Article

Dirac Equation for General Spin Particles Including Bosons

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Abstract

We demonstrate (show) that the Dirac equation – which is universally assumed to represent only spin $+\frac{1}{2}$ particles; can be manipulated using legal mathematical operations – *starting from the Dirac equation* – so that it describes any general spin particle. If our approach is acceptable and is what *Nature* employs, then, as currently obtaining, one will not need a unique and separate equation to describe particles of different spins, but only one equation is what is needed – the *General Spin Dirac Equation*. This approach is more economic and very much in the spirit of unification – *i.e.*, the tie-ing together into a single unified garment – a number of phenomenon (or facets of physical and natural reality) using a single principle, which, in the present case is the bunching together into one theory (equation), all spin particles into the *General Spin Dirac Equation*.

Keywords: Dirac equation; Majorana equation; Bhabha equation; Relativistic wave equations.

"*Somewhere, something incredible is waiting to be known.*"

– **Carl Edward Sagan** (1934 – 1996)

1 Introduction

Studies of higher spin fields is currently an active field of research (cf. Iso et al. 2008, Wagenaar & Rijken 2009, Krishnan et al. 2014, Rivelles 2015). Majorana (1932) made the first attempt at extending the Dirac (1928*a*,*b*) equation so that it can explain particles with spin other than spin $+\frac{1}{2}$, the problem of constructing a covariant and consistent equation for higher-spin fields is still only partially solved (Bekaert et al. 2009). This problem has turned out to be among the most intriguing and challenging problems of all in Quantum Field Theory (QFT) (Bekaert et al. 2009). We here suggest a most trivial generalization of the Dirac (1928*a*,*b*) equation where it is transformed into a general spin equation.

As currently understood and presented in the wider (research) literature and in most – if not all – textbooks on the planet that deal with the Dirac equation, this equation is said to describe only spin $+\frac{1}{2}$ particles and nothing else. After Majorana (1932), Dirac (1936) made the second attempt at extending his equation so that it can explain particles with spin other than spin $+\frac{1}{2}$. Dirac (1936)'s efforts where followed up by Fierz (1939) and latter by Fierz & Pauli (1939). Also – theoretical physicists, William Rarità (1907 – 1999) and Julian Seymour Schwinger (1918 – 1994) constructed the Raritā-Schwinger equation which is a relativistic field equation which is assumed to explain spin $+\frac{3}{2}$ Fermions (Raritā & Schwinger 1941). In addition, there exists other attempts at a general spin equation (Bhabha 1944, 1949, Bargmann & Wigner 1948, Dowker 1967, Hurley 1972, Vaklev et al. 1979, Baisya 1995). What all these aforecited attempts including those that we have not mentioned is that – the resulting equation is fundamentally different from the original Dirac (1928*a*,*b*). With better clarity and insight in the present than before (Nyambuya 2009, 2013), we write down a general spin Dirac equation. The equation that we write down is not fundamentally different from Dirac's original equation.

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Written down in its usual covariant form, the original Dirac (1928*a*,*b*) equation is given by:

$$\imath\hbar\gamma^{\mu}\partial_{\mu}\psi = \mathrm{m}_0 c\psi, \tag{1.1}$$

where m_0 is the rest-mass of the particle, c is the speed of light in vacuum, ∂_μ are the four partial derivatives of space and time, ψ is the Dirac four component wavefunction (*i.e.* 4×1 "vector" field) and:

$$\gamma^0 = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{1.2}$$

are the 4×4 Dirac gamma matrices with I and 0 being the 2×2 identity and null matrices respectively. Throughout this reading, the Greek indices will be understood to mean $(\mu, \nu, ... = 0, 1, 2, 3)$ and lower case English alphabet indices $(i, j, k... = 1, 2, 3)$.

