e/r^2)\{2\cos\theta\mathbf{i} + 2\sin\theta\mathbf{j}\} \quad \text{----- (9.2a)}$$

Check this for  $\theta = \pi/2 = 90^\circ$ :

$$\mathbf{E} = (1/(4\pi\epsilon_0))(e/r^2)\{2\cos(\pi/2)\mathbf{i} + 2\sin(\pi/2)\mathbf{j}\} = (1/(4\pi\epsilon_0))(e/r^2)\{0 + 2\mathbf{j}\}$$

$$= (1/(4\pi\epsilon_0))(e/r^2)\{2\mathbf{j}\}$$

which represents the force on a unit positive charge at P pulled upwards by the negative charge at  $r\mathbf{j}$  and repelled upwards by the positive charge at  $r(-\mathbf{j})$

Check this for  $\theta = 3\pi/2 = 270^\circ$ :

$$\mathbf{E} = (1/(4\pi\epsilon_0))(e/r^2)\{2\cos(3\pi/2)\mathbf{i} + 2\sin(3\pi/2)\mathbf{j}\} = (1/(4\pi\epsilon_0))(e/r^2)\{0 + (-2)\mathbf{j}\}$$

$$= (1/(4\pi\epsilon_0))(e/r^2)\{(-2)\mathbf{j}\}$$

which represents the force on a unit positive charge at P pulled downwards by the negative charge at  $r(-\mathbf{j})$  and repelled downwards by the positive charge at  $r\mathbf{j}$

Table of  $\mathbf{E}$  as a function of angle  $\theta$  for some values of  $\theta$ .

$\theta$		$\mathbf{E}$	
$0^\circ$	0	$(1/(4\pi\epsilon_0))(e/r^2)\{(2x-1)\mathbf{i} + (2x-0)\mathbf{j}\}$	$(1/(4\pi\epsilon_0))(e/r^2)\{2\mathbf{i}\}$
$90^\circ$	$\pi/2$	$(1/(4\pi\epsilon_0))(e/r^2)\{(2x-0)\mathbf{i} + (2x-1)\mathbf{j}\}$	$(1/(4\pi\epsilon_0))(e/r^2)\{2\mathbf{j}\}$
$180^\circ$	$\pi$	$(1/(4\pi\epsilon_0))(e/r^2)\{(2x-1)\mathbf{i} + (2x-0)\mathbf{j}\}$	$(1/(4\pi\epsilon_0))(e/r^2)\{-2\mathbf{i}\}$
$270^\circ$	$3\pi/2$	$(1/(4\pi\epsilon_0))(e/r^2)\{(2x-0)\mathbf{i} + (2x-1)\mathbf{j}\}$	$(1/(4\pi\epsilon_0))(e/r^2)\{-2\mathbf{j}\}$
$360^\circ$	$2\pi$	$(1/(4\pi\epsilon_0))(e/r^2)\{(2x-1)\mathbf{i} + (2x-0)\mathbf{j}\}$	$(1/(4\pi\epsilon_0))(e/r^2)\{2\mathbf{i}\}$

9.3 Electric field of a photon - component of the electric field in the y direction.

$$\mathbf{E} = (1/(4\pi\epsilon_0))(e/r^2)\{2\cos\theta\mathbf{i} + 2\sin\theta\mathbf{j}\} \quad \text{----- (9.2a)}$$

Using equation (9.2a) we can see that  $\mathbf{E}_y$  the component of  $\mathbf{E}$  in the y direction is the term in  $\mathbf{j}$  the unit vector in the y direction:

$$\mathbf{E}_y = (2e/(4\pi\epsilon_0r^2))\sin\theta\mathbf{j} \quad \text{----- (9.3a)}$$

This can also be written as

$$\mathbf{E}_y = E_0\sin\theta\mathbf{j}$$

or

$$E_y = E_0\sin\theta \quad \text{----- (9.3b)}$$

where  $E_0 = (2e/(4\pi\epsilon_0r^2))$  is the maximum value of the electric field component in the y direction which occurs when both the positive and negative charges are on the y axis.

As the photon components rotate at the speed of light and the photon advances along the x axis also at the speed of light we can see that the electric field component  $\mathbf{E}_y$  will map out a sinusoidal curve in the xy plane.

The distance travelled along the x axis is the same as the distance travelled by the photon components in their orbit about their centre of mass as shown here:

If we take the following variables:

$d$  = distance travelled by a photon component at speed  $c$  in time  $t$ ; and

$D$  = distance travelled by the photon at speed  $c$  also in time  $t$

then

$d$  = speed x time =  $ct$

$D$  = speed x time =  $ct$

so  $D = d$

$\mathbf{E}_y$  is shown in Figure 9.3a.

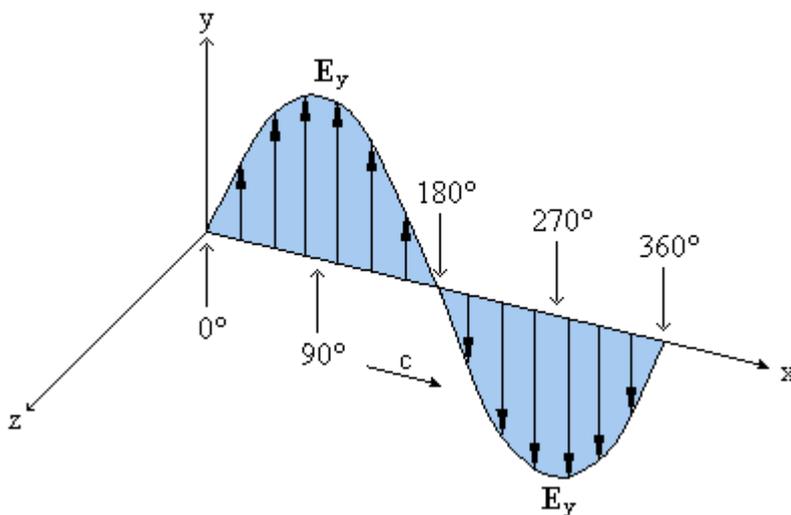


Figure 9.3a.

*9.4 Electric field of a photon - component of the electric field in the y direction as a function of x, the distance travelled by the photon along the x axis.*

Because the speed of the photon and the speed of its components are equal, the distance travelled along the x axis is the same as the distance travelled by one of the photon components around its orbit about the photon centre.

If the measurement of the photon position starts at  $x = 0$  when the position of the negative photon component is at  $\theta = 0$ , then  $x = r\theta$  where  $r$  is the radius of the photon component orbit and  $\theta$  is measured in radians.

$$\theta = r\theta/r = x/r = 2\pi x/2\pi r = 2\pi x/\lambda$$

So from:

$$E_y = E_0 \sin\theta \quad \text{----- (9.3b)}$$

$$E_y = E_0 \sin 2\pi[x/\lambda] \quad \text{----- (9.4a)}$$

*9.5 Electric field of a photon - component of the electric field in the y direction as a function of t, the time travelled by the photon along the x axis.*

Using:

distance travelled ( $x$ ) = speed ( $c$ ) x time travelled ( $t$ ),  $x = ct$

speed of the wave ( $c$ ) = frequency ( $\nu$ ) x wavelength ( $\lambda$ ),  $c = \nu\lambda$

$$x/\lambda = ct/\lambda = \nu\lambda t/\lambda = \nu t$$

So from:

$$E_y = E_0 \sin 2\pi[x/\lambda] \quad \text{----- (9.4a)}$$

or

$$E_y = E_0 \sin 2\pi[\nu t] \quad \text{----- (9.5a)}$$

$$E_y = E_0 \sin 2\pi[x/\lambda] \quad \text{or} \quad E_y = E_0 \sin 2\pi[\nu t]$$

## 10. Magnetic field of a photon.

### 10.1 Magnetic field of a photon - derivation of $B_z$ - left hand side of Maxwell-Faraday equation - area vector in positive $z$ direction.

