Conjecture that there exist an infinity of even numbers n for which n^2 is a Harshad-Coman number

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Abstract. In a previous paper I defined the notion of Harshad-Coman numbers as the numbers n with the property that (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer. In this paper I conjecture that there exist an infinity of even numbers n for which n^2 is a Harshad-Coman number and I also make a classification in four classes of all the even numbers.

Definition:

The Harshad-Coman numbers are the numbers n with the property that m = (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer.

The sequence of Harshad-Coman numbers:

: 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 19, 20, 21, 22, 25, 28, 31, 37, 40, 41, 43, 46, 49, 51, 55, 61, 64, 71, 73, 81, 82, 85, 91, 101, 103, 109, 110, 111, 112, 113, 115, 118, 121 (...)

Conjecture:

There exist an infinity of even numbers n such that n^2 is a Harshad-Coman number.

The sequence of even numbers n for which n^2 is a Harshad-Coman number:

: 2, 8, 14, 20, 46, 80, 84, 92 (...)

Indeed:

: (2^2 – 1)/(s(2^2) – 1) = 3/3 = 1, integer;
: (8^2 – 1)/(s(8^2) – 1) = 63/9 = 7, integer;
: (14^2 – 1)/(s(14^2) – 1) = 195/15 = 13, integer;
: (20^2 – 1)/(s(20^2) – 1) = 399/21 = 19, integer;
: (46^2 – 1)/(s(46^2) – 1) = 2115/9 = 235, integer;
: (80^2 – 1)/(s(80^2) – 1) = 6399/9 = 711, integer;
: (84^2 – 1)/(s(84^2) – 1) = 7055/17 = 415, integer;
: (92^2 – 1)/(s(92^2) – 1) = 8463/21 = 403, integer.

Classification of even numbers in 4 classes:

I. Even numbers n whose squares n^2 are Harshad-Coman numbers:
: 2, 8, 14, 20, 46, 64, 80 (...)

II. Even numbers n whose squares n^2 lead to an m rational:
: 4, 6, 12, 16, 18, 22, 26, 28, 30, 32, 34, 38, 40, 44, 48, 50, 52, 56, 58, 60, 62, 68, 70, 76, 82, 86, 90, 94, 98 (...),
for which m is equal to: 2.5, 4.375, 17.875, 21.25, 40.375, 40.25, 37.5, 43.5, 112.375, 170.5, 96.25, 120.25, 66.625, 107.5, 287.875, 416.5, 225.25, 261.25, 224.2, 449.875, 213.5, 227.5, 308.2, 408.25, 240.625, 373.5, 308.125, 1012.375, 368.125, 533.5 (...)

III. Even numbers n whose squares n^2 lead to an m irrational:
: 24, 42, 54, 66, 72, 74, 78, 88, 96 (...),
for which m is equal to: 33.(...)(a period of 48 digits), 103.(...)(a period of 48 digits), 171.(...)(a period of 48 digits), 256.(...)(a period of 48 digits), 304.(...)(a period of 48 digits), 260.(...)(a period of 6 digits), 357.(...)(a period of 48 digits), 368.(...)(a period of 6 digits), 542.(...)(a period of 48 digits) (...)

Note that the period obtained for 74 is the same with the one obtained for 88, i.e. 714285 and the period obtained for 24 is the same with the one obtained for 78, i.e. 823529411764705882352941176470588235294117647058.

IV. Even numbers n whose squares n^2 lead to an expression without meaning because implies the number zero as denominator:
: 10, 100, 1000, 10000, 100000, 1000000 (...)