Anti Bohr – Quantum theory and causality.

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Abstract. The political attitude and the ideology of a very small elite of physicists (Niels Bohr, Werner Heisenberg, Max Born and view other) played a major role in the construction of the Copenhagen Interpretation of quantum mechanics in the 1920s. Lastly, the hegemonic standard acausal Copenhagen Interpretation of quantum mechanics abandoned the principle of causality in quantum mechanics and opened a very wide door to mysticism, logical fallacies and wishful thinking in physics and in science as such. Historically, the Second International Congress for the Unity of Science (Copenhagen, June 21-26, 1936) tried to solve the problem of causality within physics but without a success. Thus far, 80 years after the Second International Congress for the Unity of Science this contribution at the Linnaeus University in 2016 in Växjö Sweden will make an end too Bohr's and Heisenberg's dogma of non-causality within quantum mechanics and re-establish the unrestricted validity of the principle of causality at quantum level and under conditions of relativity theory by mathematizing the relationship between cause and effect in the form of the mathematical formula of the causal relationship k. In contrast to Bohr, Heisenberg and other representatives of the Copenhagen interpretation quantum mechanics, a realistic interpretation of quantum theory grounded on the unrestricted validity of the principle of causality will expel any kind of mysticism from physics and enable a quantization of the gravitational field.

Keywords: Quantum theory, Relativity theory, Unified field theory, Causality

1 Introduction

The theory of causality is deeply connected with our understanding of objective reality, the causal investigations and the explanatory ambitions of objective reality especially by physical sciences.

1.1 Causality and philosophy

A good deal of the theoretical work in the theory of causality has been developed by several, well known philosophers. One of the first documented attempts to present a rigorous theory of causation came from the Greek philosopher and scientist Aristotle (384–322 BC). Aristotle developed a theory of causality commonly referred to as *the doctrine of four causes*. Many aspects and general features of Aristotle's logical concept of causality are meanwhile extensively and critically debated in secondary

Submitted to http://www.vixra.org/ on Sunday, November 13, 2016 11:19:33 AM

literature. Among other outstanding authors who worked on the problem of Causality David Hume is still present. Hume's (1711-1776) skeptical conception of causality is commonly known as *the regularity theory of causation*. According to Hume [1],

“we may define a cause to be *an object, followed by another,* and where all the objects similar to the first are followed by objects similar to the second. Or in other words where, *if the first object had not been, the second never had existed*.”

Roughly speaking, Hume’s understanding of causality is grounded on the *post hoc ergo propter hoc fallacy*. A day follows the night but because of this the day is not the cause or a cause of the night and vice versa. Hume’s claim that “if the first object had not been, the second never had existed” is widely used as the foundation of the counterfactual analysis of causation (i. e. David Lewis [2]). Paul-Henri Thiry, Baron d'Holbach (1723-1789), a philosopher of the French Enlightenment, notorious for his atheism and criticisms of Christianity, developed in his philosophical writings an one sided, mechanistic and deterministic theory of causality in which causality is grounded on an uninterrupted succession of causes and effects.

“L’univers, ce vaste assemblage de tout ce qui existe, ne nous offre partout que de la matière et du mouvement : son ensemble ne nous montre *qu’une chaîne immense et non interrompue de causes et d’effets* : quelques-unes de ces causes nous sont connues … d’autres nous sont inconnues …” [3]

In broken English:

“The universe, that vast assemblage of every thing that exists, presents only matter and motion: the whole offers to our contemplation, nothing but an immense, an uninterrupted succession of causes and effects; *some of these causes are known to us, … others are unknown to us ...* ”

d’Holbach links cause and effect to changes as such:

“Une cause, est un être qui en met un autre en mouvement, ou qui produit quelque changement en lui. L’effet est le changement qu’un corps produit dans un autre à l’aide du mouvement.” [4]

In broken English:

“A cause is a being which puts another in motion, or which produces some change in it. The effect is the change produced in one body, by the motion or presence of another.”

The 19th Century German philosopher, G.W.F. Hegel (1770–1831) provided a very abstract and idealistic philosophical account of the nature of causality [5] while relying on the dialectical method.

“Daher hat zwar die Ursache eine Wirkung, und ist zugleich selbst Wirkung, und die Wirkung hat nicht nur eine Ursache, sondern ist auch selbst Ursache. Aber die Wirkung, welche die Ursache hat, und die Wirkung, die sie ist – ebenso die Ursache, welche die Wirkung hat, und die Ursache, die sie ist -, sind verschieden.“ [6]

In broken English:

‚Therefore, though the cause has an effect and is at the same time itself effect, and the effect not only has a cause but is also itself cause, yet the effect which the cause has, and the effect which the cause is, are different, as are also the cause which the effect has, and the cause which the effect is.‘

Some other authors (e.g. Reichenbach [7], Suppes [8], Salmon [9]) preferred a probabilistic approach to the theory of causation. Stiehle himself is following Hegel. “Eine Einheit von Gegensätzen verkörpert die Beziehung von Ursache und Wirkung …” [10]. Barukčić is of the position that the dualism and unity between cause and effect is the foundation of the relationship between cause and effect. “Ohne einander kein Gegeneinander. Ursache und Wirkung bilden innerhalb dieses Zusammenhangs Gegensätze.“ [11] We shall not discuss neither these nor other best known theories of causality in detail. Still, the relationship between cause and effect is not solved, neither philosophically nor mathematically. The aim of this publication is to characterize the relationship between cause and effect while using the tools of probability theory. The motivation for probabilistic approaches to causation is of fundamental and far reaching importance, since such an approach, if successful, would be compatible with quantum theory while achieving a closer match with commonsense judgements about causation too. The attempt to analyze and understand causation in terms of probability theory cannot be successful without addressing a couple of preliminary issues. What is a cause or what is the cause, what is an effect or what is the effect? In principle, may an effect occur in the absence of a cause? And the other way, may an effect fail to occur in the presence of a cause? In so far, *what does constitute a causal relation*? On the other hand, if it is unclear what does constitute the causal relation, can we answer the question, *what does not constitute a causal relation*. Can a cause as such be independent from its own effect and vice versa, under conditions where *a deterministic causal relationship* is assumed?

1.2. Causality and mathematics

The concept of independence is of fundamental importance in probability theory and in science as such. Historically, the mathematical concept of independence is backgrounded by De Moivre too. De Moivre defines independence of events in the following way.

“Two Events are independent, when they have no connexion one with the other, and that the happening of one neither forwards nor obstructs the happening of the other. Two events are dependent, when they are so connected together as that the Probability of either's happening is altered by the happening of the other.” [12] In last consequence, De Moivre mathematizes independence in the form of an example as follows:

“those two Events being *independent*, the Probability of their both happening will be 1/13 × 1/13 = 1/169” [13]

In the following, one of the first detailed mathematical trials to mathematize co-relation can be ascribed to the French physicist Auguste Bravais [14],[15] (1811-1863), Francis Galton (1822 – 1911), the 1909 knighted English Victorian statistician and anthropologist, and at the end to Karl Pearson (1857 – 1936). In fact, following Karl Pearson himself Bravais developed a complete theory of correlation. “The fundamental theorems of correlation were for the first time and almost exhaustively discussed by BRAVAIS (‘Analyse mathernatique sur les probabilités des erreurs de situation d’un point.’ Mémoires par divers Savans, T. IX., Paris, 1846, pp. 255-332) nearly half a century ago. He deals completely with the correlation of two and three variables.” [16] Pearson's alternative mathematical account to non-causation is presented especially by his publication of the correlation coefficient [16], first conceived by Francis Galton [17], and Person’s publication of the mean square contingency [18] as

$$\phi^2 \equiv \frac{\left(\left(a \times d\right)-\left(b \times c\right)\right) \times \left(\left(a \times d\right)-\left(b \times c\right)\right)}{\left(a+b\right) \times \left(c+d\right) \times \left(a+c\right) \times \left(b+d\right)} \tag{1}$$

introduced by Pearson as a response to Yules [19] association of two attributes. Pearson’s ongoing battle against causation was motivated by the goal to exterminate any kind causation from statistics and science as such. Altogether, according to Pearson, “We are now in a position, I think, to appreciate the scientific value of the word cause. Scientifically, cause ... is meaningless ...” [20]. In general, “there is ... no true cause and effect” [21]. The reader who is still not hardly impressed by Pearson denial of any kind of causation, may consider Pearson’s concept of causality as follows. “No phenomena are causal” [22]. Finally, “The wider view of the universe sees all phenomena as correlated, but not causally related.” [23] Thus far, for Pearson, causation is not the major issue and Pearson’s approach the problem of causation can be summarized by his demand that “... there is association but not causation.” [24]. Pearson’s methodological reductionism of causation to correlation became a heritage of modern statistics and mathematics and had some crucial epistemological and ontological implications. Generations of scientists, mathematicians and philosophers were influenced by Pearson’s rejection of any kind of causality and the substitution of causation by correlation. In summary, Pearson's very transparent and dogmatic hostility towards causation is derived from his own philosophical point of view and still not abandoned by mathematics and statistics. “Pearson’s philosophy discouraged him from looking too far behind phenomena” [25]. In particular, neither Pearson’s correlation coefficient nor his mean square contingency can be regarded as the mathematical formula of the causal relationship k. Pearson failed to provide or to derive a self-consistent mathematical proof of a mathematical formula of the causal relationship [26]. Meanwhile many publications demonstrated that correlation is not identical with causation. One of the many very convincing [27], easy to read and yet innovative contributions to this topic was published by Sober. The sea levels in Venice and the bread prices in Britain have increased steadily with time with the consequence that higher than average bread prices tend to be associated with higher than average sea levels. Sober found a highly significant correlation between the sea levels in Venice and the bread prices in Britain, in other words a highly significant correlation between two causally independent processes. Finally, the fundamental mathematical and historical breakthrough in the concept of independence an thus far the mathematical foundation of causality can be ascribed to the measure-theoretic contributions to the mathematics of probability theory by the 20th-century Russian mathematician Andrei Nikolajewitsch Kolmogorow (1903-1987). In fact, it is insightful to view some of Kolmogorow’s theorectical approaches to the concept of independence. “The concept of mutual independence of two or more experiments holds, in a certain sense, a central position in the theory of probability.” [28] And rightly too. However, Kolmogorov's axiomatization of the Theory of Probability is a cornerstone of the assimilation of measure theory to probability theory. The concept of independence is still of strategic and central importance. “In consequence, one of the most important problems in the philosophy of the natural sciences is - in addition to the well-known one regarding the essence of the concept of probability itself - to make precise the premises which would make it possible to regard any given real events as independent.” [29] Thus far, neither Heisenberg’s uncertainty principle, nor Bell’s theorem nor CHSH-inequality have refuted the law of independence [30] of probability theory or Kolmogorov's probability calculus.