In the next section, we shall "see" that one can easily write down a general spin equation from the original Dirac equation and this equation represents a particle of spin $\frac{1}{2}s$ where ($s = \pm 1, \pm 2, \pm 3 \ \ldots$ *etc*). This single equation applies to both Bosons and Fermions. In-order to make this future task (of writing down the general spin Dirac equation) much easier – especially, the understanding of how to determined what spin a particular equation explains, it is perhaps wise and instructive for us to *ask-and-answer* the question: *Why is the Dirac equation said to represent (describe) a particle of spin 1/2?*

To do this, we shall – in the subsequent section; follow Dr. William O. Straub's presentation[2]. This approach is the standard approach that is used to demonstrate the fact that the Dirac equation – indeed – is, an equation that represents (describes) spin $+\frac{1}{2}$ particles. In §(3), we will motivate the need for Dirac particle that – separately – has a conserved spin and conserved orbital angular momentum. Lastly, in §(4), we will give a general discussion.

2 Spin of the Dirac Particle

The Dirac equation (1.1) can be re-written in the Schrödinger formulation as $(\mathcal{H}\psi = \mathcal{E}\psi)$ where $\mathcal{H}$ and $\mathcal{E}$ are the energy and Hamiltonian operators respectively. In this Schrödinger formulation, $\mathcal{H}$, will be such that it is given by:

$$\mathcal{H} = \gamma^0 \mathrm{m}_0 c^2 - \imath\hbar c\gamma^0\gamma^j\partial_j, \tag{2.1}$$

and $(\mathcal{E} = i\hbar\partial/\partial t)$.

Now, according to the quantum mechanical equation governing the evolution of any quantum operator $\mathcal{Q}$, we have:

$$\imath\hbar\frac{\partial\mathcal{Q}}{\partial t} = \mathcal{Q}\mathcal{H} - \mathcal{H}\mathcal{Q} = [\mathcal{Q}, \mathcal{H}]. \tag{2.2}$$

If:

$$[\mathcal{Q}, \mathcal{H}] \equiv 0, \tag{2.3}$$

then, the quantum mechanical observable corresponding to the operator $\mathcal{Q}$ is a conserved physical quantity *e.g.*, momentum with the operator $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$.

With this [equation (2.2)] in mind, Dirac asked himself the natural question – what the "strange" new γ-matrices appearing in his equation really represent. What are they? In-order to answer this question, he decided to have a "look" at the quantum mechanical orbital angular momentum operator:

$$\mathcal{L}_i = (\boldsymbol{r}\times\boldsymbol{p})_i = -\imath\hbar\epsilon_{ijk}x_j\partial_k, \tag{2.4}$$

[2]Straub, W. O.; Paper Title : "*Weyl's Spinor and Dirac's Equation*", `http://www.weylmann.com/`. Visited on this day: Nov. 2, 2015@12h38GMT + 2

where, ϵ_{ijk} is the completely-antisymmetric three dimensional *Levi-Civita* tensor. In the above definition of $\mathcal{L}_i$ the momentum operator $\boldsymbol{p}$ is the usual quantum mechanical operator, *i.e.*:

$$\boldsymbol{p} = -\imath\hbar\boldsymbol{\nabla} \qquad \Rightarrow \qquad p_i = \imath\hbar\partial_i. \tag{2.5}$$

From this definition of $\mathcal{L}_i$ given in (2.4), it follows from (2.2) that $\imath\hbar\partial\mathcal{L}_i/\partial t = [\mathcal{L}_i, \mathcal{H}]$, will be such that:

$$\imath\hbar\frac{\partial\mathcal{L}_i}{\partial t} = -\imath\hbar \mathrm{m}_0 c^2 \epsilon_{ijk}\left[x_j\partial_k, \gamma^0\right] + \hbar^2 c\epsilon_{ijk}\left[x_j\partial_k, \gamma^0\gamma^l\partial_l\right]. \tag{2.6}$$

Because – the term $\gamma^0 \mathrm{m}_0 c^2$ is a constant containing no term in p_i, it follows from this fact that $(\epsilon_{ijk}\left[x_j\partial_k, \gamma^0\right] \equiv 0)$, hence (2.6) will reduce to:

$$\imath\hbar\frac{\partial\mathcal{L}_i}{\partial t} = \hbar^2 c\epsilon_{ijk}\gamma^0\gamma^l\left[x_j\partial_k, \partial_l\right] = \hbar^2 c\epsilon_{ijk}\gamma^0\gamma^l\left(x_j\partial_k\partial_l - \partial_l x_j\partial_k\right). \tag{2.7}$$

