Scope of the Study

I look at a [somewhat latent] vehicle of contagious co-movement, i.e. pure co-movement between assets/markets as well as groups of markets (or “styles”), in excess of their normal or fundamental correlation, if any. Explaining the nature and origin of crisis per se extends beyond the intended scope, and has been addressed extensively in the literature. Nor do I look at normal correlations or covariances, i.e. those likely to be symmetric across states of nature. The Shleifer & Barberis (forthcoming 2003) paper does a fair job along these lines. Finally, the crisis literature normally looks at currency crises. My framework applies to just about any asset types, currencies included, that have a significant interactive or network component to their value. This perspective is consistent with Krugman (2001) suggesting that new models of crises should focus on assets at large, while currencies might not play nearly as important a part as conventionally maintained.

Where It All Began

The present co-movement theme traces its origin all the way back to my earlier conjecture looking into intra-industry trade in information. Information is viewed as an input underlying the production of some ultimate value—which could indeed span a variety of information-intensive products. Thus, the original theory emerged as a largely abstract hypothesis building on modern trade theory. There are two important results in trade theory that I looked at. First, according to the celebrated Heckscher-Ohlin finding, trade could be explained in terms of relative resource endowments or scarcities, which distribution could be effectively smoothened via spatial trade. Economies will specialize in products that are intensive in inputs which are in relative abundance, and will import the rest of products—ones whose underlying inputs are relatively scarce. Moreover, even though it might not be possible to literally eliminate or smoothen out resource scarcity across economies (resources might easily be nontradable—like climate, territory, etc.), still trade in products will increase the supply of each and every product in each country. As the Heckscher-Ohlin-Samuelson theorem (HOS) predicts, relative prices will converge materially as a result of trade. That pertains to product prices. However, since the initial scarcity of resources is no longer as relevant, input prices will converge too. The HOS result thus maintains trade to be a vehicle of effective convergence across the endowments or opportunity sets.
Moreover, the ‘new’ trade literature pays a close attention to *intra*-industry trade, i.e. trade in similar products.

I applied these results to *information (knowledge) as an input*. All players have some information, and together they hold all the information available. Similar ‘vintages’ of information might be used to produce similar products. Implications of trade in these similar products (implied *intra*-industry trade in the underlying information) could then be studied.

**Application**

I was then challenged to come up with an example of a specific industry where information could be used as an input. A natural candidate was markets for financial assets. Moreover, since I was interested in *intra*-industry trade, it was natural to consider some pattern of holding many similar assets—which I initially dubbed ‘intra-type investing’ and later learned to be referred to as ‘style investing’ in the literature. Studying the implications of holding a full-blown portfolio capturing intra-style holding (over and above the conventional diversification) naturally led me to focus on contagion, or comovement across markets during episodes of crisis. To qualify my scope, it is important to stress that I did not intend to explain crisis per se—it’s origins, mechanisms, or timing. What I did look into was contagion, or a strong vehicle of excess comovement, given that we have crisis. I therefore study contagion as a variable conditioned on crisis, without endogenizing the latter condition.

**Comovement**

The literature suggests several alternative stories or mechanisms of comovement. Some of them build on rather strong assumptions, like herding agents or fundamental linkages (e.g. trade) between the affected markets. However, there is very little trade “South-South” (i.e. between the LDCs or emerging markets), and still they are the first to get involved in a bandwagon. As far as herding is concerned, even a 10-player setting of interactive optimization suggests an unwieldy complexity, which only explodes for \( n>10 \) cases. (At any rate, it’s not been done in the quantum physics to date: there are no numerical, let alone analytical, solutions modeling the behaviors of better than 10 interacting particles). Alternatively, large players like funds might herd, in so far as their managers face the moral hazard of finding themselves unemployed if they underperform the average or the market—so that mimicking might pay off. However, moral hazard applies to *agent* type large players only, while it would be interesting to draw more general implications (for *principal* type large investors). Otherwise, it is entirely reasonable that players each having incomplete information, might heed the perceived ‘insiders.’ But, it is exactly when we have the
specialty type players (style investors) that such herding becomes irrelevant: Supposedly, there are the insiders within their styles—whence, among other things, our focus on style investing. Finally, the literature conjectures comovement might be due to exposure to some common sources of shock. Again, such fundamental sources might or might not be there—my model does not hinge on an assumption like that. In a sense, models like CAPM do feature a ‘common source of shock’ (swings in the economy at large, as denoted by the market premium) and the varying sensitivities thereto (the individual betas). So, CAPM might do the job? But the most plausible candidate for ‘common source of shock’ is information (news) applying to similar assets or markets—or indeed intra-industry information as an input!

