Dirac Equation for the Proton (II)

Why Fractional Charges for Muster Mark's Quarks?

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The present reading is the second in a series where we suggest a Dirac equation for the Proton. Despite its great success in explaining the physical world as we know it, in its bare form, not only is the Dirac equation at loss but fails to account *e.g.* for the following: (1) Why inside hadrons there are three, not four or five quarks; (2) Why quarks have fractional charges; (3) Why the gyromagnetic ratio of the Proton is not equal to two as the Dirac equation requires. In the present reading, we make an attempt to answer the second question of why quarks have fractional charges. We actually calculate the exact values of the charges of these quarks.

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INTRODUCTION

In our previous instalment [1] [hereafter Paper (I)], we not only gave an exposition of the Curved Spacetime (CST) Dirac theory since it was set-forth [2]; but went on to argue how this system of equations allows one to account for the three quarks found in hadrons – the Proton in the current investigations. In the present reading, we will demonstrate that this seemingly banal system of equations [3] can account for the fractional charges of quarks.

In Paper (I), we wrote down the equations that should govern our three-system of quarks and this equation is:

$$\left[\imath\hbar q_j A^{\mu}\gamma^{\mu}_{(a_j)}\partial_{\mu} - \mathrm{m}_0 c\right]|\psi_j\rangle = 0, \quad \text{for} \quad (j = 1, 2, 3). \tag{1}$$

Our relatively simple and trivial task in the present reading is to demonstrate that:

$$q_j := \left[\underbrace{\pm\frac{1}{3}}_{q_1}, \underbrace{\mp\frac{2}{3}}_{q_2}, \underbrace{\mp\frac{2}{3}}_{q_3}\right]. \tag{2}$$

For this, we need a system of three equations connecting these q's.

From the normalisation condition of the wavefunction, we did demonstrate in Paper (I) (equation 30), that:

$$\sum_{j=1}^{3} q_j^2 = 1 \quad \Rightarrow \quad q_1^2 + q_2^2 + q_3^2 = 1. \tag{3}$$

In-order to solve for q_1, q_2 and q_3, we need two more equations. In the subsequent section, we will deduce the other two equations and the solution of the resulting system of equations.

FRACTIONAL QUARK CHARGES

The quark model with fractional electronic charges was first put forward independently by Gellmann [4] and Zweig [5, 6]. As presently understood, the quark model comprises six flavors of quarks $[(u, d); (c, s); (t, b)]$ which come in three generations with the first generation comprising the *up* (u) and *down* (d) quark, the second generation comprises the *charm* (c) and *strange* (s) quark, and finally, the third generation comprises the *top* (t) and *botton* (b) quark. The first generation quarks (u, d) are the lightest in mass while the mass of the second generation (c, s) lies in the intermediate range and the third generation quarks (t, b) are the most massive. All the quarks are spin $1/2$ particles and have their anti-quark partners. Why these quarks have fractional electronic charges is an open question which if answered adequately, this will certainly unlock a wealth of understanding as to the nature of these mysterious "prisoners serving an eternal life sentence in the prison halls of the Proton". As already said, in the present reading, we want to calculate the electronic charges of quarks from the proposed *Theory of the Proton.*

We shall calculate the values of the q's first from the perspective of the CST-Dirac equations as presented in Paper (I). In this calculation, we shall obtain an unprecedented fit with reality and thereafter, we shall make the same computation under the pure Dirac theory [7, 8]. In the pure Dirac theory, we shall demonstrate that the Dirac theory yield prediction that are not in conformity with reality. Naturally, this is to be taken as a strong indicator that the proposed CST-Dirac equations [3] have something to do with reality.

CST-Dirac Equations

In Paper (I) (equation 31), we did argue as part of the derivation of the equations governing quarks, that:

$$\gamma^{\mu}_{(a)} = \pm \sum_{j=1}^{3} q_j \gamma^{\mu}_{(a_j)}. \tag{4}$$

We are going to use this equation (4) to deduce the two other equations that we need in-order to solve for the q's.