We need to obtain the magnetic field  $\mathbf{B}_z$  associated with the changing electric field derived and shown in equation (9.3a):

$$\mathbf{E}_y = (2e/(4\pi\epsilon_0 r^2))\sin\theta\mathbf{j} \quad \text{----- (9.3a)}$$

We can do this by applying the Maxwell-Faraday equation which is one of the four Maxwell-Heaviside equations (normally called Maxwell's equations) relating a changing electric field  $\mathbf{E}$  to a changing magnetic field  $\mathbf{B}$ :

in differential form the Maxwell-Faraday equation is:

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad \text{or} \quad \text{curl } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

in integral form:

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

Equation 10.1a

$\mathbf{E} \cdot d\mathbf{l}$  = (magnitude of vector  $\mathbf{E}$ ) x (magnitude of length vector  $d\mathbf{l}$ ) x cosine(angle between  $\mathbf{E}$  and  $d\mathbf{l}$ )

=  $E \times dl \times \cos(\text{angle between } \mathbf{E} \text{ and } d\mathbf{l})$

For the left hand side (LHS) of the integral form (Equation 10.1a) we take a rectangular closed path in the  $xy$  plane and calculate the line integral around that path [Reference 3]. Please see Figure 10.1a below.

The direction to travel around the rectangular closed path is chosen to be clockwise looking in the direction of increasing positive  $z$ . The vector defining the area within the closed path is the size of the area multiplied by a unit vector at right angles to the plane of the area.

As there are two possible vectors at right angles to the area inside the closed path Equation 10.1a uses the convention that we choose the vector direction shown by the direction of your right thumb when the fingers of your right hand curl round in the direction taken by the closed path. In this section the vector points in the direction of increasing  $z$ . This coincides with  $\mathbf{B}$  also pointing in the direction of increasing  $z$ .

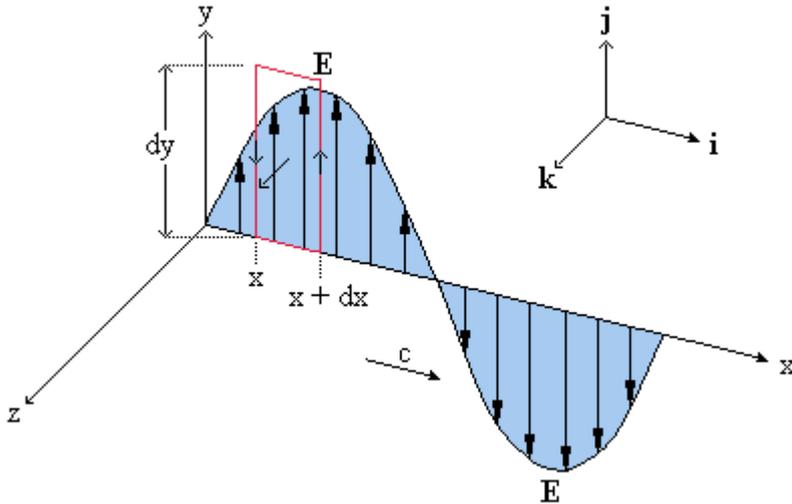


Figure 10.1a.

In the following example:

$(E_y)$  points in the same direction as the positive y axis and therefore the unit vector  $\mathbf{j}$ .

$(E_y)_x$  means the value of  $E_y$  at the point x along the x axis.

$(E_y)_{x+dx}$  means the value of  $E_y$  at the point a distance dx further along the x axis than the point x.

dx is the length of the closed path in the x direction.

dy is the length of the closed path in the y direction.

$\mathbf{i}$  is a unit vector in the direction of increasing x.

$\mathbf{j}$  is a unit vector in the direction of increasing y.

$\mathbf{k}$  is a unit vector in the direction of increasing z.

$$\mathbf{j} \cdot \mathbf{j} = 1 \times 1 \times \cos(0) = 1$$

$$\mathbf{j} \cdot \mathbf{i} = 1 \times 1 \times \cos(\pi/2) = 0$$

LHS = left hand side of Equation 10.1a

$$\begin{aligned} \text{LHS} = & \{[(E_y)_{x+dx}](\mathbf{j}) \cdot [(dy)](\mathbf{j})\} + \{[(E_y)](\mathbf{j}) \cdot (dx)(-\mathbf{i})\} \\ & + \{[(E_y)_x](\mathbf{j}) \cdot [(dy)(-\mathbf{j})]\} + \{[(E_y)](\mathbf{j}) \cdot (dx)(\mathbf{i})\} \end{aligned}$$

$$\begin{aligned} \text{LHS} = & \{[(E_y)_{x+dx}][(dy)](\mathbf{j} \cdot \mathbf{j})\} - \{[(E_y)](dx)(\mathbf{j} \cdot \mathbf{i})\} \\ & - \{[(E_y)_x][(dy)(\mathbf{j} \cdot \mathbf{j})]\} + \{[(E_y)](dx)(\mathbf{j} \cdot \mathbf{i})\} \end{aligned}$$

$$\begin{aligned} \text{LHS} = & \{(E_y)_{x+dx}(dy)\cos(0)\} - \{(E_y)(dx)\cos(\pi/2)\} \\ & - \{(E_y)_x(dy)\cos(0)\} + \{(E_y)(dx)\cos(\pi/2)\} \end{aligned}$$

with  $\cos(\pi/2) = 0$  and  $\cos(0) = 1$  we obtain:

$$\text{LHS} = (E_y)_{x+dx}(dy) - (E_y)_x(dy) = \{(E_y)_{x+dx} - (E_y)_x\}dy$$

$\{(E_y)_{x+dx} - (E_y)_x\}$  is the rate of change of  $E_y$  with x multiplied by the length over which the change occurs which is written as  $\{(\partial E_y / \partial x)dx\}$

$$\text{LHS} = \{(\partial E_y / \partial x)dx\}dy = (\partial E_y / \partial x) dx dy$$

10.2 Magnetic field of a photon - derivation of  $B_z$  - right hand side of Maxwell-Faraday equation.

The magnetic field vector  $\mathbf{B}$  written in terms of its components and unit direction vectors at right angles to each other is:

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

and the area vector may be written as:  $d\mathbf{S} = dx dy \mathbf{k}$

In the following we need to use this:

the dot product between two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is  $|\mathbf{a}| |\mathbf{b}| \cos\theta$  where  $|\mathbf{a}|$  is the magnitude of vector  $\mathbf{a}$  and  $|\mathbf{b}|$  is the magnitude of vector  $\mathbf{b}$  and  $\theta$  is the angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$ . If two vectors are at right angles to each other the dot product between them is zero because  $\cos(\pi/2) = 0$ .