From the commutation relation of position (x_i) and momentum $(-\imath\hbar\partial_j)$ due to the Heisenberg (1927) uncertainty principle, namely $(-\imath\hbar\left[x_i, \partial_j\right] = -\imath\hbar\delta_{ij})$ where δ_{ij} is the usual Kronecker-delta function, it follows that (2.7) if we substitute $(\partial_l x_j = x_j\partial_l + \delta_{lj})$ into (2.7), this equation is going to reduce to:

$$\imath\hbar\frac{\partial\mathcal{L}_i}{\partial t} = \hbar^2 c\epsilon_{ijk}\gamma^0\gamma^l\underbrace{\left(x_j\partial_k\partial_l - x_j\partial_l\partial_k\right)} + \hbar^2 c\epsilon_{ijk}\gamma^0\gamma^l\delta_{lj}\partial_k. \tag{2.8}$$

The term with the under-brace[3] vanishes identically, that is to say: $(x_j\partial_k\partial_l - x_j\partial_l\partial_k \equiv 0)$; and $(\epsilon_{ijk}\gamma^0\gamma^l\delta_{lj} = \epsilon_{ilk}\gamma^0\gamma^l)$, it follows that (2.8) will reduce to:

$$\imath\hbar\frac{\partial\mathcal{L}_i}{\partial t} = \hbar^2 c\epsilon_{ilk}\gamma^0\gamma^l\partial_k. \tag{2.9}$$

Since this result (2.9) is non-zero, it follows from the dynamical evolution theorem (2.3) of Quantum Mechanics (QM) that none of the angular momentum components $\mathcal{L}_i$ are – for the Dirac particle – going to be constants of motion. This result greatly bothered the great and agile mind of Paul Dirac. For example, a non-conserved angular momentum would mean spiral orbits – at the very least, this is very disturbing because it does not tally with observations. The miniature beauty that Dirac had had the rare privilege to discover and, the first human being to "see" with his beautiful and great mind – this – had to be salvaged[4] somehow.

Now - *enter spin*! Dirac figured that *Subtle Nature* must conserve something redolent with orbital angular momentum, and he considered adding something to $\mathcal{L}_i$ that would satisfy the desired conservation criterion, *i.e.*: call this unknown, mysterious and arcane quantity $\mathcal{S}_i$ and demand that:

$$\imath\hbar\frac{\partial\left(\mathcal{L}_i + \mathcal{S}_i\right)}{\partial t} \equiv 0. \tag{2.10}$$

Solving (2.10) for $\mathcal{S}_i$, Dirac arrived at:

$$\mathcal{S}_i = \frac{1}{2}\hbar\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} = \frac{1}{2}\hbar\gamma^5\gamma^i. \tag{2.11}$$

Realising that: (1) the matrices σ_i are Pauli matrices and they had been *ad hocly* introduced into physics to account for the spin of the Electron (Uhlenbeck *&* Goudsmit 1925); (2) and that, his equation when taken in the non-relativistic limit, it would account for the then unexplained gyromagnetic ratio ($\mathrm{g} = 2$) of the Electron and this same equation emerged with σ_i explaining the Electron's spin, Dirac seized the golden moment and forthwith identified $\mathcal{S}_i$ with the ψ-particle's spin. The factor $\frac{1}{2}\hbar$ in $\mathcal{S}_i$ implies that

[3] When we get to §(**??**), the momentum p_i and position x^i will be required to submit to a new type of *Lie-Algebra*. This new *Lie-Algebra* will be such that the term in the under-brace will be such that $(x_j\partial_k\partial_l - x_j\partial_l\partial_k = -\partial_k)$, so that $\imath\hbar\partial\mathcal{L}_i/\partial t = 0$.

[4] Such is the indispensable attitude of the greatest theoretical physicists that ever graced the face of planet Earth – beauty must and is to be preserved; this is an ideal for which they will live for, and if needs be, it is an ideal for which they will give-up their life by taking a gamble to find that unknown quantity that restores the beauty glimpsed!

the Dirac particle carries spin $+\frac{1}{2}$, hence, the Dirac equation (1.1) is an equation for a particle with spin $+\frac{1}{2}$!