1.3. Causality and physics

At least *the law of independence* is the point where (quantum) physics meet probability theory and vice versa. According to Einstein, the law of independence is the foundation of physical sciences. “Ohne die Annahme einer … Unabhängigkeit der … Dinge voneinander … wäre physikalisches Denken … nicht möglich.“ [31] Einstein’s position in broken English: “Without the assumption of ... independence of ... things from each other ... physical thinking ... is not possible.” Einstein is elaborating on the principle of locality as follows: „Für die relative *Unabhängigkeit* räumlich distanter Dinge (A und B) ist die Idee characteristisch: äussere Beeinflussung von A hat keinen unmittelbaren Einfluss auf B; dies ist als «*Prinzip der Nahewirkung*» bekannt, das nur in der Feld-Theorie konsequent angewendet ist. Völlige Aufhebung dieses Grundsatzes würde die Idee von der Existenz (quasi-) abgeschlossener Systeme und damit die Aufstellung empirisch prüfbarer Gesetze in dem uns geläufigen Sinne unmöglich machen.“ [32] In broken English: ‘For the relative independence of spatially distant things (A and B) the following principle is characteristic: any external influence of A has no direct influence on B; This is known as a ‘principle of locality’ which is only applied consistently in field theory. This principle completely abolished would disable the possibility of the existence of (nearly-) closed systems and the establishment of empirically verifiable laws in the common sense.’ A further position Einstein’s on the principle of locality is the following: “But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S2 is independent of what is done with the system S1, which is spatially separated from the former.” [33] Einstein is linking the principle of locality to the law of independence of probability theory. Due to Einstein, both are identical. However, Heisenberg's uncertainty principle, endorsed especially by the founding fathers of the so-called Copenhagen interpretation of quantum mechanics, Bohr, Born and other plays an important role in many discussions on the theoretical implications of quantum mechanics. In particular the consistency of *the principle of causality* is one striking aspect of Heisenberg's uncertainty principle and the Copenhagen dominated quantum mechanics. According to Heisenberg and his own uncertainty principle, quantum mechanics has refuted the principle of causality definitely. “... so wird durch die Quantenmechanik die Ungültigkeit des Kausalgesetzes definitiv festgestellt“ [34]. Bohr supported Heisenberg's position. In fact, Bohr in his striving to find a common ground for a causal description for physics and our knowledge in general, addressed the assembly of scientists by demanding that the “so-called indeterminacy relations explicitly bear out the limitation of causal analysis“[35] In fact, a characteristic feature of Bohr's point of view and his special account of the principle of causality is the demand that “physics ... forces us to replace ... causality by ... complementarity“ [36] Deep doubts about the unlimited validity of the principle of causality were implied by the Copenhagen dominated interpretation of quantum mechanics. Historically, a so-called *Second International Congress for the Unity of Science* organized in Copenhagen (June 21-26, 1936) was dedicated to the problem of causality in physics but did succeed to find a solution. “The Second International Congress for the Unity of Science was to deal primary with the problem of causality“[37] Under these circumstances, we are faced with the necessity of a radical revision of the foundation for explanation and description of natural phenomena. Among Einstein and many others too, Hans Reichenbach (1891-1953) states this straightforward as “Quantenmechanik [*hat, author*] zu Zweifeln an der unumschränkten Gültigkeit des Kausalprinzips geführt“ [38] Independent of Heisenberg’s uncertainty, Bell’s theorem too excludes causality. “The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality … It is the requirement of locality … that creates the essential difficulty .” [39] Indeed, in theoretical quantum mechanics, principles and theorems (no-go theorem) like Heisenberg's uncertainty principle, Bell's theorem, the CHSH inequality et cetera state with respect to the principle of causality are demanding that a deterministic relationship between cause and effect is physically not possible. Today, these quantum mechanical no-go theorems are already at or just before the level of accepted wisdom. Just a minority of dissenters try to dispute these no-go wisdom. In particular, wrong scientific positions shouldn't make it through history. Today, the refutation of the main no-go principles of the Copenhagen dominated interpretation of quantum mechanics like Heisenberg's uncertainty principle [40],[41],[42] Bell's theorem [43],[44] the CHSH inequality [43],[44] are simply being ignored or not being referred to by scientists. Thus far, a solution of the problem of a deterministic relationship between cause and effect at quantum level is not in sight. With regard to new conclusions and insights, this paper is organized as follows. In the section, *Material and methods*, I will give some basic quantum mechanical and mathematical definitions and a terminological distinction only as much as is necessary for a better understanding of this paper due to the immense amount of literature known. In the section, *Axioms*, I will introduce the most simple and the most abstract fundamental statements which are taken to be true without any further proof and which equally serve as the starting point from which the theorems in the section Results are logically derived. It is important to note that an axiom in one system may be only a theorem in another system and vice versa. The relationship between cause and effect will be fused into a single mathematical formula while using the language of quantum theory and equally following a deductive-hypothetical approach. In the section *Discussion*, the meaning of the result and the relationship to concrete problems will be discussed. In the following of this investigation I will restrict myself to a one-dimensional treatment and the discrete case in order to decrease the amount of notation needed, since in all cases, whether the observable has a discrete or continuous set of eigenvalues, the generalization to four (i.e. quantum mechanics) or n-dimensions (i.e. quantum field theory) will be equally simple.

2 Material and methods

2.1 Definitions

Definition 1. Bernoulli trials

A Bernoulli trial (or binomial trial) denotes a random experiment with exactly two possible outcomes, *either* a concrete eigenvalue *or* not a concrete eigenvalue i. e. all but the concrete eigenvalue. The mathematical formalization of the Bernoulli trial is denoted as the Bernoulli process. A random experiment may consists of performing n Bernoulli trials, each with the probability $p({}_{j}e_{t})$ as associated with the eigenvalue ${}_{j}e_{t}$, i. e. it is $t = +1, ..., +N$.

Definition 2. Bernoulli observable

Let a Bernoulli random variable or a Bernoulli quantum mechanical observable be associated with a quantum mechanical operator. Let the Bernoulli quantum mechanical observable be determined by the fact that the same observable can take only two eigenvalues *either* +1 *or* +0 associated with some adequate probabilities and eigenfunctions.

Property. In the language of set theory we obtain i. e. ${}_{O}C_t = \{+0, +1\}$.

Definition 3. The expectation value of an effect

Let ${}_{j}e_t$ (an effect) denote an eigenvalue of a quantum observable ${}_{R}E_t$. More precisely, let ${}_{R}E_t$ denote the set of all possible eigenvalues ${}_{j}e_t$ at one single Bernoulli trial t, i. e. all possible outcomes of a measurement. Let $E({}_{R}E_t)$ denote the denote the expectation value of the quantum observable ${}_{R}E_t$. Let $\Psi({}_{R}E_t)$ denote the wave function of ${}_{R}E_t$. Let $\Psi^{*}({}_{R}E_t)$ denote the complex-conjugate of the wave function of ${}_{R}E_t$. Let $c({}_{j}e_t)$ denote the complex coefficient as associated with the eigenvalue ${}_{j}e_t$ while satisfying some normalization condition. Let $c^{*}({}_{j}e_t)$ denote complex conjugate of the complex coefficient as associated with the eigenvalue ${}_{j}e_t$. Let $\psi({}_{j}e_t)$ denote the eigenfunction as associated with an eigenvalue ${}_{j}e_t$ while satisfying some normalization condition. Let $\psi^{*}({}_{j}e_t)$ denote complex conjugate of the eigenfunction as associated with the eigenvalue ${}_{j}e_t$ while satisfying normalization. Let $p({}_{j}e_t)$ denote the probability as associated with the eigenvalue ${}_{j}e_t$. Let $E({}_{j}e_t)$ denote the expectation value of an eigenvalue ${}_{j}e_t$. Let $\sigma({}_{j}e_t)^2$ denote the variance of an eigenvalue ${}_{j}e_t$. Let $\sigma({}_{j}e_t)$ denote the standard deviation of an eigenvalue ${}_{j}e_t$. In general, it is

$$E\left({}_{j}e_t\right) \equiv {}_{j}e_t \times p\left({}_{j}e_t\right) \equiv c\left({}_{j}e_t\right) \times {}_{j}e_t \times c^{*}\left({}_{j}e_t\right) \equiv \psi\left({}_{j}e_t\right) \times {}_{j}e_t \times \psi^{*}\left({}_{j}e_t\right) \equiv c\left({}_{j}e_t\right) \times \psi\left({}_{j}e_t\right) \tag{2}$$

Properties.