Theory

In my paper, I come up with a model that incorporates the notion of implied trade in information on the one hand, and the role of style investing on the other, in an attempt at capturing just about every stylized fact on contagion all within a minimalist yet powerful framework. Shleifer and Barberis (forthcoming 2003) is the more recent known theoretical treatment of the subject, even though style investing has been around as an established practice for a few decades now. However, I manage to avoid some of the overly strong assumptions they employ, such as discrete (or perfectly defined) styles. In fact, I show how style investing (intra-type holding, intra-industry trade) is effectively implied on a macro level, without there being any conscious or prior design. Put differently, investors need not knowingly engage in any style investing, for there to emerge the same consequences.

Apparently, then, my framework stems from an altogether distinct and different origin than Shleifer & Barberis (2003). For one thing, they do not seek to rationalize style investing per se (other than by showing it might prove profitable ex post), while I treat it at length from the standpoint of the intra-industry trade notion. As a consequence, I deploy a microeconomic analysis building on elasticities of substitution (constant ES, for the most part), which enables me to arrive at the results that cannot be captured by merely assuming perfectly defined styles. For one, as S&B themselves recognize, perfectly defined styles do not exist, because there will inevitably be some rich combinatorics of overlapping styles. By deploying elasticities of substitution, I arrive at a latent mechanism that holds for assets that may be related as closer substitutes, imperfect substitutes, or even neutrals (independent values); that is, my setting allows for a whole continuum of ‘styleness’—from distinct or unique styles to arbitrarily fuzzy or overlapping ones.
**Investors are Rational**

I impose no exotic behaviorist assumptions on rationality. Suppose individuals are rational, in that they mind their best self-interests and commit no systematic optimization errors. (Imperfect information per se being a standalone dimension over and above bounded rationality.) However, rationality does not amount to heeding/watching the ‘fundamentals’ *only*. Agents do observe fundamentals; suppose they in fact have perfect information on fundamentals, which arrives continually and is the same for the investors and the researcher that studies their behavior (so that there are no unobservables that serves as basis for some criticism of CAPM). However, if these rational players anticipate that the demand for the asset will deteriorate (because other players will likely be withdrawing), they will not hesitate to abandon this (otherwise fundamentally sound) asset. They will do so in an effort to minimize losses, which is a more binding direction of optimization for risk-averse individuals. (As the curvature of a nonlinear utility function suggests). Therefore, the players are rational so long as they maximize utility, rather than merely hold based on fundamentals only. In so far as there is a pure *interactive* component to value formation (i.e. in excess of information or news shaping the *fundamental* part of the value), externality and ‘*strategic rationality*’ cannot be ignored.

Now, moving with the trend is normally ascribed to *positive* feedback traders; negative feedback traders will be expected never to fail to tap into undervaluation. However, one has to distinguish between *minor* or short-run oscillations versus a major crash. Likewise, minor inflation or NAIRU unemployment rate cannot possibly suggest the same implications as *hyperinflation* or *mass* unemployment. A major crash could change the incentives and behavior (relative to routine undervaluation), in that the agents might perceive some kind of an *end* game horizon. For repeated games, it is reasonable that the final game involves very different strategies than the interior games did. A final game might pertain to a scenario whereby the network is perceived prone to decay in major ways. There is no way to cash in on undervaluation, because the price might take just too long to recover (if ever). The market need not become so thin as to vanish for good, though; yet, the recovery horizon could be just too long compared to the investor’s profile or liquidity preference (which defines how long-term or short-term a player she is). Therefore, investors will likely keep trading so long as an *end game horizon* (which is moving rather than fixed) is perceived to be long enough or uncertain. In a sense, what information or news players could be timing for is that relevant to end game horizon. Any major decisions on the part of larger players could be read by the rest as revealing some kind of ‘insider’ information as to a final game horizon.
Value Structure