While Dirac was able to explain and "demystify" spin in this way (*i.e.*, as demonstrated above), we are of the strong view that the non-independent conservation of spin and orbital angular momentum is problematic insofar as the stability of the Dirac particle is concerned. We shall elucidate on this matter and proffer a solution in future reading. In the subsequent section, we shall demonstrate how one can modify the Dirac equation so that it represents a general spin particle other than spin zero particles.

3 Modification to General Spin Equation

What we would like to do now is to demonstrate that – starting from an acceptable premise and from there-on applying logic, the Dirac equation can be modified so that it represents a general spin particle. To do this, we need first to ask ourself how the spin of any general particle can be represented. We are certain that the reader will – *without any qualms* – agree that if as given in (2.11), S_i duly represents a spin $+\frac{1}{2}$ particle, then, the spin $\mathcal{S}_i(s)$ of any general particle may as-well be represented by $\mathcal{S}_i(s)$, such that:

$$\mathcal{S}_i(s) = \frac{1}{2}s_i\hbar \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad \text{for} \quad (i = 1, 2, 3), \tag{3.1}$$

where $(s_i = \pm 1, \pm 2, \pm 3, \dots etc)$. If $(s_i = s;\ \forall i = 1, 2, 3)$, then, when s is even, we have a Boson and when is s odd, we have a Fermion.

If one agrees to the above said that $\mathcal{S}_i(s)$ is the spin of any general particle, then, they will invariably agree that if $\mathcal{L}_i(s)$ is the corresponding orbital angular momentum operator such that the total orbital angular momentum operator $[\mathcal{J}(s) = \mathcal{L}_i(s) + \mathcal{S}_i(s)]$ commutes with the Hamiltonian $\mathcal{H}(s)$, then, the equation:

$$\imath\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}(s)\psi, \tag{3.2}$$

should be the Dirac equation of a general spin particle. If we define $\mathcal{L}_i(s)$ and $\mathcal{H}(s)$, such that:

$$\mathcal{L}_i(s) = -\imath\hbar\epsilon_{ijk}x_j\gamma_k s_k\partial_k, \tag{3.3}$$

and:

$$\mathcal{H}(s) = \gamma^0 \mathrm{m}_0 c^2 - \imath\hbar c\gamma^0\gamma^j s^j\partial_j, \tag{3.4}$$

then, one can – as has been done in the previous section – demonstrate that:

$$[\mathcal{J}_i(s), \mathcal{H}(s)] = 0. \tag{3.5}$$

This fact (3.5) has "manually" (but in a less unclear manner) been demonstrated in the reading Nyambuya (2009).

Written explicitly, it follows from the foregoing that the Dirac equation for a general spin particle is such that:

$$\imath\hbar\gamma^0\frac{\partial\psi}{\partial ct} + \imath\hbar s^j\gamma^j\frac{\partial\psi}{\partial x^j} = \mathrm{m}_0 c\psi. \tag{3.6}$$

Accordingly, this equation must apply to both Bosons and Fermions whose spin is non-zero. Using the usual quantum mechanical spin ladder operators, one can show that if, s_i, where to change from one value to the next, it will do so by increasing or decreasing by one integral unit – that is to say: $(s_{i+1} - s_i = 1)$. From this, it follows that a Boson will stay a Boson and a Fermion will stay a Fermion, there this not transmutation from Boson to Fermion or *vise-versa*. Additionally, since the Dirac state is one such that $(s_i = +1)$, it follows from this $(s_i = \pm 1, \pm 2, \pm 3, \dots etc)$.

4 General Discussion

We have herein demonstrated that the Dirac equation – which is universally assumed to describe only spin $+\frac{1}{2}$ particles; can be manipulated using legal mathematical operations – *starting from the Dirac equation* – so that it describes any general spin particle. Most if not all approaches (*e.g.* Majorana 1932, Dirac 1936, Fierz *&* Pauli 1939, Rarità *&* Schwinger 1941, *etc*) aimed at achieving a general spin particle involve a fundamental change in the Dirac equation. The present approach is unique in that regard in that there is no fundamental change and meaning in the original Dirac equation.