So  $(\mathbf{i} \cdot \mathbf{k}) = (\mathbf{j} \cdot \mathbf{k}) = 0$  and  $(\mathbf{k} \cdot \mathbf{k}) = 1$

The right hand side of the Maxwell-Faraday equation above (Equation 10.1a) is:

$$- (\partial\mathbf{B}/\partial t) \cdot d\mathbf{S} = -(\partial(B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})/\partial t) \cdot (dx dy \mathbf{k})$$

$$- (\partial\mathbf{B}/\partial t) \cdot d\mathbf{S} = - \partial\{(B_x (\mathbf{i} \cdot \mathbf{k}) + B_y (\mathbf{j} \cdot \mathbf{k}) + B_z (\mathbf{k} \cdot \mathbf{k}))\}/\partial t \cdot (dx dy)$$

$$- (\partial\mathbf{B}/\partial t) \cdot d\mathbf{S} = - (\partial B_z / \partial t) dx dy$$

Equating the left and right hand sides of the Maxwell-Faraday equation we obtain:

$$(\partial E_y / \partial x) dx dy = - (\partial B_z / \partial t) dx dy$$

Since  $dx dy$  may represent any area:

$$(\partial E_y / \partial x) = - (\partial B_z / \partial t) \quad \text{----- (10.1a)}$$

The right hand side of this equation gives a fluctuating magnetic field in the  $z$  direction which is shown in Figure 10.2a.

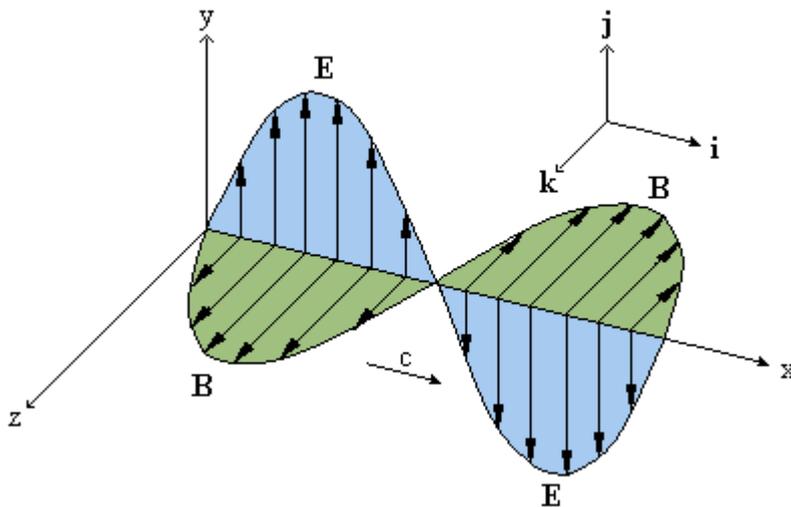


Figure 10.2a.

10.3 Magnetic field of a photon - derivation of  $B_z$  - left hand side of Maxwell-Faraday equation - area vector in negative  $z$  direction.

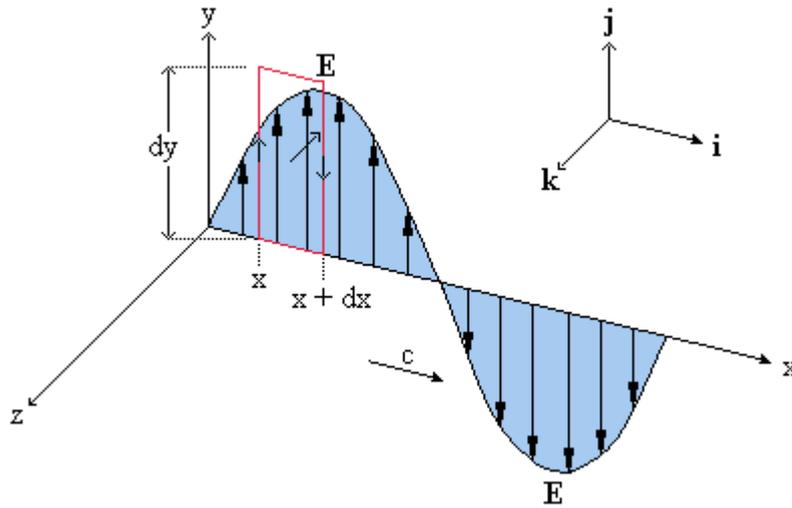


Figure 10.3a.

The area vector in this section is:  $d\mathbf{S} = dx dy (-\mathbf{k})$

LHS = left hand side of Equation 10.1a

$$\text{LHS} = \{[(E_y)_x](\mathbf{j}) \cdot [(dy)](\mathbf{j})\} + \{[(E_y)](\mathbf{j}) \cdot (dx)(\mathbf{i})\} \\ + \{[(E_y)_{x+dx}](\mathbf{j}) \cdot [(dy)(-\mathbf{j})]\} + \{[(E_y)](\mathbf{j}) \cdot (dx)(-\mathbf{i})\}$$

$$\text{LHS} = \{[(E_y)_x][(dy)](\mathbf{j} \cdot \mathbf{j})\} + \{[(E_y)](dx)(\mathbf{j} \cdot \mathbf{i})\} \\ - \{[(E_y)_{x+dx}][(dy)(\mathbf{j} \cdot \mathbf{j})]\} - \{[(E_y)](dx)(\mathbf{j} \cdot \mathbf{i})\}$$

$$\text{LHS} = \{(E_y)_x(dy)\cos(0)\} + \{(E_y)(dx)\cos(\pi/2)\} \\ - \{(E_y)_{x+dx}(dy)\cos(0)\} - \{(E_y)(dx)\cos(\pi/2)\}$$

with  $\cos(\pi/2) = 0$  and  $\cos(0) = 1$  we obtain:

$$\text{LHS} = (E_y)_x(dy) - (E_y)_{x+dx}(dy) = \{(E_y)_x - (E_y)_{x+dx}\}dy = [-\{(E_y)_{x+dx} - (E_y)_x\}]dy$$

$\{(E_y)_{x+dx} - (E_y)_x\}$  is the rate of change of  $E_y$  with  $x$  multiplied by the length over which the change occurs which is written as  $\{(\partial E_y/\partial x)dx\}$

$$\text{so } -\{[(E_y)_{x+dx} - (E_y)_x]\} = -\{(\partial E_y/\partial x)dx\}$$

$$\text{LHS} = -\{(\partial E_y/\partial x)dx\}dy = -(\partial E_y/\partial x) dx dy$$

In this section, the right hand side of the Maxwell-Faraday equation above (Equation 10.1a) is:

$$-(\partial \mathbf{B}/\partial t) \cdot d\mathbf{S} = -(\partial (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})/\partial t) \cdot (dx dy (-\mathbf{k}))$$

$$-(\partial \mathbf{B}/\partial t) \cdot d\mathbf{S} = \partial \{(B_x (\mathbf{i} \cdot \mathbf{k}) + B_y (\mathbf{j} \cdot \mathbf{k}) + B_z (\mathbf{k} \cdot \mathbf{k})\}/\partial t) \cdot (dx dy)$$

$$-(\partial \mathbf{B}/\partial t) \cdot d\mathbf{S} = (\partial B_z/\partial t) dx dy$$

Equating the left and right hand sides of the Maxwell-Faraday equation we obtain:

$$-(\partial E_y/\partial x) dx dy = (\partial B_z/\partial t) dx dy$$

Since  $dx dy$  may represent any area:

$$-(\partial E_y/\partial x) = (\partial B_z/\partial t)$$

$$(\partial E_y/\partial x) = -(\partial B_z/\partial t) \quad \text{----- (10.1a)}$$

which is the same as the result from section 10.2. So if we take the path the opposite way round we still get the same answer.

#### 10.4 Magnetic field of a photon - particular solutions of $(\partial E_y/\partial x) = -(\partial B_z/\partial t)$ .

