

## Three conjectures regarding Poulet numbers and Harshad numbers

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**Abstract.** In this paper I make the following three conjectures: (I) If  $P$  is both a Poulet number and a Harshad number, then the number  $P - 1$  is also a Harshad number; (II) If  $P$  is a Poulet number divisible by 5 under the condition that the sum of the digits of  $P - 1$  is not divisible by 5 then  $P - 1$  is a Harshad number; (III) There exist an infinity of Harshad numbers of the form  $P - 1$ , where  $P$  is a Poulet number.

In this paper I make three conjectures regarding Poulet numbers and Harshad numbers (for the sequence of Poulet numbers see A001567 and for the sequence of Harshad numbers see A005349 in OEIS).

### Conjecture 1:

If  $P$  is both a Poulet number and a Harshad number, then the number  $P - 1$  is also a Harshad number.

#### The sequence of the numbers $P$ :

(The conjecture is verified for the first 8 numbers which are both Poulet and Harshad)

: 645, 1387, 1729, 1905, 2465, 2821, 8911, 30121 (...)  
(indeed, 644, 1386, 1728, 1904, 2464, 2820, 8910, 30120 are also Harshad numbers)

### Conjecture 2:

If  $P$  is a Poulet number divisible by 5 under the condition that the sum of the digits of  $P - 1$  is not divisible by 5 then  $P - 1$  is a Harshad number.

#### The sequence of the numbers $P$ :

(The conjecture is verified for the first 10 Poulet numbers divisible by 5 which respect the condition mentioned)

: 645, 1105, 1905, 2465, 10585, 11305, 16705, 34945,  
41665, 62745 (...)  
(indeed, 644, 1104, 1904, 2464, 10584, 11304, 16704,  
34944, 41664, 62744 are Harshad numbers)

Note: three other Poulet numbers up to 62745 are divisible by 5, i.e. 18705, 39865 and 55245 (but the sum of the digits of 18704, 39864 respectively 55244 is divisible by 5).

**Conjecture 3:**

There exist an infinity of Harshad numbers of the form  $H = P - 1$ , where  $P$  is a Poulet number.

**The sequence of the numbers  $H$ :**

: 644, 1104, 1386, 1728, 1904, 2464, 2700, 2820, 3276,  
4032, 4368, 4680, 5460, 6600, 8320, 8480, 8910,  
10260, 10584, 11304, 13740, 14490, 15708, 15840,  
16704, 18720, 23000, 25760, 29340, 30888, 31608,  
31620, 33152, 34944, 41040, 41664, 46656, 49140,  
49980, 52632, 57420, 62744, 68100 (...)