The very same Dirac equation in the same form – *except the introduction of the numbers*, s_i; here describes both Bosons and Fermions. From a vantage point of unity, this is an appealing aspect of the present approach. Unification requires the explanation of a wide range of phenomenon using a minimal number principles. An explanation of a diverse of phenomenon using a single principle (equation), as has been proposed herein – this is the desideratum of the purest soul of the searching theoretical physicist. If our approach is acceptable and is what *Nature* employs, then, as currently obtaining, one will not need a unique and separate equation to describe particles of different spins, but only one equation is what is needed – the *General Spin Dirac Equation* (3.6).

In the present scheme – in general, the spin of a particle is here given by:

$$\boldsymbol{S} = \frac{1}{2}s_1\hbar\boldsymbol{\sigma}^1 + \frac{1}{2}s_2\hbar\boldsymbol{\sigma}^2 + \frac{1}{2}s_3\hbar\boldsymbol{\sigma}^3. \tag{4.1}$$

The associated spin quantum numbers $(s_1, s_2, s_3 : s_i)$ are in general not equal but are different *i.e.*: $(s_1 \neq s_2)$, $(s_1 \neq s_3)$ and $(s_2 \neq s_3)$. If this is so, then, for the Electron and all other particles whose spin we known, we have found that the spin is a fixed physical quantity. Each time we measure the spin of particle, we always arbitrarily measure the spin along any one of the three axises (x, y, z). If the spin where different along these these axises, then, we would find that spin is not a fixed quantity. The fact that experiments reveal that the spin is a fixed physical quantity, this – for the present paradigm – suggests that the spin quantum number s_i is to be set equal for the three axises, *i.e.* $(s_i = s)$. So doing, it follows that equation (3.6) may – be written as:

$$\imath\hbar\gamma^0\frac{\partial\psi}{\partial ct} + \imath\hbar s\gamma^j\frac{\partial\psi}{\partial x^j} = \mathrm{m}_0 c\psi. \tag{4.2}$$

this equation (4.2) represents – *in the present paradigm* – a particle of spin $\frac{1}{2}s$. The corresponding Einstein energy-momentum equation for the particle described by the relativistic equation (4.2), is such that:

$$E_s^2 = s^2p^2c^2 + \mathrm{m}_{0s}^2c^4, \tag{4.3}$$

where E_s and m_{0s} are the energy and rest-mass of a particle of spin $\frac{1}{2}s$ respectively. This energy equation (4.3) was first written down in the reading Nyambuya (2009). The group velocity $v_{\rm g}$ of matter waves described by (4.3) will have to be:

$$v_{\rm g} = \frac{1}{s}\frac{\partial E_s}{\partial p}. \tag{4.4}$$

According to (4.3), the usual Einstein energy-momentum equation corresponds to the case $(|s| = 1)$ *i.e.*:

$$E_1^2 = p^2c^2 + \mathrm{m}_{01}^2c^4. \tag{4.5}$$

Clearly, from (4.3) and (4.5), it follows that:

$$E_s = sE_1 \quad \text{and} \quad \mathrm{m}_{0s} = s\mathrm{m}_{01}. \tag{4.6}$$

What (4.6) implies is that higher spin particles are expected to have higher energies and rest masses and these come in integral multiples of the lowest spin state energies and masses. The spin state ($|s| = 1$) is the lowest possible spin state.

Now, before we close this reading, it is important that we mention that – in our presentation, it may appear as though we have ignored the all-important mathematical result based on the theory of representations for the Lorentz Group – Spin $(3, 1)$, namely that, the Dirac equation can represent only spin $+\frac{1}{2}$ particles. Because of this, the Dirac matrices arise in the equation and not *vice versa*. This can be observed in the relation between the spin operators and the Lorentz (algebra) generators. By the same token, higher spin representations would require of different sets of "matrices" (*e.g.* the Bhabha 1944, 1949, equation), even infinite dimensional ones (*e.g.* the Majorana 1932, equation), to correspond to faithful representations of the higher spin Lorentz algebra. These shortcomings do not apply to the approach/scheme that we have adopted.