Particular forms of  $E_y$  and  $B_z$  satisfying equation (10.1a) are:

$$E_y = E_0 \sin 2\pi[x/\lambda - vt] \text{ and } B_z = (1/c)E_0 \sin 2\pi[x/\lambda - vt] \quad \text{----- (10.4a)}$$

#### 10.5 Magnetic field of a photon - derivation of $B_z$ in particular solutions of $(\partial E_y/\partial x) = -(\partial B_z/\partial t)$

$$(\partial E_y/\partial x) = -(\partial B_z/\partial t)$$

$$-(\partial B_z/\partial t) = (\partial E_y/\partial x)$$

$$-(\partial B_z/\partial t) = (\partial(E_0 \sin 2\pi[x/\lambda - vt])/\partial x)$$

$$-(\partial B_z/\partial t) = (\partial/\partial x(E_0 \sin 2\pi[x/\lambda - vt]))$$

$$-(\partial B_z/\partial t) = (2\pi/\lambda)E_0 \cos 2\pi[x/\lambda - vt])$$

Integrating with respect to  $t$ :

$$-B_z = \int (2\pi/\lambda)E_0 \cos 2\pi[x/\lambda - vt]) dt$$

$$-B_z = (2\pi/\lambda)/(-2\pi v)E_0 \sin 2\pi[x/\lambda - vt]) + \text{constant}$$

$$-B_z = -(2\pi/\lambda)/(2\pi v)E_0 \sin 2\pi[x/\lambda - vt]) + \text{constant}$$

$$B_z = (1/v\lambda)E_0 \sin 2\pi[x/\lambda - vt]) + \text{constant}$$

$$B_z = (1/c)E_0 \sin 2\pi[x/\lambda - vt]) \text{ if we take the constant to be zero.}$$

#### 10.6 Magnetic field of a photon - particular solutions of $(\partial E_y/\partial x) = -(\partial B_z/\partial t)$ - check.

We can check that the particular solutions of  $(\partial E_y/\partial x) = -(\partial B_z/\partial t)$  shown in equations (10.4a) are consistent as follows:

$$(\partial E_y/\partial x) = E_0 \cos 2\pi[x/\lambda - vt](2\pi/\lambda)$$

$$(\partial E_y/\partial x) = (2\pi/\lambda)E_0 \cos 2\pi[x/\lambda - vt]$$

$$-(\partial B_z/\partial t) = - (1/c)E_0 \cos 2\pi[x/\lambda - vt](2\pi(-v))$$

$$-(\partial B_z/\partial t) = - (2\pi(-v))(1/c)E_0 \cos 2\pi[x/\lambda - vt]$$

$$-(\partial B_z/\partial t) = ((2\pi v)/c)E_0 \cos 2\pi[x/\lambda - vt]$$

$$-(\partial B_z/\partial t) = ((2\pi v)/(v\lambda))E_0 \cos 2\pi[x/\lambda - vt]$$

$$-(\partial B_z/\partial t) = (2\pi/\lambda)E_0 \cos 2\pi[x/\lambda - vt]$$

This shows that  $(\partial E_y/\partial x) = -(\partial B_z/\partial t)$  for the solutions

$$E_y = E_0 \sin 2\pi[x/\lambda - vt] \text{ and } B_z = (1/c)E_0 \sin 2\pi[x/\lambda - vt]$$

## 11. Magnetic field of photon components - derivation of $B_z$ using the magnetic force of each photon component.

### 11.1 Magnetic field of photon components - derivation of $B_z$ using the magnetic force of each photon component - introduction.

We used the formula for the magnetic field vector in section 6:

$$\mathbf{B} = (\mu_0/4\pi) \times (q_1 \mathbf{v}_1 \times \mathbf{r}_1 / r_1^3) \quad \text{----- (6.1a)}$$

$\mathbf{B}$  = magnetic field vector (magnetic flux density vector) at a point a distance and direction  $\mathbf{r}_1$  away from a charge  $q_1$  moving with velocity  $\mathbf{v}_1$ .

In SI units  $\mathbf{B}$  is measured in  $\text{Wb m}^{-2} = (\text{kg m}^2 \text{C}^{-1} \text{s}^{-1}) \text{m}^{-2} = \text{kg C}^{-1} \text{s}^{-1} = \text{T}$   
 C = Coulomb, T = Tesla, Wb = Weber.

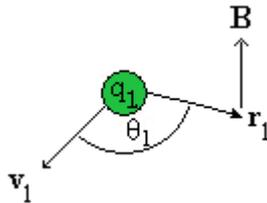


Figure 6.1a.

$\mathbf{v}_1 \times \mathbf{r}_1$  is the vector cross product which takes account of the magnitude and direction of both  $\mathbf{v}_1$  and  $\mathbf{r}_1$  and the angle  $\theta_1$  between them. It also defines a vector at right angles (in a right handed sense) to both  $\mathbf{v}_1$  and  $\mathbf{r}_1$  to show the direction of  $\mathbf{B}$ .  $r_1$  is the magnitude of the vector  $\mathbf{r}_1$  and denotes the distance but not the direction of the point where  $\mathbf{B}$  is to be evaluated from the charge  $q_1$ .

Without using vectors the above equation (6.1a) becomes:

$$B = (\mu_0/4\pi) \times ((q_1 v_1 \sin\theta_1) / r_1^2) \quad \text{----- (6.1b)}$$

where  $B$  is now the magnitude of  $\mathbf{B}$ , whereas  $\mathbf{B}$  has direction as well as magnitude.

11.2 Magnetic field of photon components - derivation of  $B_z$  using the magnetic force of each photon component - geometry.

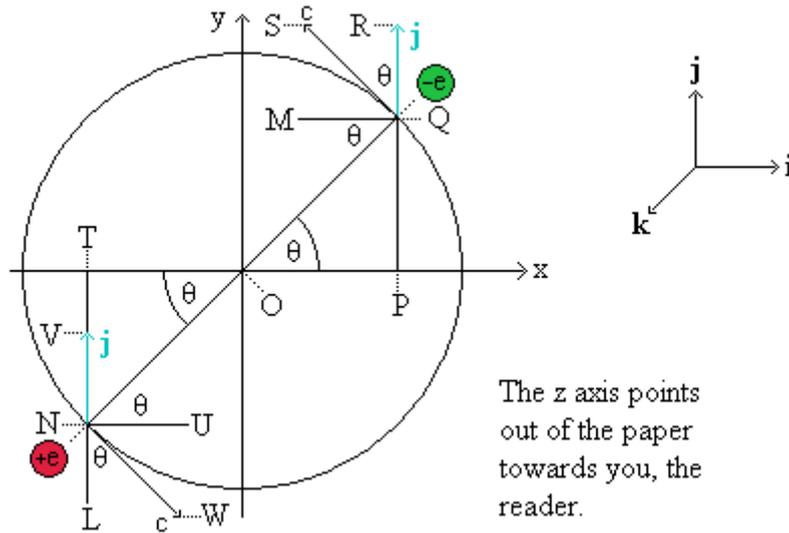


Figure 11.2a.

We start with  $\angle POQ = \theta$

$MQ$  is parallel to  $OP$  (by construction) so  $\angle OQM = \angle POQ = \theta$  (alternate angles)

$\angle MQS = \pi/2 - \theta$  as  $QS$  (tangent to the circle) is perpendicular to  $OQ$

$\angle SQR = \angle MQR - \angle MQS = \pi/2 - (\pi/2 - \theta) = \pi/2 - \pi/2 + \theta = \theta$

$\angle TON = \angle POQ = \theta$  (vertically opposite angles)

$NU$  is parallel to  $OT$  (by construction) so  $\angle UNO = \angle TON = \theta$  (alternate angles)

$\angle WNU = \pi/2 - \theta$  as  $NW$  (tangent to the circle) is perpendicular to  $ON$

$\angle LNW = \angle LNU - \angle WNU = \pi/2 - (\pi/2 - \theta) = \pi/2 - \pi/2 + \theta = \theta$

11.3 Magnetic field of photon components - derivation of  $B_z$  using the magnetic force of each photon component - velocities.