I chose to model interactive or strategic value/price formation by deploying the notion of network. Dowd & Greenaway (1993) suggest an illuminating perspective on currency areas and dynamics thereof, by treating currencies as networks. Currency value will be higher, the larger the network; however, even if network decays, there is still some fundamental component left over not accounted for by interaction or externality:

\[
U = (a + b \log N) \int_0^T \exp{-r(t - T)} dt = (a + bn)/r.
\]

Our value function \( V = a_i + b_i \log(N_i + 1) \) features the fundamental value component (CAPM?) and the pure network component, respectively. I will get back to it after I outline the essential intuition behind our diagrammatic analysis building on elasticities of substitution.

Contagion (Downside Comovement)

On the one hand, contagious comovement would supposedly be due to some kind of complementarity—albeit spurious, but anyway occurring on the downside. For simplicity, consider a two-asset case first. Suppose they constitute a distinct style, or are perfect substitutes. The isoquant would in this case be linear, and its curvature will increase for any departure from perfect substitutability.

![Figure 8.a](image1.png) Lower semi-complementarity on the downside. ![Figure 8.b](image2.png) Mixed strategy equilibrium is not restored on recovery (no complementarity on the upside).

Reasonably, the lower threshold for assets is full neutrality: I do not perceive literal complementarity as very suggestive, necessary, or ‘interesting’ for the asset case, the way it applies to commodities. Or, if that were the case, the story could be over: We have complementarity, we have comovement.
Consider, for simplicity, a two-asset or two-market case (see Figure 8a above). For perfect complements, the budget line that kisses the isoquant everywhere could suggest multiple equilibria (indeed, an infinity of choices), unless the slopes are so different as to assure unique corner solutions. But, if both assets (or markets) are believed to be bound for en-masse withdrawal (as a style), substitution effect between them will be irrelevant and in fact will be nil. This is consistent with the convention in Shleifer & Barberis (2003) who suggest equal weights within styles. Admittedly, they maintain it for any state of nature, whereas I argue it will hold in crisis (end game) or under deterioration only. Indeed, on the upside (normal times when news improve for a style), investors might not only increase their exposure to both assets, but do so unequally: we simply do not know how the agent will choose on the linear isoquant, and there is no reason to anticipate any particular choices as more likely than other mixes. Under a minor undervaluation or otherwise deterioration of price, the budget line does not reduce all the way down to zero or minimum, so substitution effect might be of some relevance. Not so in crisis (end game), though, when the budget line rapidly converges to zero level: we are going to abandon the markets (the style) anyway, so interim substitution effect is irrelevant and is nil.

Such asymmetry of substitution effect between the normal upside and end game downside suggests that in the latter case (and only then), assets will behave as perfect complements (commove in terms of network size and prices). Their fundamental relationship (substitutability) is irrelevant: effectively they behave as complements. To crystallize this intuition:

1. Irrelevance of substitution effect amounts to ‘fixed proportions.’
2. Fixed proportions (Leontief function) suffices for perfect complementarity, which in turn captures [weak] complementarity.
3. Irrelevance of substitution effect is assured within a style (i.e. for perfect substitutes).
4. Therefore, perfect substitutability amounts to a potential for comovement, which materializes in the end game (in crisis, whatever its causes), and on the downside (significant undervaluation, or growing book-to-market ratio, or huge deterioration of value) being close to end game setups asymptotically.

To draw a bottomline, style investing does account for much of contagion. However, unlike S&B, my setting implies there is significant excess comovement on the downside over and above whatever symmetric cross-correlation they maintain. Excessive or pure comovement during crisis is an observed phenomenon, and is the focus of my study. Granted, as will be shown, mine is a latent mechanism building on effective relationships ex post, whereas theirs is a vehicle building on straightforward prior design. Importantly, I stress the relevant relationships between properties,
without necessarily knowing these properties per se. In particular, my modeling enables me to study the effective relationships between markets (elasticities of substitution) without knowing much about their own behaviors or otherwise cardinal parameters that might be accountable for crises. My emphasis on ordinal properties while assuming away cardinality, proves quite in line with the modern microeconomics legacy.