Since $[\gamma^0_{(a)} = \gamma^0_{(a_j)} = \gamma^0]$, this means that – for (4), if we take the $(\mu = 0)$-component, we will have:

$$\gamma^0 = \left(\pm \sum_{j=1}^{3} q_j \right) \gamma^0, \tag{5}$$

in-turn, equation (5) implies:

$$\sum_{j=1}^{3} q_j = \pm 1 \quad \Rightarrow \quad q_1 + q_2 + q_3 = \pm 1. \tag{6}$$

Equation (6) is the second equation of the the three equations that we need.

Now, for the third and last equation, this now depends on the nature of the Proton's spacetime. By the nature of the Proton's spacetime, we mean the configuration of the Proton's spacetime – is it quadratic $(a = 1)$, paraboloid $(a = 2)$ or hyperboloid $(a = 3)$. If the Proton's spacetime is quadratic [*i.e.*, $(a = 1)$], then, the sort for third equation will be:

$$q_2 = q_3. \tag{7}$$

Equations (3), (6) and (7) are our three system of equations and according to (6), there are two cases to be considered: the first of which is $(\sum_{j=1}^{3} q_j = +1)$, and the second of which is $(\sum_{j=1}^{3} q_j = -1)$. The resulting solutions are presented in Table (I).

TABLE I: **Solutions for the q's for a Quadratic Particle**

Case	Quark Charges			Particle
	q_1	q_2	q_3	
$\sum_{j=1}^{3} q_j = +1$	$-1/3$	$+2/3$	$+2/3$	Proton
$\sum_{j=1}^{3} q_j = -1$	$+1/3$	$-2/3$	$-2/3$	Positron

The perfect fit of the q's with the electronic charges of the quarks found in the Proton leave no room for one to think otherwise, but to take this opportunity to identify the q's as representing quark charges. This points to these equation having something to do with the nature of quarks.

Original Dirac Equation

If the Dirac theory is to be applied in its bare form as we have down with the CST-Dirac equations [3], then for the $\gamma^{\mu}_{(a)}$-matrices, we will have:

$$\gamma^{\mu}_{(a)} = \gamma^{\mu} \qquad [\forall\, a = (1, 2, 3)]. \tag{8}$$

Because of this, there will be two equations for the q's and these will be equation (3) and (6), *i.e.*, two systems of equations with three unknowns. Under such a scenario, since the q's are also the probability coefficients of the wavefunction and give that they can vary, in the case that they are not fixed by the internal logic of the theory, the will vary in a random fashion and taking values the satisfy the two system of equations.

Given that these same coefficients also determine the electronic charge of the three quarks, it would mean that the electronic charges of these quarks will have to vary accordingly. This variation will however not affect the overall electronic charge of the Proton as this is fixed by equation (6). Further, this variation will not lead to a violation of the Law of Conservation of Electronic Charge either. As far as reality dictates to us, the is not in tandem, hence, under the proposed model, the Dirac theory alone (*i.e.*, without the two extra equations obtained under the CST-Dirac equations) is not sufficient to account for the electronic charges of the quarks. The three configurations of spacetime as represented by the three matrices $\gamma^{\mu}_{(a)}$ allows for there to be three equations with three unknowns hence fixing once and for all, the values of the q, hence the electronic charges of quarks. This fixation of the q's, is without precedent – in exact accord with experimental philosophy.

PERTINENT QUESTIONS

Given that the proposed theory of the Proton has been able to explain at a relatively satisfactory level why there has to be three quarks inside the Proton and why these quarks need to have the very fractional electronic charges that they have, the reader is probably persuaded in the possible relevance of this theory in Particle physics. No other system of equations in the literature has been able to do this and it may very well be the first time that a theory does this. Because of this and given that the present paper is just the second in series (or about eight papers), some pertinent questions will arise regarding this theory. Below we present some of the these pertinent questions:

(1). **Gyromagnetic Ratio of the Proton:** If the Dirac equation can explain the Proton, especially the fact that it has to have three quarks residing inside it with these quarks having the charges that they are observed to have, why then and how does it come about that the Proton's gyromagnetic ratio is different from the predicted Dirac value of 2?