Under conditions where $p({}_{j}e_t) = 1$ it is $E({}_{j}e_t) = {}_{j}e_t$. Further, from the definition above it follows that

$$c\left({}_{j}e_t\right) \equiv \frac{E\left({}_{j}e_t\right)}{\psi\left({}_{j}e_t\right)} \tag{3}$$

According to mathematical statistics, the proof of following relationships can be found in literature. In general, it is

$$E\left({}_{j}e_t{}^{2}\right) \equiv c\left({}_{j}e_t\right) \times {}_{j}e_t{}^{2} \times c^{*}\left({}_{j}e_t\right) = \psi\left({}_{j}e_t\right) \times {}_{j}e_t{}^{2} \times \psi^{*}\left({}_{j}e_t\right) = {}_{j}e_t{}^{2} \times p\left({}_{j}e_t\right) \tag{4}$$

The variance $\sigma({}_{j}e_t)^2$ of an eigenvalue ${}_{j}e_t$ follows as

$$\sigma\left({}_j e_t\right)^2 \equiv E\left({}_j e_t{}^2\right) - E\left({}_j e_t\right)^2 = \left({}_j e_t{}^2 \times p\left({}_j e_t\right)\right) - \left({}_j e_t \times p\left({}_j e_t\right)\right)^2 \tag{5}$$

which is equivalent with

$$\sigma\left({}_j e_t\right)^2 \equiv \left({}_j e_t{}^2 \times p\left({}_j e_t\right)\right) - \left({}_j e_t \times p\left({}_j e_t\right)\right)^2 = {}_j e_t{}^2 \times \left(p\left({}_j e_t\right) \times \left(1 - p\left({}_j e_t\right)\right)\right) \tag{6}$$

From this relationship, the eigenvalue ${}_j e_t$ can be derived as

$$\left|{}_j e_t\right| \equiv \sqrt[2]{\frac{\sigma\left({}_j e_t\right)^2}{p\left({}_j e_t\right) \times \left(1 - p\left({}_j e_t\right)\right)}} \equiv \frac{\sigma\left({}_j e_t\right)}{\sqrt[2]{p\left({}_j e_t\right) \times \left(1 - p\left({}_j e_t\right)\right)}} \tag{7}$$

while the standard deviation $\sigma({}_j e_t)$ of an eigenvalue ${}_j e_t$ is defined as

$$\sigma\left({}_j e_t\right) \equiv \sqrt[2]{\left({}_j e_t{}^2 \times p\left({}_j e_t\right)\right) - \left({}_j e_t \times p\left({}_j e_t\right)\right)^2} = \sqrt[2]{{}_j e_t{}^2 \times \left(p\left({}_j e_t\right) \times \left(1 - p\left({}_j e_t\right)\right)\right)} \tag{8}$$

The definition of variance leads to the equation that

$$E\left({}_j e_t{}^2\right) \equiv E\left({}_j e_t\right)^2 + \sigma\left({}_j e_t\right)^2 \tag{9}$$

which can be rearranged and yields *the normalisation of the variance* in general as

$$\frac{E\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} + \frac{\sigma\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} \equiv 1 \tag{10}$$

This relationship can be simplified as

$$\frac{E\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} + \frac{\sigma\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} \equiv \frac{\left({}_j e_t \times p\left({}_j e_t\right)\right)^2}{\left({}_j e_t{}^2 \times p\left({}_j e_t\right)\right)} + \frac{\sigma\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} \equiv p\left({}_j e_t\right) + \frac{\sigma\left({}_j e_t\right)^2}{E\left({}_j e_t{}^2\right)} = 1 \tag{11}$$

The probability $p({}_j e_t)$ as associated with an eigenvalue ${}_j e_t$ can be calculated in general as

$$p\left({}_{j}e_{t}\right) \equiv 1-\frac{\sigma\left({}_{j}e_{t}\right)^{2}}{E\left({}_{j}e_{t}{}^{2}\right)} \tag{12}$$

or with respect to the Chebyshev's inequality [45] as

$$p\left({}_{j}e_{t}\right) \equiv p(\left|{}_{j}e_{t}-E\left({}_{j}e_{t}\right)\right| <> \sqrt[2]{E\left({}_{j}e_{t}{}^{2}\right)} \times \sigma\left({}_{j}e_{t}\right)) \equiv 1-\frac{\sigma\left({}_{j}e_{t}\right)^{2}}{\sigma\left({}_{j}e_{t}\right)^{2} \times E\left({}_{j}e_{t}{}^{2}\right)} \equiv 1-\frac{1}{E\left({}_{j}e_{t}{}^{2}\right)} \tag{13}$$

while the associated eigenfunction $\psi({}_{j}e_{t})$ follows from the relationship

$$p\left({}_{j}e_{t}\right) \equiv \psi\left({}_{j}e_{t}\right) \times \psi^{*}\left({}_{j}e_{t}\right) \equiv 1-\frac{\sigma\left({}_{j}e_{t}\right)^{2}}{E\left({}_{j}e_{t}{}^{2}\right)} \tag{14}$$

as

$$\psi\left({}_{j}e_{t}\right) \equiv \frac{p\left({}_{j}e_{t}\right)}{\psi^{*}\left({}_{j}e_{t}\right)} \equiv \frac{1}{\psi^{*}\left({}_{j}e_{t}\right)}-\frac{\sigma\left({}_{j}e_{t}\right)^{2}}{\psi^{*}\left({}_{j}e_{t}\right) \times E\left({}_{j}e_{t}{}^{2}\right)} \tag{15}$$

Definition 4. The expectation value of a cause
Let ${}_{i}c_{t}$ (a cause) denote an eigenvalue of a quantum observable ${}_{O}C_{t}$. In particular, let ${}_{O}C_{t}$ denote the set of all possible eigenvalues ${}_{i}c_{t}$ at one single Bernoulli trial t, i. e. all possible outcomes of a measurement. Let $E({}_{O}C_{t})$ denote the denote the expectation value of the quantum observable ${}_{O}C_{t}$. Let $\Psi({}_{O}C_{t})$ denote the wave function of ${}_{O}C_{t}$. Let $\Psi^{*}({}_{O}C_{t})$ denote the complex-conjugate of the wave function of ${}_{O}C_{t}$. Let $c({}_{i}c_{t})$ denote the complex coefficient as associated with the eigenvalue ${}_{i}c_{t}$ while satisfying some normalization condition. Let $c^{*}({}_{i}c_{t})$ denote complex conjugate of the complex coefficient as associated with the eigenvalue ${}_{i}c_{t}$. Let $\psi({}_{i}c_{t})$ denote the eigenfunction as associated with an eigenvalue ${}_{i}c_{t}$ while satisfying some normalization condition. Let $\psi^{*}({}_{i}c_{t})$ denote complex conjugate of the eigenfunction as associated with the eigenvalue ${}_{i}c_{t}$ while satisfying normalization. Let $p({}_{i}c_{t})$ denote the probability as associated with the eigenvalue ${}_{i}c_{t}$. Let $E({}_{i}c_{t})$ denote the expectation value of an eigenvalue ${}_{i}c_{t}$. Let $\sigma({}_{i}c_{t})^{2}$ denote the variance of an eigenvalue ${}_{i}c_{t}$. Let $\sigma({}_{i}c_{t})$ denote the standard deviation of an eigenvalue ${}_{i}c_{t}$. In general, it is

$$E\left({}_{i}c_{t}\right) \equiv c\left({}_{i}c_{t}\right) \times {}_{i}c_{t} \times c^{*}\left({}_{i}c_{t}\right) = \psi\left({}_{i}c_{t}\right) \times {}_{i}c_{t} \times \psi^{*}\left({}_{i}c_{t}\right) = {}_{i}c_{t} \times p\left({}_{i}c_{t}\right) \tag{16}$$

Properties.
Under conditions where $p({}_ic_t) = 1$ it is $E({}_ic_t) = {}_ic_t$. According to mathematical statistics, the following relationships are proofed as correct.

$$E\left({}_ic_t^{\,2}\right) \equiv c\left({}_ic_t\right) \times {}_ic_t^{\,2} \times c^*\left({}_ic_t\right) = \psi\left({}_ic_t\right) \times {}_ic_t^{\,2} \times \psi^*\left({}_ic_t\right) = {}_ic_t^{\,2} \times p\left({}_ic_t\right) \tag{17}$$

The variance of an eigenvalue ${}_ic_t$ follows as

$$\sigma\left({}_ic_t\right)^2 \equiv E\left({}_ic_t^{\,2}\right) - E\left({}_ic_t\right)^2 = \left({}_ic_t^{\,2} \times p\left({}_ic_t\right)\right) - \left({}_ic_t \times p\left({}_ic_t\right)\right)^2 \tag{18}$$

which is equivalent with

$$\sigma\left({}_ic_t\right)^2 \equiv \left({}_ic_t^{\,2} \times p\left({}_ic_t\right)\right) - \left({}_ic_t \times p\left({}_ic_t\right)\right)^2 = {}_ic_t^{\,2} \times \left(p\left({}_ic_t\right) \times \left(1 - p\left({}_ic_t\right)\right)\right) \tag{19}$$

From this relationship, the eigenvalue ${}_ic_t$ can be derived as

$$\left|{}_ic_t\right| \equiv \sqrt[2]{\frac{\sigma\left({}_ic_t\right)^2}{p\left({}_ic_t\right) \times \left(1 - p\left({}_ic_t\right)\right)}} \equiv \frac{\sigma\left({}_ic_t\right)}{\sqrt[2]{p\left({}_ic_t\right) \times \left(1 - p\left({}_ic_t\right)\right)}} \tag{20}$$

while the standard deviation of an eigenvalue ${}_ic_t$ is defined as

$$\sigma\left({}_ic_t\right) \equiv \sqrt[2]{\left({}_ic_t^{\,2} \times p\left({}_ic_t\right)\right) - \left({}_ic_t \times p\left({}_ic_t\right)\right)^2} = \sqrt[2]{{}_ic_t^{\,2} \times \left(p\left({}_ic_t\right) \times \left(1 - p\left({}_ic_t\right)\right)\right)} \tag{21}$$