**Formalizing the Intuition**

In the previous section, I featured the basic intuition behind the relationships between assets (see Figure 8a). In fact, this same result could be shown formally, by employing the modern duality theory [e.g. Varian 1992, Ch. 6]. Microeconomic theory maintains a crucial duality between maximizing the utility or production function (direct or money-metric or its analogues like our value function) and minimizing the cost or expenditure function. In particular, it can be shown that the curvatures of their indifference curves are inversely related. Put differently, there is an inverse relationship between their elasticities of substitution. In particular, if the value function’s ES (defined with respect to quantities) is closer to substitutability, then the expenditure or loss function’s ES (defined with respect to prices or the loss Lagrange multipliers) will be closer to complementarity. Now, if we define the value function in terms of value improvement and the loss function in terms of value deterioration, then perfect substitutability on the upside (value isoquants) implies perfect complementarity on the downside (loss isoquants). But that’s exactly the point we maintained as a case for our asymmetry, or an irrelevance of substitution effect in crisis or on the downside! So long as the assets are perfect substitutes, they will remain that if their values are expected to improve, yet will act like perfect complements if their value is expected to deteriorate significantly.

By employing this duality-theoretic result, our intuition on the end game horizon becomes but supportive. We have suggested that crisis differs dramatically from routine, minor drops in value: reverting to mean which shapes the negative feedback trading incentives, does not hold near end game. However, the central result on asymmetry does not hinge upon this notion of end game horizon, anyway.

Technically, duality is perfectly defined for the less complex constraint sets. Remarkably, we have a single constraint (budget constraint). Incorporating another constraint, say, for an end game horizon criterion, might compromise duality to an extent. Which is one other reason to maintain the final game criterion as but a supportive pillar: there is a rationale behind choosing not to formalize it.
There is no way duality (and/or the asymmetry) could possibly be obtained or even conjectured without explicitly employing a formal analysis of elasticities of substitution. No wonder, the S&B paper fails to build on these tools, as it fails to incorporate the elasticities-based modeling and instead defines styles as some kind of ‘natural’ categories. They implicitly maintain perfect substitutability within styles and perfect neutrality across styles, which strong assumption does not stand up to reality checks and, more importantly, overlooks some central results. Moreover, it is exactly that intra-industry trade intuition that motivated the use of elasticities of substitution in conceptualizing the story and in formal modeling. Our manner of motivating style investment to be viewed at the crossroad of two fields could suggest some overlaps between these, as well as yield some implications relevant and applicable to both. It was made possible by looking at them through the underlying microeconomics common to both. To draw a bottom line, the Shleifer & Barberis study spots normal or symmetric correlations, without giving any account of contagious comovement in excess of these.

From Perfect Substitutes to a General Case

Arguably, engaging in style investing is largely the prerogative of large investors like funds. It would be interesting to know just how the masses of small individual players (not herding via membership of the same fund) could account for comovement. Moreover, as I pointed out elsewhere, perfect substitutability (discrete styles) is a fiction anyway. (Which would call for some kind of behavioral assumptions of bounded rationality to make a model like S&B quite stand up). I now show that assets/markets could be imperfect substitutes or even independents (neutrals), and still be prone to contagious comovement. The key here is the value structure, or the importance of the interactive component (in excess of bare-bones fundamental value). I will demonstrate how perfect substitutability (ideal style) results effectively from a value structure, without there being any a priori or actual style designs at work.

Value Structure

The individual value function as per each asset or network is, $V = a_i + b_i \log (N_i + 1)$. Assuming additive separability, the total value function is, $V' = \sum_i V_i = \sum_i a_i + \sum_i b_i \log N_i$. Indeed, this amounts to a CES function, with the [constant] elasticity of substitution equal to near zero (logs implying the neutrality case). I now deploy a modelling tool as in Hansen (1985), who suggested that a representative agent level accounts for higher elasticity of substitution (or sharp
swings in labor supply not attributable to tantamount shocks) that cannot be explained based on the individual utilities alone. That paper studied the labor market; I found the treatment could be adapted to our setting.