Referring to Figure 11.2a:

the negative component ( $-e$ ) at the point  $Q$  is travelling at speed  $c$  in the direction

$-\mathbf{i} \cos(\angle MQS) + \mathbf{j} \cos(\angle SQR)$

$-\mathbf{i} \cos(\pi/2 - \theta) + \mathbf{j} \cos(\theta)$

so its velocity is:

$\mathbf{v}_{-e} = c(-\mathbf{i} \cos(\pi/2 - \theta) + \mathbf{j} \cos(\theta))$

the positive component ( $+e$ ) at the point  $N$  is travelling at speed  $c$  in the direction

$+\mathbf{i} \cos(\angle WNU) - \mathbf{j} \cos(\angle LNW)$

$+\mathbf{i} \cos(\pi/2 - \theta) - \mathbf{j} \cos(\theta)$

so its velocity is:

$\mathbf{v}_{+e} = c(+\mathbf{i} \cos(\pi/2 - \theta) - \mathbf{j} \cos(\theta))$

11.4 Magnetic field of photon components - derivation of  $B_z$  using the magnetic force of each photon component - calculation.

Removing the suffixes from Equation (6.1a) we obtain:

$$\mathbf{B} = (\mu_0/4\pi) \times (q \mathbf{v} \times \mathbf{r} / r^3)$$

We take  $\mathbf{r}$  to be  $\mathbf{j}$ , the unit vector (magnitude  $j = 1$ ) in the direction of increasing  $y$ .

$$\text{So } r^3 = j^3 = 1^3 = 1$$

We use:

$$\mathbf{j} \times \mathbf{j} = \mathbf{0}$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

Using the trigonometric compound angle formula:  $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$$\cos(\pi/2 - \theta) = \cos(\pi/2) \cos\theta + \sin(\pi/2) \sin\theta$$

$$\cos(\pi/2 - \theta) = (0 \times \cos\theta) + (1 \times \sin\theta)$$

$$\cos(\pi/2 - \theta) = \sin\theta$$

$$\mathbf{B} (+e) = (\mu_0/4\pi) \times ((+e) \mathbf{v}_{+e} \times \mathbf{j})$$

$$\mathbf{B} (+e) = (\mu_0/4\pi) \times (e c [+ \mathbf{i} \cos(\pi/2 - \theta) - \mathbf{j} \cos(\theta)] \times \mathbf{j})$$

$$\mathbf{B} (+e) = (\mu_0/4\pi) \times (e c \mathbf{k} \cos(\pi/2 - \theta))$$

$$\mathbf{B} (+e) = (\mu_0/4\pi) \times (e c \mathbf{k} \sin\theta) \quad \text{----- (11.3a)}$$

$$\mathbf{B} (-e) = (\mu_0/4\pi) \times ((-e) \mathbf{v}_{-e} \times \mathbf{j})$$

$$\mathbf{B} (-e) = (\mu_0/4\pi) \times ((-e) c [- \mathbf{i} \cos(\pi/2 - \theta) + \mathbf{j} \cos(\theta)] \times \mathbf{j})$$

$$\mathbf{B} (-e) = (\mu_0/4\pi) \times ((-e) c - \mathbf{k} \cos(\pi/2 - \theta))$$

$$\mathbf{B} (-e) = (\mu_0/4\pi) \times (e c \mathbf{k} \sin\theta) \quad \text{----- (11.3b)}$$

Equations (11.3a) and (11.3b) are equal and point in a direction at right angles to the directions of both;  
the movement of the photon charged components; and  
the fixed direction ( $\mathbf{j}$ ) used to calculate the component of the electric field of a photon in the  $y$  direction in section (9.3).

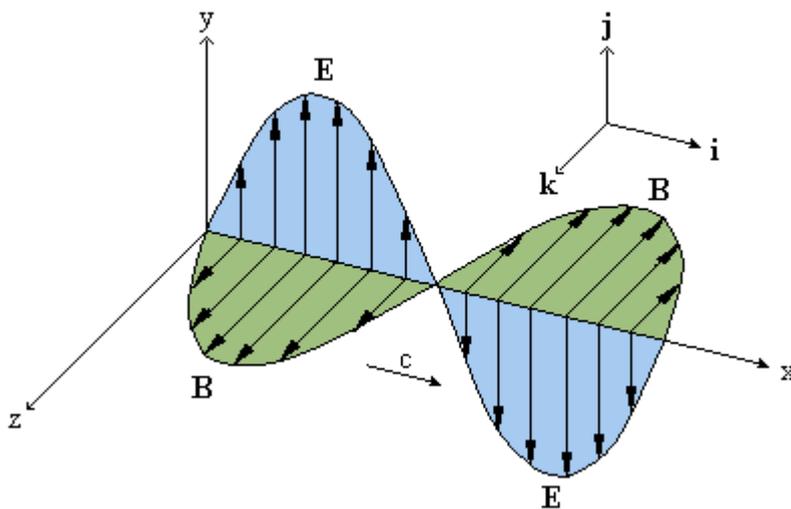


Figure 10.2a.

## 12. Photon polarisation.

We can describe a photon as being plane polarised if the plane containing the orbit of its charged components is coincident with a plane containing the axis along which it travels and an axis at right angles to that axis.

For example in Figure 12.1a the plane containing the orbit of the photon's components is the xy plane, which is the same as the plane containing the direction of travel (x axis) and the y axis which is at right angles to the x axis.

Photon polarisation is usually defined by the plane containing the oscillations of the photon's electric field vector and the direction of photon travel. This is the same as the plane of rotation of the photon's components as suggested in this paper and the photon's direction of travel.

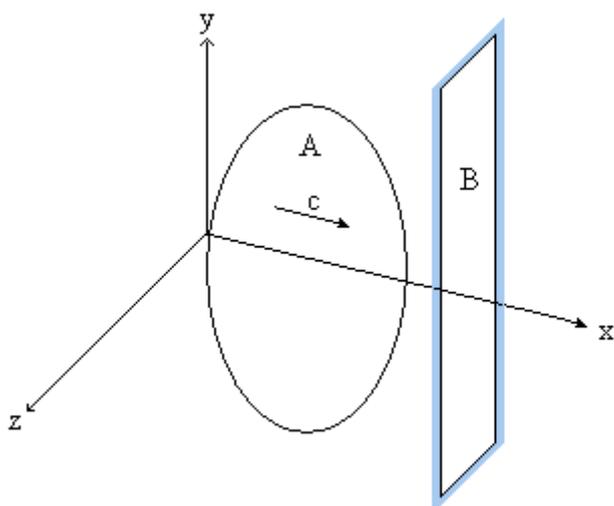


Figure 12.1a.

In Figure 12.1a the photon A will probably (quantum variation may, for example, alter the photon components orbit size) pass through the slit B because its components rotate in a plane that lines up with the opening in slit B.