Definition 5. The co-variance of cause and effect

The covariance of two different eigenvalues ${}_ic_t$ and ${}_je_t$, denoted by $\sigma({}_ic_t\ ,\ {}_je_t)$, is known to be defined as

$$\sigma\left({}_ic_t, {}_je_t\right) \equiv E\left({}_ic_t, {}_je_t\right) - \left(E\left({}_ic_t\right) \times E\left({}_je_t\right)\right) = \left(\left({}_ic_t \times {}_je_t\right) \times p\left({}_ic_t, {}_je_t\right)\right) - \left(\left({}_ic_t \times p\left({}_ic_t\right)\right) \times \left({}_je_t \times p\left({}_je_t\right)\right)\right) \tag{22}$$

where $E({}_ic_t\ ,\ {}_je_t)$ denotes the expectation value of the two different eigenvalues ${}_ic_t$ and ${}_je_t$. This equation can be simplified as

$$\sigma\left({}_ic_t, {}_je_t\right) \equiv \left({}_ic_t \times {}_je_t\right) \times \left(p\left({}_ic_t, {}_je_t\right)\right) - \left(p\left({}_ic_t\right) \times p\left({}_je_t\right)\right) \tag{23}$$

where $p({}_ic_t\,,\,{}_je_t)$ denotes the joint probability function between the eigenvalues ${}_ic_t$ and ${}_je_t$. The joint probability function between the eigenvalues ${}_ic_t$ and ${}_je_t$ can be equal to zero. From this relationship, the product of the eigenvalues ${}_ic_t$ and ${}_je_t$ can be derived as

$$ {}_ic_t \times {}_je_t \equiv \frac{\sigma\left({}_ic_t\,,\,{}_je_t\right)}{\left(p\left({}_ic_t\,,\,{}_je_t\right)\right)-\left(p\left({}_ic_t\right)\times p\left({}_je_t\right)\right)} \tag{24} $$

It is easy to extend these definitions to n-dimensional cases.

Definition 6. The causal relationship $k({}_ic_t, {}_ie_t)$

The deterministic relationship between cause and effect (even at quantum level) is determined by the mathematical formula of the causal relationship $k({}_ic_t, {}_ie_t)$ as

$$ k\left({}_ic_t\,,\,{}_je_t\right) \equiv \frac{\left(p\left({}_ic_t\,,\,{}_je_t\right)\right)-\left(p\left({}_ic_t\right)\times p\left({}_je_t\right)\right)}{\sqrt[2]{p\left({}_ic_t\right)\times\left(1-p\left({}_ic_t\right)\right)\times p\left({}_je_t\right)\times\left(1-p\left({}_je_t\right)\right)}} = \frac{\sigma\left({}_ic_t\,,\,{}_je_t\right)}{\sigma\left({}_ic_t\right)\times\sigma\left({}_je_t\right)} \tag{25} $$

Definition 7. The commutation relation

Today, the Copenhagen dominated interpretation of quantum mechanics, mostly regarded as synonymous with indeterminism, has posed innumerable problems to scientist and challenges at least our imagination. In fact, the question of what kind of reality the Copenhagen dominated interpretation of quantum mechanics describes, however, is controversial. In particular, one pillar of the mathematical formalism of the Copenhagen dominated interpretation of quantum mechanics is *the canonical commutation relation* which is attributed to Max Born [46]. The canonical commutation relation is by definition such that

$$ [X,p] = i\times\frac{h}{2\times\pi} \tag{26} $$

where X denotes the position operator, p denotes the momentum operator, i is the imaginary unit, h denotes Planck's constant while π denotes the mathematical constant pi. The Copenhagen dominated interpretation of quantum mechanics reduces more or less the whole quantum mechanics to the relationship between position and momentum and forces us to study objective reality only through this one and only scientific eye. The implications of the at least to some extent unfair Copenhagen dominated monocular vision (loss of one scientific eye) of objective reality has raised many questions about the basic concepts of objective reality and especially about *the principle of causality*. In the land of the blind, the one-eyed scientist may be a king. Still, the one-sided and restricted Copenhagen dominated approach to objective reality is neither necessary nor mathematically the only possible way, '*how to see the world*'. Finally, a radically different perspective is posed if we try to assure compatibility between quantum mechanics and general relativity from the beginning. Thus far, we define a quantum mechanical operator of curvature, denoted as C, of preliminary unknown (mathematical) structure. Further, we define a quantum mechanical operator denoted as E, of preliminary unknown (mathematical) structure. In this way, the following relationship may hold:

$$C = C = C + 0 = C - X + X \tag{27}$$

where C denotes *the quantum mechanical operator of curvature* and X denotes the position operator. We define the *anti operator* of the position operator X, denoted as $\underline{X}$, (the *'local hidden variable'* of X) such that

$$\underline{X} = C - X \tag{28}$$

In other words, there is no third between the position operator X and the anti position operator $\underline{X}$, a third is not given (Aristotle's principle of the exluded middle (*principium exlusii tertii*)). In the context of the curvature operator C all but X is regarded as $\underline{X}$. We obtain

$$C = X + \underline{X} \tag{29}$$

The commutation relation changes to

$$[C, p] = i \times \frac{h}{2 \times \pi} \tag{30}$$

where C denotes the curvature operator, p denotes the momentum operator, i is the imaginary unit, h denotes Planck's constant while π denotes the mathematical constant pi. In other words, it is

$$[(X + \underline{X}), p] = i \times \frac{h}{2 \times \pi} \tag{31}$$

In the same respect, we define a quantum mechanical operator E such that

$$E = p + \underline{p} \tag{32}$$

where p is the momentum operator and $\underline{p}$ denotes the anti-momentum operator. Under conditions of general relativity matter (*momentum*) causes space-time how to curve (*curvature*) while curved space-time (*curvature*) causes matter how to move (*momentum*) which is the epistemological background of the saying *curvature equals momentum and vice versa*. This principle is valid under conditions of quantum mechanics too. Thus far, the general relativity compatible commutation relation follows as

$$[C,E]=\left[(X+\underline{X}),(p+\underline{p})\right]=i\times\frac{h}{2\times\pi} \quad (33)$$

In fact, only under conditions, where the *anti position operator* $\underline{X}$ = 0 and where the *anti momentum operator* $\underline{p}$=0 we obtain the one-sided and restricted Copenhagen dominated view of the world as

$$[C,E]=\left[(X+\underline{X=0}),(p+\underline{p=0})\right]=[X,p]=i\times\frac{h}{2\times\pi} \quad (34)$$

but not in general. The question is, what is the concrete mathematical structure of the operators C and E. In the following lines of this paper, the operator *C may denote a cause while the operator E may denote an effect* as postulated before.

Definition 8. The wave function

Let $f({}_{O}C_{t})$ denote any kind of a (complex, composite et cetera) mathematical function of preliminary unknown properties. In this context, the element ${}_{O}C_{t}$ is called the argument of the (mathematical) function f. For each argument ${}_{O}C_{t}$, a corresponding function value y in given and abbreviated such that

$$y=f({}_{O}C_{t}) \quad (35)$$

In this context, ${}_{O}C_{t}$ can denote (the expectation value of) a random variable, a (quantum mechanical) observable, a quantum mechanical operator, a tensor (of general relativity) or any other mathematical object. Let $\Psi({}_{O}C_{t})$ denote the wave function of ${}_{O}C_{t}$. In general, we define the wave function of ${}_{O}C_{t}$ as

$$\Psi({}_{O}C_{t})\equiv{}_{O}C_{t}\equiv{}_{O}C_{t}\times 1\equiv{}_{O}C_{t}\times\frac{f({}_{O}C_{t})}{f({}_{O}C_{t})}\equiv\frac{{}_{O}C_{t}}{f({}_{O}C_{t})}\times f({}_{O}C_{t}) \quad (36)$$

*Remark 1.*Such a definition of the wave function is a theoretical attempt to provide a contribution to solve the problems as related with the wave functions as such. Whether such a definition of the wave function makes any sense or not is a point of further research. The solution of the problem of cause and effect as presented in this publication is independent of the previous definition of the wave function. In fact, the above definition of the wave function is following *the simple chain rule* in Leibniz (1646-1716) notation knowing to be defined something like

$$dz\equiv\frac{dz}{dy}\times dy \quad (37)$$

2.2.1 Axiom I (Lex identitatis)

$$+1 = +1 \tag{38}$$

2.2.2 Axiom II (Lex negationis)

$$\frac{+1}{+0} \equiv +\infty \tag{39}$$

2.2.3 Axiom III (Lex contradictionis)

$$\frac{+0}{+0} \equiv +1 \tag{40}$$

3 Results

3.1 Theorem: The relationship between the complex coefficient and probability

Thesis (Claim).
In general, based on the definition before, we obtain the following relationship. It is

$$c({}_{i}c_{t}) \times \psi({}_{i}c_{t}) = \left(\frac{c({}_{i}c_{t}) \times \psi({}_{i}c_{t})}{c^{*}({}_{i}c_{t}) \times \psi^{*}({}_{i}c_{t})}\right) \times c^{*}({}_{i}c_{t}) \times \psi^{*}({}_{i}c_{t}) \tag{41}$$