I will assume each investor holds the full-blown portfolio consisting of all assets out there—albeit some with a zero weight in the portfolio. So, assume there exists some allocation rule \( \alpha' \) which is a vector or matrix of asset weights. For simplicity, let’s study the two-network case first. The expected value of allocating between these would be as follows:

\[
EV = a(a + b \log (N + 1)) + (1 - a)b \log (0 + 1) = \\
\alpha a + ab \log (N + 1) + a - \alpha a + b \log (0 + 1) - \alpha b \log (0 + 1) = \\
a + ab \log (N + 1)
\]

However, since,

\[
N_1 + N_2 = N
\]

\[
N_1 = \alpha N ,
\]

\[
N_2 = N - N_1 = (1 - \alpha)N
\]

the resulting value is,

\[
EV_1 = a_1 + ab_1 \log (N + 1) = a_1 + \frac{N_1}{N} b_1 \log (N + 1) = a_1 + B_1 N_1 ,
\]

\[
B_1 = \frac{b_1 \log (N + 1)}{N} = const
\]

The total value CES is,

\[
V = \sum_i^m V_i = \sum_i^m a_i + \sum_i^m B_i N_i
\]

What this suggests rather unequivocally is that, even though individual value functions imply neutrality, the representative level assures linearity in \( N \)'s, or perfect substitutability. Moreover, it can be shown that this result holds for any scale (or risk aversion) other than \( \log N \) (maximum risk aversion):
\[ V = a_i + b_i N^\beta \implies \]

\[ EV = a \alpha(a + b(N + 1)^\beta + (1 - \alpha)(a + b) = a + b + ab[(N + 1)^\beta - 1] \alpha = \frac{N}{N} \]

\[ EV = a + b[1 + \alpha(1 + N + 1)^\beta - 1] = a + b[1 + cN_1] = \frac{(N + 1)^\beta - 1}{N} \]

\[ EV = a + b[cN_1 + 1] = (a + b) + bcN_1 = k_i + k_2N_i, k_1, and k, const = –QED! \]

Which suggests there will exist potential for comovement on the downside for any elasticity of substitution or degree of similarity between assets, even between styles (i.e. assets with zero ES, or neutrals).

**Diversification**

Krugman (1999) suggests that networks can hardly be the candidate sources of increasing returns to scale (that both the ‘new trade theory’ and the ‘new growth theory’ maintain to be the linchpin of modern trade and growth). He observes that, for the most part, networks exhibit positive yet decreasing returns to scale. Incidentally, our modeling fully complies with this stylized fact:

\[ V = a + b \log N, \]

\[ FOC : \frac{\partial V}{\partial N} = \frac{b}{N} > 0, \]

\[ SOC : \frac{\partial^2 V}{\partial N^2} = -\frac{b}{N^2} < 0. \]

However, the presence of decreasing returns to scale would suggest a rationale for holding as many networks as possible, which would actually justify a continuous CES:

\[ \dot{V} = \int_0^1 a_i + b_i \log N_i,di \]

In this light, what are the implications for diversification? Evidently, covariance does not apply conceptually to generic interactive components of value, \( COV(N_i, N_j) = 0, \forall i \neq j \)
So, the more important the pure network component of value, the less applicable the conventional
diversification is (and accordingly, the more rationale behind style investing). A rethinking of
diversification could pertain to the above-discussed diminishing returns to scale reason.

That said, how consistent is this style investing notion with the benefits of
diversification? On the one hand, style or intra-type investing is quite at odds with [inter-type]
diversification. However, in our setting, the investors are maximizing value rather than minimizing
risk. Moreover, it can be shown that style investing—holding many similar assets—comes quite in
line with diversification, even if we hold many perfect substitutes. If perfect substitutability can be
proxied as a unity correlation coefficient, \( \rho = 1 \iff r_{ij} = 1 \), then the style portfolio variance is this,

\[
VAR_{PORTFOLIO} = \alpha^2 VAR_i + (1 - \alpha)^2 VAR_j + 2\alpha_r (VAR_i VAR_j)^{1/2} = VAR_i = VAR_j
\]