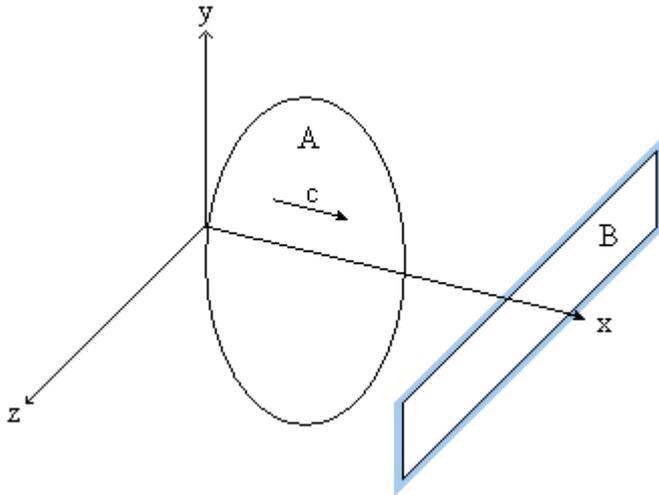


Figure 12.1b.

In Figure 12.1b the photon A is very unlikely to pass through the slit B because its components do not rotate in a plane that lines up with the opening in slit B.

It is possible that the photon's energy may fluctuate during a time  $\Delta t$  by an amount  $\Delta E$  in the region of slit B so that its radius decreases and the photon is small enough to pass through the slit before reverting back its standard energy.  $\Delta E$  and  $\Delta t$  are related by the time-energy Heisenberg uncertainty relation [Reference 4] as:

$$\Delta E \Delta t \geq \hbar/4\pi$$

### 13. Double slit experiment.

#### 13.1 Double slit experiment - introduction.

Originally the double slit experiment was used to show that light was a wave form. Light from a source was shone through two small holes close together in one screen so that the light fell on a second screen.

This was used to show wave properties of light (such as interference) on the second screen caused by the two light 'sources' coming through the two holes in the first screen and spreading out and then overlapping each other to create regions of light (peaks) and dark (troughs).

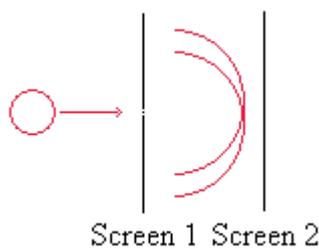


Figure 13.1a.

#### 13.2 Double slit experiment - one slit/hole.

Although the photon as pictured in this paper is not a single particle it can nevertheless pass through a single hole as indicated in Figure 13.2a.

The red and green circles indicate the positive and negative components and the forward movement (translation of the centre of mass) is accompanied by a sideways movement (rotation about the centre of mass).

Single photon, single slit - plan view (4 time intervals). Forward movement of the photon is upwards in each frame.

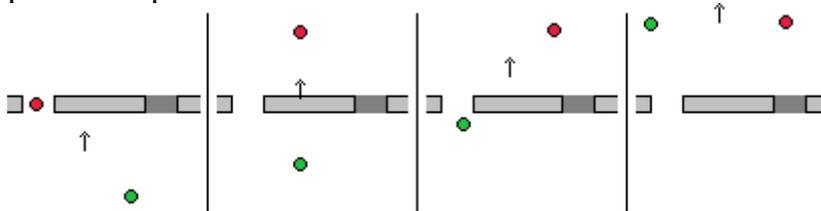


Figure 13.2a.

### 13.3 Double slit experiment - two slits/holes.

Because the photon as pictured in this paper is not a single particle it can pass through two different holes at about the same time as indicated in Figure 13.3a.

The red and green circles indicate the positive and negative components and the forward movement (translation of the centre of mass) is accompanied by a sideways movement (rotation about the centre of mass).

Single photon, double slit - plan view (4 time intervals). Forward movement of the photon is upwards in each frame.

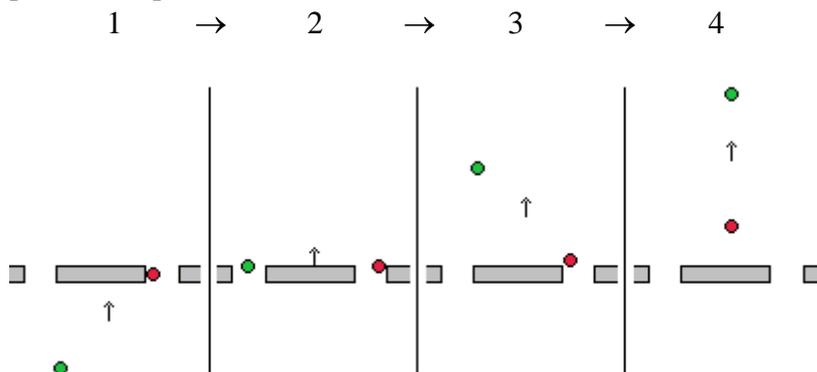


Figure 13.3a.

### 13.4 Double slit experiment - hole to hole distance and component to component distance - basics.

Quantum uncertainty means that the distance between the components in a single photon is not rigidly fixed, but we still need to know that the distance between the slits/holes is compatible with the distance between the two components of the photon at the time that the photon moves through the two holes.

Here is an example of a double slit experiment using visible light:

Spacing between slits  $\approx 10^{-5}$  m

Wavelength of visible light

$\approx 400 \times 10^{-9}$  m  $\approx 0.4 \times 10^{-6}$  m

The wavelength of a photon matches the circumference of the orbit of the components (equation 1.5a). The circumference of the orbit is  $\pi$  times the diameter of the orbit, therefore

wavelength =  $\pi$  times the diameter  $\approx 0.4 \times 10^{-6}$  m

diameter  $\approx (0.4 \times 10^{-6})/\pi$  m  $\approx (0.13 \times 10^{-6})$  m  $\approx 10^{-7}$  m

So the hole/slit to hole/slit distance is  $\approx 10^{-5}$  m while the component to component distance in the photon's 'free state' is  $\approx 10^{-7}$  m.

We are using the term 'free state' to mean that in this state the photon is not constrained.

What is a photon? Photon kinetic and electromagnetic structure simplified and explained and how one photon can go through two different holes at the same time. 14th November 2016. © Colin James 2016. Page 43 out of 51.

*13.5 Double slit experiment - hole to hole distance and component to component distance - energy calculation.*

To calculate the energy of a photon with a wavelength of visible light  
 $\approx 0.4 \times 10^{-6} \text{ m}$   
 we use the following relations:

$$v\lambda = c \text{ (frequency x wavelength = speed)}$$

$$\Rightarrow v = c/\lambda$$

$$E = hv = (hc)/\lambda$$

*13.6 Double slit experiment - hole to hole distance and component to component distance - energy, time and distance calculations.*

It is possible for the energy of a photon (and therefore the mass and thus the diameter of the orbit) to vary for a short time. The variation on energy is  $\Delta E$  and the variation on time interval is  $\Delta t$  and they are related by the time-energy Heisenberg uncertainty relation [Reference 4] as:

$$\Delta E \Delta t \geq h/4\pi$$

So the product of variation in energy and time interval is at least as great as  $h/4\pi$

If we decrease the energy of the photon  $E$  to  $E/400 = \Delta E$   
 then

$$\Delta E \Delta t \geq h/4\pi$$

$$\Rightarrow (hc)/400\lambda \times \Delta t \geq h/4\pi$$

$$\Rightarrow \Delta t \geq (h/4\pi) \times 400\lambda/(hc)$$

$$\Rightarrow \Delta t \geq 400\lambda/(4\pi c)$$

$$\Rightarrow \Delta t \geq (400 \times 2\pi r)/(4\pi c)$$

$$\Rightarrow \Delta t \geq (400 \times 2r)/(4c)$$

$$\Rightarrow \Delta t \geq (100 \times 2r)/c$$

$2r =$  original diameter before quantum fluctuation  $\approx 10^{-7} \text{ m}$   
 required diameter during quantum fluctuation  $\approx (100 \times 2r) \approx (100 \times 10^{-7}) \text{ m} \approx 10^{-5} \text{ m}$   
 Wavelength during quantum fluctuation  $\approx \pi \times 10^{-5} \text{ m} \approx 3 \times 10^{-5} \text{ m}$  ----- (13.6a)