Proof by contradiction.
The starting point of this theorem is axiom I. Thus far, it is

$$+1 = +1 \tag{42}$$

After multiplication, we obtain

$$c\left({}_{i}c_{t}\right)\times c^{*}\left({}_{i}c_{t}\right)=c\left({}_{i}c_{t}\right)\times c^{*}\left({}_{i}c_{t}\right)=p\left({}_{i}c_{t}\right) \tag{43}$$

or

$$c\left({}_{i}c_{t}\right)\times c^{*}\left({}_{i}c_{t}\right)=\psi\left({}_{i}c_{t}\right)\times \psi^{*}\left({}_{i}c_{t}\right)=p\left({}_{i}c_{t}\right) \tag{44}$$

From this equation it follows that

$$\psi\left({}_{i}c_{t}\right)=\frac{c^{*}\left({}_{i}c_{t}\right)}{\psi^{*}\left({}_{i}c_{t}\right)}\times c\left({}_{i}c_{t}\right)=\frac{p\left({}_{i}c_{t}\right)}{\psi^{*}\left({}_{i}c_{t}\right)} \tag{45}$$

In other words, it is equally

$$c\left({}_{i}c_{t}\right)=\frac{\psi^{*}\left({}_{i}c_{t}\right)}{c^{*}\left({}_{i}c_{t}\right)}\times \psi\left({}_{i}c_{t}\right)=\frac{p\left({}_{i}c_{t}\right)}{c^{*}\left({}_{i}c_{t}\right)} \tag{46}$$

At the end, we obtain

$$c\left({}_{i}x_{t}\right)\times\psi\left({}_{i}c_{t}\right)=\left(\frac{\psi^{*}\left({}_{i}c_{t}\right)}{c^{*}\left({}_{i}c_{t}\right)}\times\psi\left({}_{i}c_{t}\right)\right)\times\left(\frac{c^{*}\left({}_{i}c_{t}\right)}{\psi^{*}\left({}_{i}c_{t}\right)}\times c\left({}_{i}c_{t}\right)\right)=\left(\frac{c\left({}_{i}c_{t}\right)\times\psi\left({}_{i}c_{t}\right)}{c^{*}\left({}_{i}c_{t}\right)\times\psi^{*}\left({}_{i}c_{t}\right)}\right)\times c^{*}\left({}_{i}c_{t}\right)\times\psi^{*}\left({}_{i}c_{t}\right) \tag{47}$$

Quod erat demonstrandum.

3.2 Theorem: The collapse of the wave function $\Psi({}_{O}C_{t})$

Thesis (Claim).
In general, the collapse of the wave function is determined by the equation

$$\left(c\left({}_{i}c_{t}\right)\times\psi\left({}_{i}c_{t}\right)\right)=\Psi\left({}_{O}C_{t}\right)\times\left(1-\frac{\left(c\left({}_{1}\underline{c}_{t}\right)\times\psi\left({}_{1}\underline{c}_{t}\right)\right)}{\Psi\left({}_{O}C_{t}\right)}\right) \tag{48}$$

Proof by contradiction.

As a starting point in understanding the collapse of the wave function of quantum mechanics, we start with axiom I. Thus far, it is

$$+1 = +1 \tag{49}$$

After multiplication by the wave function, we obtain

$$+1 \times \Psi({}_{O}C_t) = +1 \times \Psi({}_{O}C_t) \tag{50}$$

or

$$\Psi({}_{O}C_t) = \Psi({}_{O}C_t) \tag{51}$$

According to the so-called *'expansion postulate'*, a fundamental postulate of quantum mechanics, the equation before changes to

$$\Psi({}_{O}C_t) = \left(c({}_{1}c_t) \times \psi({}_{1}c_t)\right) + \left(c({}_{2}c_t) \times \psi({}_{2}c_t)\right) + \left(c({}_{3}c_t) \times \psi({}_{3}c_t)\right) + \dots \tag{52}$$

which can be simplified as

$$\begin{aligned} \Psi({}_{O}C_t) &= \left(c({}_{1}c_t) \times \psi({}_{1}c_t)\right) + \underbrace{\left(c({}_{2}c_t) \times \psi({}_{2}c_t)\right) + \left(c({}_{3}c_t) \times \psi({}_{3}c_t)\right) + \dots}_{} \\ \Psi({}_{R}C_t) &= \left(c({}_{1}c_t) \times \psi({}_{1}c_t)\right) + \left(c({}_{1}\underline{c}_t) \times \psi({}_{1}\underline{c}_t)\right) \end{aligned} \tag{53}$$

where $c({}_{1}c_t)$ denotes the complex coefficient of anti ${}_{1}c_t$ and $\psi({}_{1}\underline{c}_t)$ denotes the anti-eigenfunction. Rearranging equation before yields

$$\frac{\Psi({}_{O}C_t)}{\Psi({}_{O}C_t)} = \frac{\left(c({}_{1}c_t) \times \psi({}_{1}c_t)\right)}{\Psi({}_{O}C_t)} + \frac{\left(c({}_{1}\underline{c}_t) \times \psi({}_{1}\underline{c}_t)\right)}{\Psi({}_{O}C_t)} = 1 \tag{54}$$

In other words, it is

$$\frac{\left(c\left({}_{1}c_{t}\right)\times\psi\left({}_{1}c_{t}\right)\right)}{\Psi\left({}_{O}C_{t}\right)}=1-\frac{\left(c\left({}_{1}\underline{c}_{t}\right)\times\psi\left({}_{1}\underline{c}_{t}\right)\right)}{\Psi\left({}_{O}C_{t}\right)} \tag{55}$$

or

$$\left(c\left({}_{1}c_{t}\right)\times\psi\left({}_{1}c_{t}\right)\right)=\Psi\left({}_{O}C_{t}\right)\times\left(1-\frac{\left(c\left({}_{1}\underline{c}_{t}\right)\times\psi\left({}_{1}\underline{c}_{t}\right)\right)}{\Psi\left({}_{O}C_{t}\right)}\right) \tag{56}$$

Quod erat demonstrandum.

Remark 2. The term

$$\left(c\left({}_{1}c_{t}\right)\times\psi\left({}_{1}c_{t}\right)\right) \tag{57}$$

denotes the situation after the collapse of the wave function $\Psi({}_{O}C_{t})$, while the wave function itself denotes the situation before the collapse of itself into an eigenvalue and an eigenfunction. Under some circumstances, the collapse of the wave function may pass over into the Lorenz transformation as

$$\left(1-\frac{\left(c\left({}_{1}\underline{c}_{t}\right)\times\psi\left({}_{1}\underline{c}_{t}\right)\right)}{\Psi\left({}_{O}C_{t}\right)}\right)=1-\frac{v^{2}}{c^{2}} \tag{58}$$

The notion *collapse of the wave function* is discussed by philosophers since ancient times under the notion negation or negation of the negation.

3.3 Theorem: The expansion postulate of the wave function $\Psi({}_{R}E_{t})$

Thesis (Claim).

In general, the generic state $\Psi({}_{R}E_{t})$ can be expressed as a *superposition of eigenstates* $\psi({}_{i}e_{t})$. In other words, every wave function $\Psi({}_{R}E_{t})$ can be expanded as a series involving all of the eigenfunctions $\psi({}_ie_t)$ of an operator ${}_RE_t$ (the expansion postulate) such that

$$\Psi\left({}_RE_t\right)=\sum_{i=+1}^{i=+N}\left(c\left({}_ie_t\right)\times\psi\left({}_ie_t\right)\right) \tag{59}$$

Proof by contradiction.

Again, we start with axiom I. Thus far, it is

$$+1=+1 \tag{60}$$

After multiplication by the expectation value, we obtain

$$+1\times E\left({}_RE_t\right)=+1\times E\left({}_RE_t\right) \tag{61}$$

or

$$E\left({}_RE_t\right)=E\left({}_RE_t\right) \tag{62}$$

According to mathematical statistics and probability theory, this equation is equivalent with

$$E\left({}_RE_t\right)=E\left({}_1e_t\right)+E\left({}_2e_t\right)+E\left({}_3e_t\right)+\ldots \tag{63}$$

Multiplying equation by the expectation value $E({}_ie_t)$ of particular eigenvalue ${}_i\underline{e}_t$ by the associated eigenfunction $\psi({}_ie_t)$ without changing the equation before it follows that

$$E\left({}_RE_t\right)=\left(E\left({}_1e_t\right)\times\frac{\psi\left({}_1e_t\right)}{\psi\left({}_1e_t\right)}\right)+\left(E\left({}_2e_t\right)\times\frac{\psi\left({}_2e_t\right)}{\psi\left({}_2e_t\right)}\right)+\left(E\left({}_3e_t\right)\times\frac{\psi\left({}_3e_t\right)}{\psi\left({}_3e_t\right)}\right)+\ldots \tag{64}$$

or that

$$\frac{E\left({}_RE_t\right)}{\Psi\left({}_RE_t\right)}\times\Psi\left({}_RE_t\right)=\left(\frac{E\left({}_1e_t\right)}{\psi\left({}_1e_t\right)}\times\psi\left({}_1e_t\right)\right)+\left(\frac{E\left({}_2e_t\right)}{\psi\left({}_2e_t\right)}\times\psi\left({}_2e_t\right)\right)+\left(\frac{E\left({}_3e_t\right)}{\psi\left({}_3e_t\right)}\times\psi\left({}_3e_t\right)\right)+\ldots \tag{65}$$

Due to our general definition of $c({}_ie_t) = E({}_ie_t)/\ \psi({}_ie_t)$, the equation before changes to

$$c\left({}_{R}E_{t}\right)\times\Psi\left({}_{R}E_{t}\right)=\left(c\left({}_{1}e_{t}\right)\times\psi\left({}_{1}e_{t}\right)\right)+\left(c\left({}_{2}e_{t}\right)\times\psi\left({}_{2}e_{t}\right)\right)+\left(c\left({}_{3}e_{t}\right)\times\psi\left({}_{3}e_{t}\right)\right)+\ldots \tag{66}$$

Especially under conditions where $c({}_{R}E_{t})=1$, the expansion postulate of the wave function follows as

$$\Psi\left({}_{R}E_{t}\right)=\left(c\left({}_{1}e_{t}\right)\times\psi\left({}_{1}e_{t}\right)\right)+\left(c\left({}_{2}e_{t}\right)\times\psi\left({}_{2}e_{t}\right)\right)+\left(c\left({}_{3}e_{t}\right)\times\psi\left({}_{3}e_{t}\right)\right)+\ldots \tag{67}$$

or as

$$\Psi\left({}_{R}E_{t}\right)=\sum_{i=+1}^{i=+N}\left(c\left({}_{i}e_{t}\right)\times\psi\left({}_{i}e_{t}\right)\right) \tag{68}$$

Quod erat demonstrandum.

