Conjecture that there exist an infinity of odd numbers n for which n^2 is a Harshad-Coman number

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Abstract. In a previous paper I defined the notion of Harshad-Coman numbers as the numbers n with the property that (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer. In this paper I conjecture that there exist an infinity of odd numbers n for which n^2 is a Harshad-Coman number and I also make a classification in three classes of all the odd numbers greater than 1.

Definition:

The Harshad-Coman numbers are the numbers n with the property that m = (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer.

The sequence of Harshad-Coman numbers:

: 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 19, 20, 21, 22, 25, 28, 31, 37, 40, 41, 43, 46, 49, 51, 55, 61, 64, 71, 73, 81, 82, 85, 91, 101, 103, 109, 110, 111, 112, 113, 115, 118, 121 (...)

Conjecture:

There exist an infinity of odd numbers n such that n^2 is a Harshad-Coman number.

The sequence of odd numbers n for which n^2 is a Harshad-Coman number:

: 3, 5, 7, 9, 11, 15, 17, 19, 21, 25, 29, 31, 33 (...)

Indeed:

: (3^2 – 1)/(s(3^2) – 1) = 8/8 = 1, integer;
: (5^2 – 1)/(s(5^2) – 1) = 24/6 = 4, integer;
: (7^2 – 1)/(s(7^2) – 1) = 48/12 = 4, integer;
: (9^2 – 1)/(s(9^2) – 1) = 80/8 = 10, integer;
: (11^2 – 1)/(s(11^2) – 1) = 120/3 = 40, integer;
: (15^2 – 1)/(s(15^2) – 1) = 224/8 = 28, integer;
: (17^2 – 1)/(s(17^2) – 1) = 288/18 = 16, integer;
: (19^2 – 1)/(s(19^2) – 1) = 360/9 = 40, integer.
(...)

Classification of odd numbers greater than 1 in 3 classes:

I. Odd numbers n whose squares n^2 are Harshad-Coman numbers:
: 3, 5, 7, 9, 11, 15, 17, 19, 21, 25, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 59, 61, 63, 65, 67, 69, 71, 73, 77, 79, 89, 91, 97 (...)

II. Odd numbers n whose squares n^2 lead to an m rational:
: 13, 23, 83, 85, 95 (...),
for which m is equal to: 11.2, 35.2, 229.6, 481.6, 601.6 (...)

III. Odd numbers n whose squares n^2 lead to an m irrational:
: 27, 57, 63, 75, 81, 87, 93, 99 (...),
for which m is equal to: 42.(...)(a period of 48 digits), 191.(...)(a period of 48 digits), 191.(...)(a period of 6 digits), 330.(...)(a period of 48 digits), 291.(...)(a period of 6 digits), 332.(...)(a period of 6 digits), 576.(...)(a period of 48 digits) (...)

Note that the period obtained for 27 is the same with the one obtained for 75, 24 and 78 (see my previous paper "Conjecture that there exist an infinity of even numbers n for which n^2 is a Harshad-Coman number", i.e. 823529411764705882352941176470588235294117647058.

Note also that the period obtained for 57 is the same with the one obtained for 96, i.e. 058823529411764705882352941176470588235294117647 (compare it with the one from above).

Note also that the period obtained for 81 is the same with the one obtained for 72, i.e. 882352941176470588235294117647058823529411764705 (compare it with the ones from above).

Note also that the period obtained for 99 is the same with the one obtained for 54, i.e. 470588235294117647058823529411764705882352941176 (compare it with the ones from above).

Note also that the period obtained for 93 is the same with the one obtained for 63, i.e. 615384.

Open problem: is there a limited number of distinct periods that can be obtained applying to the set of natural numbers n the expression $(n - 1)/(s(n) - 1)$?

Comment:

Unlike in the case of even numbers (see the paper mentioned above) where very few squares seem to be Harshad-Coman numbers, in the case of odd numbers this seem to be the dominant rule. It is once again a case when I am noticing similarities between the behaviour of squares of odd numbers and Fermat pseudoprimes (see my paper “Conjecture that there exist an infinity of Poulet numbers which are also Harshad-Coman numbers”.