Required time of quantum fluctuation  $\Delta t \geq (10^{-5} \text{ m})/c$   
 $\Delta t \geq (10^{-5} \text{ m})/(3 \times 10^8 \text{ ms}^{-1})$   
 $\Delta t \geq 3 \times 10^{-13} \text{ s}$

In a time of  $3 \times 10^{-13} \text{ s}$  the photon travels a distance  $D$  where  $3 \times 10^{-13} \text{ s} = D/c$   
 $D = c \times (3 \times 10^{-13} \text{ s}) = (3 \times 10^8 \text{ ms}^{-1}) \times (3 \times 10^{-13} \text{ s}) = 9 \times 10^{-5} \text{ m} \approx 10^{-4} \text{ m}$

This is approximately 3 times the wavelength during quantum fluctuation shown in equation 13.6a and allows for about 3 cycles/orbits of the photon during the transition of the double slits.

## 14. Pair production.

Pair production is the change of a photon into a particle/antiparticle pair. The rest mass-energy of a particle is the same as the rest mass-energy of its antiparticle.

The particle/antiparticle pair may be, for example, an electron and a positron or more massive particles such as a proton and an antiproton.

An electron/positron pair may be created in the presence of another particle such as an atomic nucleus from a photon having sufficient energy. The energy of the photon must be at least twice the mass-energy of the electron ( $E \geq 2m_e c^2$ ).

Other particle/antiparticle pairs will need an equivalent amount of energy supplied by the photon components. Each photon component having at least the mass-energy of either the particle or its antiparticle.

With the picture of a photon discussed in this paper the production of a charged particle/antiparticle pair is straightforward as shown in Figure 14.1a. An extra particle is needed in order to conserve momentum but the main change is the decomposition of the photon into a particle/antiparticle pair.

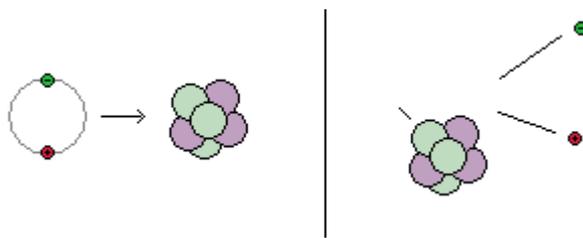


Figure 14.1a.

An uncharged particle/antiparticle pair may also be created as shown in Figure 14.1b.

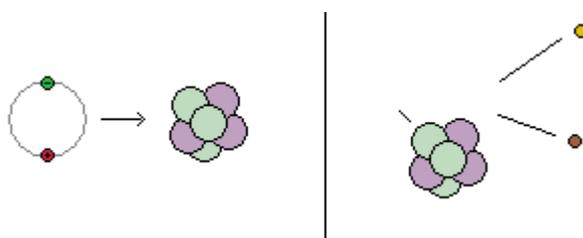


Figure 14.1b.

## 15. Force between two electrons.

### 15.1 Force between two electrons - Introduction.

If we remove the positively charged component of a photon from a photon with mass  $2m_e$  (twice the mass of an electron) we obtain the equivalent of a solitary electron represented as a particle of mass  $m_e$ , travelling at light speed  $c$ , and held in a circular orbit of radius  $r$  and angular momentum of  $m_e c r = h/4\pi$ .

We can see in this picture of the 'electron' its diffuse and dual nature - being both a particle and having a wavelength - wave-particle duality. This helps to explain why it is not possible to specify an exact location for an electron.

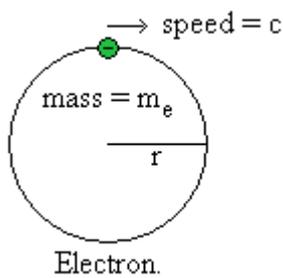


Figure 15.1a.

We can calculate the radius  $r$ , of the orbit of the 'electron' from:

$$m_e c r = h/4\pi$$

$$r = h/(4\pi m_e c)$$

$$r \approx (6.63 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}) / (4 \times 3.14 \times 9.11 \times 10^{-31} \text{ kg} \times 3.00 \times 10^8 \text{ ms}^{-1})$$

$$r \approx (6.63) / (4 \times 3.14 \times 9.11 \times 3.00) \times 10^{-11} \text{ m}$$

$$r \approx (6.63) / (343.18) \times 10^{-11} \text{ m}$$

$$r \approx 1.93 \times 10^{-13} \text{ m}$$

This is between the nuclear radius  $\sim 10^{-15} \text{ m}$  and the atomic radius  $\sim 10^{-10} \text{ m}$

### 15.2 Force between two electrons - Outward momentum flow (force).

Outward momentum flow (force) from one electron (the source electron) =  $m_e c^2 / r$

This outward momentum flow (force) spreads outwards and at a distance  $d$ , from the electron extends over the surface of spherical area such that its area density is

$$(m_e c^2 / r) / (4\pi d^2)$$

### 15.3 Force between two electrons - Interception of outward momentum flow (force) by a second electron.

The amount of the momentum flow intercepted by another electron (the target electron) is the force  $F$ , on the target electron and is characterized by an area (interaction cross-section) of size,  $\sigma$

$$F = (m_e c^2 / r) (\sigma / (4\pi d^2))$$

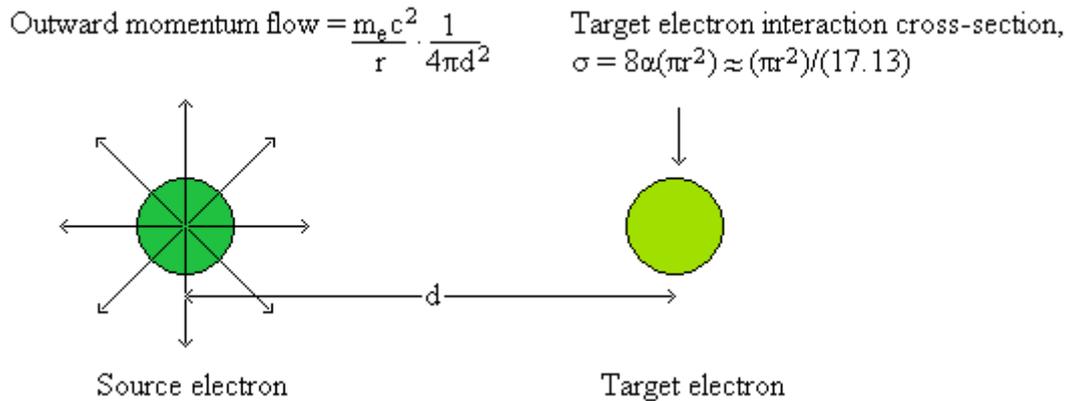


Figure 15.3a.