3.4 Theorem: The determination of the complex coefficient $c({}_{i}c_{t})$

Thesis (Claim).

In general, the complex coefficient as associated with the eigenvalue ${}_{i}c_{t}$ can be calculated as

$$c\left({}_{1}c_{t}\right)=\frac{\psi^{*}\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)}\times\psi\left({}_{1}c_{t}\right) \tag{69}$$

Proof by contradiction.

Again, we start with axiom I. Thus far, it is

$$+1=+1 \tag{70}$$

After multiplication by the complex coefficient $c({}_{1}c_{t})$, we obtain

$$+1\times c\left({}_{1}c_{t}\right)=+1\times c\left({}_{1}c_{t}\right) \tag{71}$$

or

$$c\left({}_{1}c_{t}\right)=c\left({}_{1}c_{t}\right) \tag{72}$$

Multiplying by the $c^*({}_1c_t)$, the complex conjugate of the complex coefficient as associated with the eigenvalue ${}_1c_t$ it is

$$c\left({}_{1}c_{t}\right)\times c^{*}\left({}_{1}c_{t}\right)=c\left({}_{1}c_{t}\right)\times c^{*}\left({}_{1}c_{t}\right) \tag{73}$$

Due to Born's rule, this is equivalent with

$$c\left({}_{1}c_{t}\right)\times c^{*}\left({}_{1}c_{t}\right)=\psi\left({}_{1}c_{t}\right)\times \psi^{*}\left({}_{1}c_{t}\right) \tag{74}$$

and the the complex coefficient $c({}_1c_t)$ follows as

$$c\left({}_{1}c_{t}\right)=\frac{\psi^{*}\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)}\times \psi\left({}_{1}c_{t}\right) \tag{75}$$

Quod erat demonstrandum.

3.5 Theorem: The determination of an eigenvalue ${}_\mathbf{i}\mathbf{c}_\mathbf{t}$

Thesis (Claim).
In general, the eigenvalue ${}_i\underline{c}_t$ is determined by the equation

$$ {}_{1}c_{t}=\frac{c\left({}_{1}c_{t}\right)}{\psi^{*}\left({}_{1}c_{t}\right)} \tag{76}$$

Proof by contradiction.
Again, we start with axiom I. Thus far, it is

$$+1=+1 \tag{77}$$

After multiplication by the complex coefficient $c({}_i c_t)$, we obtain

$$+1 \times c({}_1 c_t) = +1 \times c({}_1 c_t) \tag{78}$$

or

$$c({}_1 c_t) = c({}_1 c_t) \tag{79}$$

According to our definition our definition it is $E({}_i c_t) = c({}_i c_t) \times \psi({}_i c_t)$ with the consequence that $c({}_i c_t) = E({}_i c_t)/ \psi({}_i c_t)$. Thus far, the equation before is equivalent with

$$c({}_1 c_t) = \frac{E({}_1 c_t)}{\psi({}_1 c_t)} = \frac{\psi({}_1 c_t) \times {}_1 c_t \times \psi^*({}_1 c_t)}{\psi({}_1 c_t)} \tag{80}$$

Simplifying this equation, we obtain

$$c({}_1 c_t) = {}_1 c_t \times \psi^*({}_1 c_t) \tag{81}$$

Thus far, if our definition that $E({}_i c_t) = c({}_i c_t) \times \psi({}_i c_t)$ is correct it is $c({}_i c_t) = E({}_i c_t)/ \psi({}_i c_t)$ and we must accept too that

$${}_1 c_t = \frac{c({}_1 c_t)}{\psi^*({}_1 c_t)} \tag{82}$$

Quod erat demonstrandum.

3.6 Theorem: The determination of an eigenvalue $_i c_t$

Thesis (Claim).

In general, the eigenvalue ${}_i \underline{c}_t$ is determined by the equation

$$ {}_{1}c_{t} = \frac{\psi\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)} \tag{83} $$

Proof by contradiction.

Again, we start with axiom I. Thus far, it is

$$ +1 = +1 \tag{84} $$

After multiplication by the complex coefficient $c({}_{i}c_{t})$, we obtain

$$ +1 \times c\left({}_{1}c_{t}\right) = +1 \times c\left({}_{1}c_{t}\right) \tag{85} $$

or

$$ c\left({}_{1}c_{t}\right) = c\left({}_{1}c_{t}\right) \tag{86} $$

According to one of the theorems before, this equation is equivalent with

$$ {}_{1}c_{t} \times \psi^{*}\left({}_{1}c_{t}\right) = c\left({}_{1}c_{t}\right) \tag{87} $$

Due to another theorem before, the complex coefficient $c({}_{i}c_{t})$ can be substituted and the equation changes to

$$ {}_{1}c_{t} \times \psi^{*}\left({}_{1}c_{t}\right) = \frac{\psi^{*}\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)} \times \psi\left({}_{1}c_{t}\right) \tag{88} $$

Simplifying this equation, we obtain

$$ {}_{1}c_{t} = \frac{\psi\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)} \tag{\mathbf{89}} $$

Quod erat demonstrandum.

3.7 Theorem: The definition $E({}_ic_t) = c({}_ic_t) \times \psi({}_ic_t)$ is correct.

Thesis (Claim).
In general, the definition

$$E\left({}_1c_t\right) = c\left({}_1c_t\right) \times \psi\left({}_1c_t\right) \tag{90}$$

is correct.

Proof by contradiction.
Again, we start with axiom I. Thus far, it is

$$+1 = +1 \tag{91}$$

After multiplication by the complex coefficient $c({}_1c_t)$, we obtain

$$+1 \times c\left({}_1c_t\right) = +1 \times c\left({}_1c_t\right) \tag{92}$$

or

$$c\left({}_1c_t\right) = c\left({}_1c_t\right) \tag{93}$$

Multiplying by the $c^*({}_1c_t)$, the complex conjugate of the complex coefficient as associated with the eigenvalue ${}_1c_t$ it is

$$c\left({}_1c\right) \times c^*\left({}_1c_t\right) = c\left({}_1c_t\right) \times c^*\left({}_1c_t\right) \tag{94}$$

Due to Born's rule, this is equivalent with

$$c\left({}_1c_t\right) \times c^*\left({}_1c_t\right) = \psi\left({}_1c_t\right) \times \psi^*\left({}_1c_t\right) \tag{95}$$

Rearranging equation, we obtain

$$\frac{c\left({}_{1}c_{t}\right)}{\psi^{*}\left({}_{1}c_{t}\right)}=\frac{\psi\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)} \tag{96}$$

Due to our theorems before, the equation is equivalent with

$${}_{1}c_{t}=\frac{c\left({}_{1}c_{t}\right)}{\psi^{*}\left({}_{1}c_{t}\right)}=\frac{\psi\left({}_{1}c_{t}\right)}{c^{*}\left({}_{1}c_{t}\right)}={}_{1}c_{t} \tag{97}$$

and simplifies as

$${}_{1}c_{t}={}_{1}c_{t} \tag{98}$$

Subtracting ${}_{1}c_{t}$, it is

$$+{}_{1}c_{t}-{}_{1}c_{t}=+0=+1-1 \tag{99}$$

or

$$+1=+1 \tag{100}$$

Quod erat demonstrandum.

Remark 3. This proof is based on the definition $E({}_{i}c_{t})=c({}_{i}c_{t})\times\psi({}_{i}c_{t})$ and that the same is correct. Based on this definition we were able to derive a correct conclusion, i. e. +1=+1. Thus far, we must conclude, that $E({}_{i}c_{t})=c({}_{i}c_{t})\times\psi({}_{i}c_{t})$ and thus far that $c({}_{i}c_{t}) = E({}_{i}c_{t})/\psi({}_{i}c_{t})$ is indeed correct.

3.8 Theorem: The multiplication by zero is not generally valid

Thesis (Claim).
The multiplication by zero is not generally valid.

Proof by contradiction.
To demonstrate the problems as associated with the multiplications by zero we start with an incorrect *Ansatz* (claim), something which cannot be accepted at all. Thus far, let us claim that

$$+2 = +3 \tag{101}$$

Multiplying this equation by +0, we obtain

$$+2 \times 0 = +3 \times 0 \tag{102}$$

which is equivalent with the equation

$$+0 = +0 \tag{103}$$

or in other word with

$$+1 - 1 = +1 - 1 \tag{104}$$

At the end it is

$$+1 = +1 \tag{105}$$

Quod erat demonstrandum.

Remark 4. In general, it should be *impossible* to have a false conclusion if all the premises are true and no technical errors can be identified. Still, invalid arguments come in all sorts of flavors and it is difficult to be aware of the many different types. Consequently, the above type of an invalid argument +2=+3 is simply a logical fallacy. Multiplying this equation by the number +2 we obtain the equation (+2)×(+2)= (+3)×(+2) which is equivalent with +4=+6. Subtracting +4 we obtain +0=+2. Dividing by +2 it is +0=+1. From something incorrect follows something incorrect which is acceptable. Thus far, it is not the multiplication by a number as such which changes something incorrect to something correct but especially the multiplication by zero. The multiplication by zero has the potential to result in a logical fallacy especially if it is not assured that the starting point, *der Ansatz*, is correct. Because of this, the multiplication by zero must be treated very carefully and cannot be regarded as being generally valid.