(2). **Quark Confinement:** Can the proposed theory of the Proton explain why quarks appear to exist in a state of permanent confinement *i.e.*, they can not be isolated and observed as free particles?

(3). **Quark Color Charge:** Can the proposed theory of the Proton explain why quarks have color charge? More so, why these color charges are three, not four, five or anything number?

(4). **Quark Generations:** Can the proposed theory of the Proton explain why quarks come in three generations? More so, why in each generation we have two quarks? Ultimately, why are there six quarks – not seven, eight or anything number?

We will not be pre-emptive by trying to shade some light on what this proposed new theory of the Proton has to say about the above questions. All we can say is that, it appears that the answers to these questions are well within provinces of the proposed theory. In Paper (III), we will answer the question on the gyromagnetic ratio of the Proton and in Paper (IV) we will answer the question on quark confinement *etc*. These papers are at an advanced stage of preparation awaiting the reviewer and subsequent acceptance of the first two readings: Paper (I) and (II).

GENERAL DISCUSSION

Thus far, using the seemingly banal system of equations proposed in [3]), we have demonstrated three things *i.e.*:

(1). In Paper (I): How a bispinor whose components are also bispinors can be made to comprise of a system of three independent non-interacting particles.

(2). In the Present Paper: How such a system as proposed in Paper (I) will have these three sub-particle to comprise of fractional electronic charges.

(3). In the Present Paper: Why such a system as proposed in Paper (I) will have fractional electronic charges which match those found for quarks.

As to the issue of how a bispinor whose components are also bispinors can be made to comprise of a system of three independent non-interacting particles, first one needs only realise that using the rich mathematics of *Lie Algebra*, any 2×1 component object (X, Y) can always be written as a linear superposition of three linearly independent 2×1 component objects (x_j, y_j), *i.e.*:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \sum_{j=1}^{3} \begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix} \begin{pmatrix} x_j \\ y_j \end{pmatrix}, \tag{9}$$

where $[\sigma_j : j = (1,2,3)]$ are the usual Pauli matrices. As demonstrated, the sub-systems (x_j, y_j) can be interpreted as quarks.

On the same pedestal, using the *Lie Algebra*, any 3×1 component object (X, Y, Z) can always be written as a linear superposition of eight linearly independent 3×1 component objects (x_j, y_j, z_j), *i.e.*:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \sum_{j=1}^{8} \begin{pmatrix} \lambda_j & 0 & 0 \\ 0 & \lambda_j & 0 \\ 0 & 0 & \lambda_j \end{pmatrix} \begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix}, \tag{10}$$

where the λ's are the usual eight 3×3 Gell-Mann matrices. One can continue the trend and show that any $N\times 1$ component object can always be written as a linear superposition of N^2-1 linearly independent $N\times 1$ component objects.

In the case of the present Proton model whose wavefunction is a spinor of bisponrs (*i.e.*, the usual four component Dirac wavefunction), the condition given in equations (33) and/or (34) in Paper (I) is what leads to the resulting sub-systems (quarks) to behave as independent and non-interacting particles.

As to the second and third issue of how such a system (proposed Proton Model) will have these three subparticles (quarks) to comprise of fractional electronic charges and why these fractional electronic charges match those found for quarks – this is because the CST-Dirac equation has three configurations of space [these are represented by the index $a = (1,2,3)$ in the matrix $\gamma^{\mu}_{(a)}$].

If anything, what these Proton model is telling us is that the Dirac equation in its own is not enough to explain the World. It may need the other two new equations that the CST-Dirac theory has been able to unearth. In-closing, allow us to that the the present in just work in progress, the meaning of which is that, the many pieces of the puzzle are not in yet, there is more to follow. From the work done this far, all we can say is that, it appears the proposed CST-Dirac equations [3], may hold some potent seed for particle physics.

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