In other words, the resulting portfolio risk will not exceed the maximum individual risks
for perfect complements, and will be lower than that for imperfect complements, \( \rho < 1 \) ! Now, of
course it would be ideal to have uncorrelated \( \rho = 0, r_{ij} = 0 \) assets, or for that matter negatively
correlated assets. But again, risk minimization is not the whole story. Moreover, there are no
conceptual grounds to believe covariances apply to generic network components:

\[
COV(N_i, N_j) = 0. \text{ Finally, Shleifer and Barberis find that style investing has in fact outperformed}
index investing or small-cap holding patterns.}

**Emerging Markets**

What is the candidate profile of market for which the interactive value component \( b^* \log N \) is
very important? That’s emerging markets, whose fundamental value is just too uncertain (variance
high), for lack of history. The previous formal treatment could shed light on why markets with a
low or uncertain fundamental component \( a \) are all the more likely to get hooked in contagious
comovement.

It would be reasonable to think that the fundamental value is the product of relevant
[cumulative] information, whereas the network size is that of news. We can denote information as

\( I \) and news as change or time derivative \( \dot{I} \), so that \( a_j = a_j(I_j) \) and \( N_j = N_j(\dot{I}_j) \). This speaks back
to the original idea of studying [intra-industry] trade in information, the latter (information) being
an input! Indeed, the shorter the [emerging] market’s history, the more distributed its fundamental
value and the less relevant the cumulative information; on the other hand, news (or change in
information) will be all the more relevant. We could easily incorporate transaction costs and
information costs into the formal analysis, to arrive at some implications of incomplete information
or restricted response to news:

\[ V = a_i \left( I(1-t_i) \right) + (1-t_2)b_i \log \left( N_i \left( t_1 \right) \right), \]

where \( t_i \) and \( t_2 \) denote the two dimensions of
efficiency or transaction costs: quality of information and/or news, and the liquidity or stickiness of
market (mostly relevant to the network value component, or for the shorter-horizon investors).

Finally, in effect, our framework implicitly addresses a few other important results in the
finance literature. The CAPM component is captured in \( a \), while the finding by Fama & French
(1995) of the importance of size and book-to-market value ratio could be captured (and re-
motivated!) as follows. ‘Size’ pertains to network size or indeed the excess demand for the asset
(not exactly the same as capitalization in their test), whose first order effect is positive and second-
order negative. This could be in line with the Walras’ law implicating that, assuming the desirability
condition, in a GE setting, price will be all the higher, the greater the excess demand on a particular
market. This formal result could in fact rationalize our intuition on strategic rationality and end-
game horizon—both pertaining to value (network) dynamics!

The very duality result could now be put in excess demand terms: so long as positive or
growing excess demands are substitutes, negative or decaying excess demands are complements! In
other words, stable or growing networks will be related as \( \rho \leq 1 \) substitutes, but networks expected
to decrease in size will relate as \( \frac{\rho}{\rho - 1} \) complements. Upper semivariances are substitutes, in which
case lower semivariances are complements. Indeed, the generalized value structure

\[ V = a_i + b_i \log \left( N_i + 1 \right) \]

could be rethought as follows: \( a \) refers to a general equilibrium value at
zero excess demand (which is like the long-term, fundamental component), while the interactive
part \( b*\log N \) refers to the importance of nonzero excess demand, which can be positive or negative,
growing or decreasing.

Book-to-market value (as in the French and Fama [1995] criticisms of CAPM) could, in turn,
suggest either undervaluation or deteriorating value. Indeed, a growing \( \frac{BV}{MV} \) could point out to a
short end game horizon (or to downside times). Alternatively, this gap (which could go either way)
could be interpreted as an effect of transaction costs (market or institutional stickiness). In
particular, higher transaction costs could prevent an overvalued asset from deteriorating and an
undervalued market from appreciating—exactly in line with the minus sign of the $\frac{BV}{MV}$ coefficient in their study. To draw a bottom line, it is to be expected that, empirically, our model should fare about as well as these studies whose scopes are implicitly captured therein (let alone that it might rationalize or re-motivate these empirical stories conceptually). The model I suggest captures both the “book-to-market value” and the “size” as the sign and size of excess demands $N$, respectively. Moreover, the $a$ component captures the fundamental value *a la* CAPM pricing.

References