3.9 Theorem: The equivalence of multiplication and conjunction

The logical conjunction '*and*' (in set theory: intersection), denoted by the sign $\cap$, is related to the 'and' of natural languages. The conjunction of two expressions ${}_ic_t$ and ${}_je_t$ is denoted by ${}_ic_t \cap {}_je_t$, while the expressions ${}_ic_t$ and ${}_je_t$ are called the conjunctive terms of ${}_ic_t \cap {}_je_t$. Let the sign $\times$ denote the mathematical multiplication between ${}_ic_t$ and ${}_je_t$ as ${}_ic_t \times {}_je_t$. The method of truth tables is a conventional technique of proving and determining the validity of (propositional) formulas i. e. with respect to (mathematical) logic.

Thesis (Claim).

The logical conjunction and the mathematical multiplication are identical. In general, we obtain

$$ {}_i c_t \times {}_j e_t = {}_i c_t \cap {}_j e_t \tag{106} $$

Proof by contradiction.

Again, our starting point (*der Ansatz*) is axiom I. Thus far, it is

$$ +1 = +1 \tag{107} $$

Multiplying by the eigenvalue of the cause ${}_ic_t$, we obtain

$$ +1 \times {}_i c_t = +1 \times {}_i c_t \tag{108} $$

or

$$ {}_i c_t = {}_i c_t \tag{109} $$

Multiplying this equation by an eigenvalue of an effect ${}_je_t$, it is

$$ {}_{i}c_{t} \times_{j} e_{t} = {}_{i} c_{t} \times_{j} e_{t} \tag{110} $$

The normal usage of the logical conjunction in (mathematical) logic corresponds to the following truth table:

Bernoulli trial t	${}_{i}c_{t}$	${}_{j}e_{t}$	${}_{i}c_{t} \times {}_{j}e_{t}$	${}_{i}c_{t} \cap {}_{j}e_{t}$
1	1	1	1	1
2	1	0	0	0
3	0	1	0	0
4	0	0	0	0
…	.	.	.	.
N	.	.	.	.

As can be seen, the logical conjunction and the mathematical multiplication are identical. In general, we obtain

$$ {}_{i}c_{t} \times_{j} e_{t} = {}_{i} c_{t} \cap_{j} e_{t} \tag{111} $$

Quod erat demonstrandum.

3.10 Theorem: The equivalence of cause and effect

The intention of this article is not to give a review of the history of the identity law (*principium identitatis*). Since the relationship between cause and effect can be derived from the identity law it makes sense to elaborate on this point in view sentences. Hessen claims that the principle of causality can be derived from the identity law. “Eine Begründung des Kausalprinzips mit Hilfe des Identitätssatzes ist ... möglich ...“ [47] Historically, one of the first attempts to mathematize the identity law can be ascribed to Leibnitz. Gottfried Wilhelm von Leibniz (1646-1716) expressed the law of identity by claiming that everything is that what it is. “Chaque chose est ce qu’elle est. Et dans autant d’exemples qu’on voudra A est A, B est B” [48]. If we substitute the French word “*est*” by the mathematical sign “=” we obtain A=A, B=B et cetera. In ‘The problems of philosophy’ (1912) Bertrand Russell (1872-1970) himself elaborates about the law of identity in the following sense. “… three of these Principles have been singled out by tradition under the name of ‘Laws of Thought.’ They are as follows :

(1) The law of identity: ‘Whatever is, is.’
(2) The law of contradiction: ‘Nothing can both be and not be.’
(3) The law of excluded middle: ‘Everything must either be or not be.’ “ [49]. Historically, *principium contradictionis* has been used to derive the principle of causality. „Wie das Identitätsgesetz, so hat man auch das Widerspruchsgesetz bei der Begründung des Kausalprinzips verwertet.“ [50], [51]. Still, the *principium*

contradictionis as the starting point to derive the principle of causality is not generally accepted. „Übereinstimmend mit Hume ist Kant der Meinung, daß sich das Prinzip Kausalität nicht aus dem Satz des Widerspruchs ableiten läßt." [52]

Thesis (Claim).

The equivalence of cause and effect, an identity of two which are different, mathematically expressed as

$$ {}_{i}c_{t} \times {}_{j}e_{t} = {}_{i}c_{t} \cap {}_{j}e_{t} \tag{112} $$

is the foundation of a deterministic relationship between cause and effect.

Proof by contradiction.

When philosophers, mathematicians, physicist and other are engaged in deductive reasoning about the relationship between cause and effect, classic logic can be a common foundation. On a view like this, logic, as a branch of mathematics and equally as branch of philosophy, can help us to recognize the basics of causality. The method of truth tables is a technique for determining the fundamental relationship between cause and effect. In particular, to start with, we regard circumstances where there is only one cause and one effect. Again, our starting point (*der Ansatz*) is axiom I. Thus far, it is

$$ +1 = +1 \tag{113} $$

Multiplying by the eigenvalue of the cause ${}_{i}c_{t}$, we obtain

$$ +1 \times {}_{i}c_{t} = +1 \times {}_{i}c_{t} \tag{114} $$

or

$$ {}_{i}c_{t} = {}_{i}c_{t} \tag{115} $$

Multiplying this equation by an eigenvalue of an effect ${}_{j}e_{t}$, it is

$$ {}_{i}c_{t} \times {}_{j}e_{t} = {}_{i}c_{t} \times {}_{j}e_{t} \tag{116} $$

Due to the identity of multiplication and conjugation as proofed by a theorem before we obtain

$$ {}_{i}c_{t} \times {}_{j}e_{t} = {}_{i}c_{t} \cap {}_{j}e_{t} \tag{117} $$

To demonstrate the equivalence of cause and effect, the equivalence of two which are different, we perform a thought experiment. After some thought measurements, we obtained the following results:

Bernoulli trial t	**Cause ${}_{i}c_{t}$**	**Effect ${}_{j}e_{t}$**	**${}_{i}\text{Cause}_{t} \cap {}_{j}\text{Effect}_{t}$**	**Equivalence of cause and effect?**
1	**1**	**1**	**1**	**Yes.**
2	1	0	0	No.

3	0	1	0	No
4	**0**	**0**	**0**	**Yes.**
…	.	.	.	.
N	.	.	.	.

As can be seen, it is the equivalence of cause and effect or in other words $_{i}\mathbf{Cause}_{t}={}_{j}\mathbf{Effect}_{t} = {}_{i}\mathbf{Cause}_{t} \cap {}_{j}\mathbf{Effect}_{t}$ which is the foundation of a deterministic relationship between a cause and its own effect.

Quod erat demonstrandum.

Remark 5. Under circumstances, where there is only one cause and one effect, it is easy to analyze the relationship between cause and effect. In any case, the concept of causality is inseparable from the assumption of a deterministic relationship between cause and effect.

Bernoulli trial t=1

Thus far at the Bernoulli trial t=1 we measured a cause and at the same time we were able to measure an effect which is exactly what we do understand under the deterministic relationship between cause and effect. A cause has an effect and vice versa. An effect has a cause, we obtain $_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t} = 1$. Still, the co-occurrence of cause and effect alone, as demonstrated by the Bernoulli trial = 1, is not enough to explain causation. Two events which occur together must because of this not be caused by each other. A concept of a deterministic relationship between cause and effect which would rely only on the co-occurrence of cause and effect and thus far on Bernoulli trial 1 would end up at the so called *cum hoc ergo propter hoc* fallacy and is of very limited value.

Bernoulli trial t=2

Under conditions of only one cause and one effect we measured a cause but failed to detect an effect. Under circumstances where there are many causes and many effects such an measurements could be justified. Still, we are under conditions of one cause and one effect. Under these conditions, the cause was not able to produce an effect which contradicts the assumption of a deterministic relationship between cause and effect. A cause must have an effect but the cause did not had an effect. Thus far, the Bernoulli trial t=2 documents that we are not allowed to talk about a deterministic relationship between cause and effect, we obtain $_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t} = 0$.

Bernoulli trial t=3

At the Bernoulli trial t=3 we were able to detect the effect, while there was no cause to be measured. An effect exists without a cause. Still, an effect which can occur without a cause means that uncaused changes are possible or theoretically allowed. In this case, we don't need causality at all. In particular, an effect must have a cause. An effect without a cause is not an effect and vice versa. A cause without an effect is not a cause. In this case, we cannot talk about a deterministic relationship between cause and effect, we obtain ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t} = 0$.

Bernoulli trial t=4

At the Bernoulli trial t=4 there is *no cause and no effect*. If a cause must have an effect it is justified to assume that when there is no cause that there is no effect too. This measurement does not contradict the assumption of a deterministic relationship between cause and effect, still we obtain ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t} = 0$. Evidenly, whatever else it may be, the term ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t}$ alone is not enough to justify the assumption of a deterministic relationship between cause and effect. At the Bernoulli trial t=2 and t=3, we cannot talk about the deterministic relationship between cause and effect and the term ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t}$ is equal to 0. The same term ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t}$ is equal to 0 at the Bernoulli trial t=4, where we are allowed to talk about the deterministic relationship between cause and effect. Clearly, the term ${}_{i}\text{cause}_{t} \cap {}_{j}\text{effect}_{t}$ alone is not enough to explain all the possible aspects of the relationship between cause and effect. In real life, we are confronted with more than one cause and with more than one effect. Thus far, a more complex concept of causality is needed.