Perhaps, to demonstrate equation (4.6)'s potential, we shall apply this to the case of photon in a gravitational field – *i.e.*, to the gravitational bending of starlight in the vicinity of massive body. Let us assume that the photon has a vanishing rest-mass [*i.e.*, ($\mathrm{m}_0 \equiv 0$)]. We know a photon as spin $+1$ particle, this – according to (4.6) – implies that ($s = 2$) for the photon. From these facts, it follows from (4.6), that ($E = 2pc$). According to the definition of the group velocity given in (4.4), these photons will travel at the speed of light c. Actually, the definition of the group velocity given in equation (4.4) has been defined so that for the case ($\mathrm{m}_0 = 0$), we must have ($v_{\mathrm{g}} = c$).

Now, taking as suggested in the readings Nyambuya & Simango (2014), Nyambuya (2015), namely that:

1. ($E = \mathrm{m_g}c^2$) and ($p = \mathrm{m}_i c$) where $\mathrm{m_g}$ and m_i are the gravitational and inertial mass of the photon.
2. That the gravitation deflection angle δ in-accordance with Newtonian gravitational theory where the identity of gravitational and inertial mass are maintained: ($\delta_\gamma = 4\gamma G\mathcal{M}_\odot/c^2\mathcal{R}_\odot$), where ($\gamma = \mathrm{m_g}/2\mathrm{m}_i$) and $\mathcal{M}_\odot$ and $\mathcal{R}_\odot$ are the Solar mass and radius respectively.
3. For the case where the gravitational and inertial mass are identical [($\mathrm{m_g} \equiv \mathrm{m}_i$) $:\Rightarrow$ ($\gamma = 1/2$)] as is assumed in Newtonian gravitation: $\delta_N = 2G\mathcal{M}_\odot/c^2\mathcal{R}_\odot$ and as is well known, this is a factor 2 smaller than what one obtains from Einstein (1916)'s embellished General Theory of Relativity (GTR).

It follows from *items* (1) and (2) above, that:

$$\delta_s = \frac{2sG\mathcal{M}_\odot}{c^2\mathcal{R}_\odot}. \tag{4.7}$$

From this formula (4.7), it follows that the gravitational bending angle δ will depend on the spin of the photon which in this case is ($s = 2$) and gives that same formula as that obtained in Einstein (1916)'s GTR:

$$\delta_{\mathrm{GTR}} = \frac{4G\mathcal{M}_\odot}{c^2\mathcal{R}_\odot}. \tag{4.8}$$

If the present ideas are anything to go-by, it follows that the missing factor '2' between Newtonian gravitational theory and observations can be explained on the basis of the spin of the photon. Apart from this, it follows that in the case of the Solar gravitational deflections where ($\delta_\odot = 1.75''$), for deflections well in excess of this – such as the June 19, 1936 USSR eclipse result which gave: ($\delta = 2.73 \pm 0.31''$) (Mikhailov 1940, 1949), this can be explained as a being a result of higher spin photons (*i.e.*, spin $+\frac{3}{2}$ photons[5] in this case). This June 19, 1936 USSR eclipse result is about 1.6 times that predicted by Einstein's GTR and it has not been explained or taken seriously as signalling a possible deviation from Einstein's GTR. If anything, the present ideas offer hope for an explanation. What is required is to

[5] Spin $+\frac{3}{2}$ would make such a photon a Fermion and this is unheard of – but, we must be open minded to entertain this as dictated to us by the theory until such a time that we have enough evidence to rule the theory as un-physical.

dedicate an entire reading to these matters where we will look into these issues in much greater depth than the pedestrian analysis that we have just conducted here.

In-closing, we should say that, while we have written down [in a much more lucid manner than before (Nyambuya 2009, 2013)] an equation for a general spin particle, there is nothing in the theory that tells us how to achieve such higher spin particles in the laboratory or how they occur in *Nature*. Therefore, the present work may be relevant in the future if discoveries of higher spin particles are ever made. It is off-cause always good to have forehand a theory ready to explain these eventualities if they ever "*visit our world*".

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