Theoretical prediction of the fine structure constant within Quantum Electrodynamics

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Abstract: QED has predicted a relationship between the amplitude of an electron, the coupling constant (e), and the fine structure coupling constant (α); yet this prediction has never been theoretically achieved even though QED enjoyed enormous success in experimental tests that proven the predicted values of the fine structure constant. For the first time, this paper will make such a theoretical prediction regarding the relationship of (e) and (α) and constitute a new dimensionless constant (Д). Other methods of achieving theoretical predictions for a rough value of the fine structure constant will be debated as well.

Introduction

In order to obtain the value of the fine structure constant (α) experts in Quantum Electrodynamics, both theoretical and experimental, had to perform rather difficult tasks due to the lack of a theoretical prediction established that the amplitude of an electron in order to absorb or emit a photon ($e = 0.08542455$) [1] is directly connected with the constant (α). We will define this connection and try to establish another one from mathematical constants. Therefore we will provide two new ways to predict the fine structure constant.

The value of alpha has been experimentally measured to be ($\alpha^{-1} = 137.035\ 999\ 084\ (51)$) [2] in the latest experiment. The theoretical value is in near agreement but the process of determining the value of (α) is truly painstaking, see [3] for a better insight in the theoretical process of determination, the current theoretical value is ($\alpha^{-1}_{theory} = 137.035\ 999\ 174\ (35)$) [4].

The simple method

The simplest way to define the (α) constant is to slightly revise the equation:

$$(1)\ \delta F_1(q^2) \rightarrow \delta F_1(q^2) - \delta F_1(0)$$

Where ($F_1(0) = 1$) and (δF_1) is the first order correction to (F_1).

We define for electrons:

$$(2)\ F_1(q^2) = 1 + \frac{e^{Д}}{\pi}\int_0^1 dx\ dy\ dz\ \delta(x+y+z+1)\left[\log\left(\frac{m^2(1-z)^2}{m^2(1-z)^2 - q^2xy}\right) + \left(\frac{m^2(1-4z+z^2) + q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2xy + \mu^2 z}\right) - \left(\frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + \mu^2 z}\right)\right] + \mathcal{O}\left\{\left(e^{Д}\right)^2\right\}$$

$$(3)\ F_2(q^2) = \frac{e^{Д}}{Д\pi}\int_0^1 x\ dy\ dz\ \delta(x+y+z-1)\left[\frac{2m^2(1-z)}{m^2(1-z)^2 - q^2xy}\right] + \mathcal{O}\left\{\left(e^{Д}\right)^2\right\}$$

where (e) is the amplitude of the electron, mentioned above, and (Д) is a new dimensionless constant with a value ($Д = 2.0000000653643$).

Therefore:

$$(4)\ F_2(q^2 = 0) = \frac{e^{Д}}{Д\pi}\int_0^1 x\ dy\ dz\ \delta(x+y+z-1)\frac{2m^2(1-z)}{m^2(1-z)^2} = \frac{e^{Д}}{\pi}\int_0^1 dz \int_0^{1-z} dy\frac{z}{1-z} = \frac{e^{Д}}{Д\pi}$$

Therefore obtaining us the anomalous magnetic dipole moment (a_e) in a slightly changed form:

$$(5)\ a_e \equiv \frac{q-2}{2} = \frac{e^{Д}}{Д\pi} = 0.001161409695$$

which is the one loop result and (g) is the g-factor of the electron [5].

We make an obvious claim that:

$$(6)\ e^{-Д} = \alpha^{-1} = 137.035999084(49)$$

since ($Д = 2.0000000653643$) and the electron amplitude is ($e = 0.08542455$) as mentioned before. We also claim that:

$$(7)\ e^{Д} = \alpha = 0.00729735256(93)$$

Both of these claims are in agreement with the experimental and theoretical values referenced in the paper.

The complex method

The second method asserts the relationship of the fine structure constant and mathematical constants. This method is less accurate and more difficult to use but it is worth mentioning it nevertheless.