3.11 Theorem: The mathematical formula of the causal relationship k

The mathematical formula of the causal relationship is published several [26], [53], [54], [55], [56], [57], [58] times. The purpose of this publication is to provide a proof strictly from the standpoint of quantum theory.

Thesis (Claim).

The deterministic relationship between cause and effect is determined by the mathematical formula of the causal relationship $k({}_{i}c_{t}, {}_{i}e_{t})$ as

$$k\left({}_{i}c_{t}, {}_{j}e_{t}\right) \equiv \frac{\left(p\left({}_{i}c_{t}, {}_{j}e_{t}\right)\right) - \left(p\left({}_{i}c_{t}\right) \times p\left({}_{j}e_{t}\right)\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right) \times \left(1 - p\left({}_{i}c_{t}\right)\right) \times p\left({}_{j}e_{t}\right) \times \left(1 - p\left({}_{j}e_{t}\right)\right)}} = \frac{\sigma\left({}_{i}c_{t}, {}_{j}e_{t}\right)}{\sigma\left({}_{i}c_{t}\right) \times \sigma\left({}_{j}e_{t}\right)} \tag{118}$$

Proof by contradiction.

Again, our starting point (*der Ansatz*) is axiom I. Thus far, it is

$$+1 = +1 \tag{119}$$

Multiplying by the eigenvalue of the cause ${}_{i}c_{t}$, we obtain

$$+1 \times {}_{i}c_{t} = +1 \times {}_{i}c_{t} \tag{120}$$

or

$${}_{i}c_{t} = {}_{i}c_{t} \tag{121}$$

Multiplying this equation by an eigenvalue of an effect ${}_{j}e_{t}$, it is

$${}_{i}c_{t} \times {}_{j}e_{t} = {}_{i}c_{t} \times {}_{j}e_{t} \tag{122}$$

The eigenvalue of the effect ${}_{j}e_{t}$ is known to be defined as

$${}_{j}e_{t} \equiv \frac{\sigma\left({}_{j}e_{t}\right)}{\sqrt[2]{p\left({}_{j}e_{t}\right) \times \left(1 - p\left({}_{j}e_{t}\right)\right)}} \tag{123}$$

Substituting this relationship into the equation before, we obtain

$${}_{i}c_{t} \times \frac{\sigma\left({}_{j}e_{t}\right)}{\sqrt[2]{p\left({}_{j}e_{t}\right) \times \left(1 - p\left({}_{j}e_{t}\right)\right)}} = {}_{i}c_{t} \times {}_{j}e_{t} \tag{124}$$

The eigenvalue of the cause ${}_{i}c_{t}$ is known to be defined as

$${}_{i}c_{t} \equiv \frac{\sigma\left({}_{i}c_{t}\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right) \times \left(1 - p\left({}_{i}c_{t}\right)\right)}} \tag{125}$$

Substituting this relationship into the equation before, it is

$$\frac{\sigma\left({}_{i}c_{t}\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right) \times \left(1 - p\left({}_{i}c_{t}\right)\right)}} \times \frac{\sigma\left({}_{j}e_{t}\right)}{\sqrt[2]{p\left({}_{j}e_{t}\right) \times \left(1 - p\left({}_{j}e_{t}\right)\right)}} = {}_{i}c_{t} \times {}_{j}e_{t} \tag{126}$$

The product of the two eigenvalues ${}_{i}c_{t}$ and effect ${}_{j}e_{t}$ was derived as

$${}_{i}c_{t} \times {}_{j}e_{t} \equiv \frac{\sigma\left({}_{i}c_{t}, {}_{j}e_{t}\right)}{\left(p\left({}_{i}c_{t}, {}_{j}e_{t}\right)\right) - \left(p\left({}_{i}c_{t}\right) \times p\left({}_{j}e_{t}\right)\right)} \tag{127}$$

Substituting this relationship into the equation before, it follows that

$$\frac{\sigma\left({}_{i}c_{t}\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right) \times \left(1 - p\left({}_{i}c_{t}\right)\right)}} \times \frac{\sigma\left({}_{j}e_{t}\right)}{\sqrt[2]{p\left({}_{j}e_{t}\right) \times \left(1 - p\left({}_{j}e_{t}\right)\right)}} = \frac{\sigma\left({}_{i}c_{t}, {}_{j}e_{t}\right)}{\left(p\left({}_{i}c_{t}, {}_{j}e_{t}\right)\right) - \left(p\left({}_{i}c_{t}\right) \times p\left({}_{j}e_{t}\right)\right)} \tag{128}$$

The equation can be simplified as

$$\frac{\left(p\left({}_{i}c_{t},{}_{j}e_{t}\right)\right)-\left(p\left({}_{i}c_{t}\right)\times p\left({}_{j}e_{t}\right)\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right)\times\left(1-p\left({}_{i}c_{t}\right)\right)\times p\left({}_{j}e_{t}\right)\times\left(1-p\left({}_{j}e_{t}\right)\right)}}=\frac{\sigma\left({}_{i}c_{t},{}_{j}e_{t}\right)}{\sigma\left({}_{i}c_{t}\right)\times\sigma\left({}_{j}e_{t}\right)} \tag{129}$$

The mathematical formula of the causal relationship $k({}_{i}c_{t}, {}_{i}e_{t})$ follows as

$$k\left({}_{i}c_{t},{}_{j}e_{t}\right)\equiv\frac{\left(p\left({}_{i}c_{t},{}_{j}e_{t}\right)\right)-\left(p\left({}_{i}c_{t}\right)\times p\left({}_{j}e_{t}\right)\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right)\times\left(1-p\left({}_{i}c_{t}\right)\right)\times p\left({}_{j}e_{t}\right)\times\left(1-p\left({}_{j}e_{t}\right)\right)}}=\frac{\sigma\left({}_{i}c_{t},{}_{j}e_{t}\right)}{\sigma\left({}_{i}c_{t}\right)\times\sigma\left({}_{j}e_{t}\right)} \tag{130}$$

Quod erat demonstrandum.

Remark 6. The range of the causal relationship is

$$-1\leq k\left({}_{i}c_{t},{}_{j}e_{t}\right)\leq +1 \tag{131}$$

4 Discussion

This mathematical approach to the relationship between cause and effect is grounded on the law of independence known as

$$p\left({}_{i}c_{t},{}_{j}e_{t}\right)\equiv p\left({}_{i}c_{t}\right)\times p\left({}_{j}e_{t}\right) \tag{132}$$

while the mathematical formula of the causal relationship $k({}_{i}c_{t}, {}_{i}e_{t})$ even at quantum level follows as

$$k\left({}_{i}c_{t},{}_{j}e_{t}\right)\equiv\frac{\left(p\left({}_{i}c_{t},{}_{j}e_{t}\right)\right)-\left(p\left({}_{i}c_{t}\right)\times p\left({}_{j}e_{t}\right)\right)}{\sqrt[2]{p\left({}_{i}c_{t}\right)\times\left(1-p\left({}_{i}c_{t}\right)\right)\times p\left({}_{j}e_{t}\right)\times\left(1-p\left({}_{j}e_{t}\right)\right)}}=\frac{\sigma\left({}_{i}c_{t},{}_{j}e_{t}\right)}{\sigma\left({}_{i}c_{t}\right)\times\sigma\left({}_{j}e_{t}\right)} \tag{133}$$

These consideration of a deterministic relationship between cause and effect follows Einstein’s principle of locality and is primary a local-realistic approach to the relationship between cause and effect. Still, these approach to causality neither disproof nor disables non-locality. Meanwhile, the mathematical formula of causal relationship k is formulated even under conditions of general theory of relativity [59] while the electromagnetic field is geometrized [60] too. The problem of the physical meaning of the wave function [61] with the possibility to geometrize the gravitational field is solved. Still, other problems have to be considered. Especially the behavior and the meaning of Newton's gravitational constant (at quantum level) is not clear. To see the problems clearly, let us perform the following thought experiment. Two space-ship in deep space with the mass m_1 and m_2 are accelerating while the distance between both is d and the net force is F. According to Newton's law of gravitation we obtain

$$F = \frac{\gamma \times m_1 \times m_2}{d^2} \tag{134}$$

where γ denotes Newton's constant of gravitation. Rearranging equation we obtain

$$\frac{F \times d^2}{m_1 \times m_2} = \gamma \tag{135}$$

After a period of time the acceleration of both masses (i.e. space-ships) ends, the net force F becomes zero while the distance d and the masses are different from zero. We obtain

$$\frac{0 \times d^2}{m_1 \times m_2} = \gamma \tag{136}$$

or

$$0 = \gamma \tag{137}$$

Dividing by Newton's constant of gravitation, it is

$$\frac{0}{\gamma} = \frac{\gamma}{\gamma} \tag{138}$$

or

$$0 = 1 \tag{139}$$

Consequently, if we accept Newton's constant as a constant, we must accept too that +0=+1 or logical contradictions in nature. Thus far, there is already some evidence that Newton's gravitational constant is not [62], [63] a constant. Form these considerations follows that *either* an accelerated system (i.e. conditions of the general theory of relativity) cannot change into the state of a non- accelerated system (i.e. conditions of the special theory of relativity) *or* Newton's gravitational constant cannot be treated as a constant. No reasonable understanding of physical reality could expect to permit the first case. The first is not being the case, we are left with the alternative stated.

5 Conclusion

Recent years have seen a proliferation of different refinements of the basic idea to achieve a closer match with commonsense judgements about causation

6 Acknowledgement

I am very happy to have the opportunity to express my very deep gratitude to the Scientific Committee of the international conference “Quantum and Beyond (QB)”, at the Linnaeus University in Växjö, Sweden, June 13-16, 2016, an international conference devoted to quantum theory and experiment. This paper has been accepted for presentation and was presented at the conference “Quantum and Beyond (QB)”.

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