*15.4 Force between two electrons - Comparison with the electrostatic force to find  $\sigma$ , the interaction cross-section.*

If we compare this with the electrostatic force between two electrons a distance  $d$ , apart we obtain:

$$F = (m_e c^2/r)(\sigma/(4\pi d^2)) = e^2/(4\pi\epsilon_0 d^2)$$

$$(m_e c^2/r)(\sigma) = e^2/(\epsilon_0)$$

$$\sigma = (e^2 r)/(\epsilon_0 m_e c^2)$$

$$\sigma = (e^2 r^2)/(\epsilon_0 m_e c^2 r)$$

Using the angular momentum of an electron  $m_e c r = \frac{1}{2}(h/2\pi) = h/4\pi$  in:

$$\sigma = (e^2 r^2)/(\epsilon_0 m_e c^2 r)$$

$$\sigma = (e^2 r^2)/(\epsilon_0 \{m_e c r\} c)$$

$$\sigma = (e^2 r^2)/(\epsilon_0 \{h/4\pi\} c)$$

$$\sigma = (e^2 4\pi r^2)/(\epsilon_0 h c)$$

$$\sigma = (8e^2 \pi r^2)/(2\epsilon_0 h c)$$

$$\sigma = 8(e^2/(2\epsilon_0 h c))(\pi r^2)$$

$$\sigma = 8\alpha(\pi r^2) \text{ where } \alpha \text{ is the fine structure constant } \{\alpha = e^2/(2\epsilon_0 h c) \approx 1/137.036\}$$

$$\sigma = 8\alpha(\pi r^2) \approx (\pi r^2)/(17.13)$$

Alternatively:

$$\sigma = 2\alpha(4\pi r^2) \approx (4\pi r^2)/(68.52)$$

So using straightforward kinetics we obtain an interaction cross-section for the target electron which is either:

$$\sigma = 8\alpha(\pi r^2)$$

the area of the plane of the orbit of the target electron reduced by eight times the fine structure constant; or

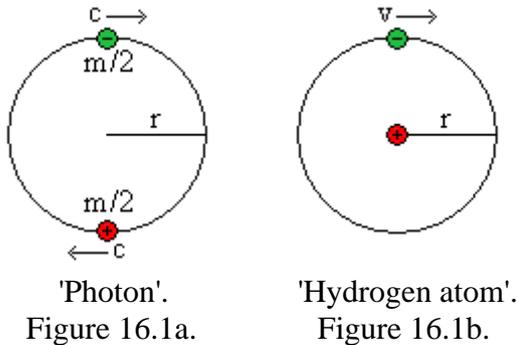
$$\sigma = 2\alpha(4\pi r^2)$$

the spherical surface area of the target electron's orbit reduced by twice the fine structure constant.

## 16. Hydrogen atom.

### 16.1 Hydrogen atom - introduction.

If we substitute a proton for the positively charged photon component we obtain a structure that resembles the lowest mass isotope of the hydrogen atom  $^1\text{H}$  and they both have the same angular momentum.



### 16.2 Hydrogen atom - angular momentum.

In the Bohr theory of the hydrogen atom (*References 5 and 6*) the angular momentum of the hydrogen atom is:

$$mvr = nh/2\pi \text{ where } n \text{ is a positive integer } (1, 2, 3 \dots) \quad \text{----- (16.2a)}$$

In the ground state of the hydrogen atom  $n = 1$  and the angular momentum is

$$mvr = h/2\pi \quad \text{-----(16.2b)}$$

This has the same form as the photon angular momentum.

$$mcr = h/2\pi \quad \text{----- (1.6c)}$$

The increase in angular momentum in equation 16.2a from  $n = 1$  to  $n = 2$  etc. suggests the interplay of one or more further photons.

One difference between the photon and the hydrogen atom is that the force containing the components is:

electrostatic for the hydrogen atom; but

$2 \times 10^{37}$  times stronger than electrostatic for the photon (as derived in section 5).

## 17. Fractional charges.

### 17.1 Fractional charges - introduction.

In section 8 we looked at the assumed vacuum pressure as it varies with radius.

If the force  $F$  (momentum flow per unit time) present at radius  $r_1$  continues inwards without loss of strength to a spherical region at radius  $r_2 = r_1/b$  then it give rise to a pressure (force per unit area)  $b^2$  times as great because the area reduces to  $(4\pi r_1^2)/b^2$

$$\begin{aligned} \text{Pressure at } r_1 &= F/(4\pi r_1^2) \\ \text{Pressure at } r_2 &= b^2 F/(4\pi r_1^2) \quad \text{----- (17.1a)} \end{aligned}$$

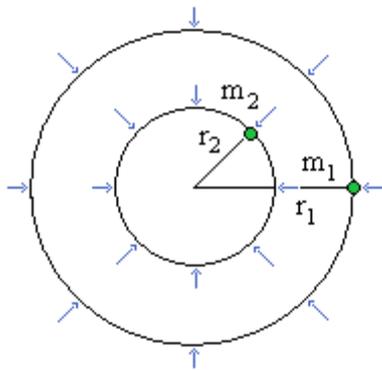


Figure 8.2a.

### 17.2 Fractional charges - radii at pressures in the ratio of 1:2:3.

Using equation 17.1a we can obtain the radii for:

pressure in the ratio	1:2:3
by setting $b^2$ to	1:2:3
and the corresponding radii	$r_1 : r_2 : r_3$
to	$r_1/\sqrt{1} : r_1/\sqrt{2} : r_1/\sqrt{3}$
equivalent to	$r_1\sqrt{(3/1)} : r_1\sqrt{(3/2)} : r_1\sqrt{(3/3)}$

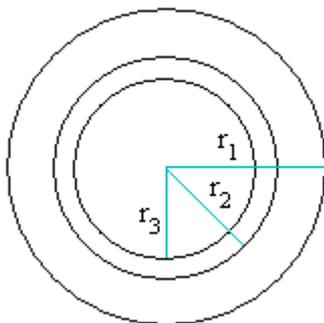


Figure 17.2a.

17.3 Fractional charges - matching pressure with outward force of orbiting particle.

The outward force of an orbiting particle of mass  $m$ , speed  $c$  and radius of orbit  $r$  is:  
 $mc^2/r$

If the angular momentum of the same particle is  $mcr = h/4\pi$

then  $mr = \text{constant}$

and  $m_1r_1 = m_2r_2 = m_3r_3 = \text{constant}$

then with radii:	$r_1/\sqrt{1} : r_1/\sqrt{2} : r_1/\sqrt{3}$
the masses are:	$m_1\sqrt{1} : m_1\sqrt{2} : m_1\sqrt{3}$
the outward forces are:	$(\sqrt{1})^2 (m_1c^2/r_1) : (\sqrt{2})^2 (m_1c^2/r_1) : (\sqrt{3})^2 (m_1c^2/r_1)$
the outward forces are:	$1(m_1c^2/r_1) : 2(m_1c^2/r_1) : 3(m_1c^2/r_1)$
the outward forces are:	$(1/3)(m_1c^2/r_1) : (2/3)(m_1c^2/r_1) : (3/3)(m_1c^2/r_1)$
the outward forces are:	$1/3 : 2/3 : 1$

Notice that:

the ratios of the outward forces match the magnitudes of:

the electric charges of the down quark : up quark : electron.

17.4 Fractional charges - matching pressure with the number of 'average momentum vacuum particles' per unit time operating singly : doubly : in threes.

Assumption of 'average momentum vacuum particles':

the force (momentum flow per unit time) involved in the vacuum pressure (force per unit area) is caused by virtual particles with a fixed average momentum.

The interaction between the 'average momentum vacuum particles' and contained (orbiting) particles such that the outward forces are in the ratios:  $1/3 : 2/3 : 1$  suggests that containment only occurs where:

'average momentum vacuum particles' operate singly : doubly : in threes.

## References.

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