I Case

We define that:

$$(8)\ \alpha \sim \frac{\alpha_c \cdot \zeta(3)}{\sqrt{5} \cdot \pi^4 \cdot (\Psi - p)}$$

Where (α_c) is the second Feigenbaum constant, ($\zeta(3)$) is the Apery constant, (Ψ) is the Reciprocal Fibonacci constant and (p) is the Porter constant.

II Case

We define that:

$$(9)\ \alpha \sim \frac{\alpha_c \cdot \zeta(3)}{\sqrt{5} \cdot \pi^4 \cdot \left(\frac{\alpha_c}{-g} + \mu\right)}$$

where (g) is the g-factor and (μ) is the Cohen constant.

Where we established that a rough value of (α) can be attained by using mathematical constants, apart from the g-factor in the second equation, which is not a mathematical constant.

Conclusions

We found new ways of defining the fine structure constant (α) in far simpler way than the previous cases, specifically the simpler method is the most efficient one since it offers accurate results in complete agreement with the latest experimental results by applying a new, dimensionless constant. We can also state than ($e^{Д} = \alpha = \alpha_G 10^{36} = \alpha_G \cdot N$) where ($\alpha_G$) is the gravitational coupling constant, defined by using two protons, and ($N = 10^{36}$) is the ratio of ($\frac{e^{Д}}{\alpha_G} = N$) where we use ($e^{Д}$) instead of ($\alpha$).

We constitute an equation to describe how the photons and electrons interact.

$$(10)\ E = \frac{q^2}{4\pi\varepsilon_0 d} \Big/ e^{Д}$$

Where we used the symbol (q) for elementary charge instead of the standard practice symbol (e) in order to avoid confusion with the amplitude of the electron (e) that is a dimensionless value.

We prove this claim by showing that:

$$(11)\ e^{Д} = \frac{q^2}{4\pi\varepsilon_0 d} \Big/ E$$

Since we photons are massless particles ($E = \frac{hc}{\lambda}$) meaning:

$$(12)\ e^{Д} = \frac{q^2}{4\pi\varepsilon_0 d} \Big/ \frac{hc}{\lambda}$$

This can be rewritten as:

$$(13)\ e^{Д} = \frac{q^2}{4\pi\varepsilon_0 d} \cdot \frac{2\pi d}{hc}$$

Changing the Planck constant (h) to the reduced Planck constant (ħ):

$$(14)\ e^{Д} = \frac{q^2}{4\pi\varepsilon_0 d} \cdot \frac{d}{\hbar c}$$

We conclude that:

$$(15)\ e^{Д} = \frac{q^2}{4\pi\varepsilon_0 \hbar c} = 0.00729735256$$

Which means that ($e = \sqrt[Д]{\frac{q^2}{4\pi\varepsilon_0 \hbar c}}$) is the relationship in question.

We should also state that the value of (α) can be roughly deduced by a third method which has the lowest accuracy.

We state that:

$$(16)\ \alpha \sim \frac{\zeta(3)}{{}^{43}/_{K} \cdot \pi^2}$$

where (K) is the Sierpinski constant. We also state that:

$$(17)\ \alpha \sim \frac{\zeta(3)}{{}^{47}/_{F} \cdot \pi^2}$$

where (F) is the Fransen Robinson constant. Finally we state that:

$$(18)\ \alpha \sim \frac{\zeta(3)}{(16 + \Omega_\lambda)\pi^2}$$

Where (Ω_λ) is the ratio between the energy density due to the cosmological constant and the critical density of the Universe. However, the value has to be $(\Omega_\lambda \approx 0.690139)$ in order for the equation to be accurate. This differs from the results attained by experts in their field which is $(\Omega_\lambda = 0.6911(62))$ [6] although to be fair, the prediction hardly has an accurate value.

The simple method is more advisable for theoretical predictions of the fine structure constant (α) due to its simplicity and accuracy.

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