Alternative representations of $\frac{x}{2}$

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Abstract: In this paper, we prove interesting alternative representations of the simple fraction x/2 where x is a real number using complex numbers.

$$\frac{x}{10^i+1} = \frac{x}{10^i+1} \cdot \frac{10^{-i}+1}{10^{-i}+1}$$

$$= \frac{x(10^{-i}+1)}{10^i+10^{-i}+2}$$

$$= \frac{x(e^{-i\cdot ln(10)}+1)}{e^{i\cdot \ln(10)}+e^{-i\cdot \ln(10)}+2}$$

$$= \frac{x\left[\left(\cos\left(ln(10)\right)-i\cdot \sin\left(ln(10)\right)\right)+1\right]}{2\left(\frac{e^{i\cdot \ln(10)}+e^{-i\cdot \ln(10)}}{2}+1\right)}$$

$$= \frac{x\cdot \cos\left(ln(10)\right)-x\cdot i\cdot \sin\left(ln(10)\right)+x}{2\left(\cos\left(ln(10)\right)+1\right)}$$

$$= \frac{x\cdot \cos\left(ln(10)\right)+x}{2\left(\cos\left(ln(10)\right)+1\right)} - \frac{x\cdot i\cdot \sin\left(ln(10)\right)}{2\left(\cos\left(ln(10)\right)+1\right)}$$

$$= \frac{x\left(\cos\left(ln(10)\right)+1\right)}{2\left(\cos\left(ln(10)\right)+1\right)} - \frac{x}{2}i\cdot \frac{\sin\left(ln(10)\right)}{\cos\left(ln(10)\right)+1}$$

$$\boxed{\frac{x}{10^i+1} = \frac{x}{2} - \frac{x}{2}i\cdot tan\left(\frac{1}{2}\ln(10)\right)} => \boxed{Re\left(\frac{x}{10^i+1}\right) = \frac{x}{2}}$$

$$\frac{x}{10^i-1} = \frac{x}{10^i-1} \cdot \frac{10^{-i}-1}{10^{-i}-1}$$

$$= \frac{x(10^{-i}-1)}{2-10^{-i}-10^i}$$

$$= \frac{x(e^{-i\cdot \ln(10)}-1)}{2-e^{-i\cdot \ln(10)}-e^{i\cdot \ln(10)}}$$

$$= \frac{x\left[\left(\cos\left(ln(10)\right)-i\cdot \sin\left(ln(10)\right)\right)-1\right]}{2-2\cdot \cos\left(ln(10)\right)}$$

$$= \frac{x\cdot \cos\left(ln(10)\right)-x\cdot i\cdot \sin\left(ln(10)\right)-x}{2\left(1-\cos\left(ln(10)\right)\right)}$$

$$= \frac{x\cdot \cos\left(ln(10)\right)-x}{2\left(1-\cos\left(ln(10)\right)\right)} - \frac{x\cdot i\cdot \sin\left(ln(10)\right)}{2\left(1-\cos\left(ln(10)\right)\right)}$$

$$= \frac{x \cdot \left(\cos\left(ln(10)\right) - 1\right)}{2\left(1 - \cos\left(ln(10)\right)\right)} - \frac{x}{2} i \cdot \frac{\sin\left(ln(10)\right)}{1 - \cos\left(ln(10)\right)}$$

$$= \frac{x}{2} \cdot \frac{\cos\left(ln(10)\right) - 1}{1 - \cos\left(ln(10)\right)} - \frac{x}{2} i \cdot cot\left(\frac{1}{2}\ln(10)\right)$$

$$\boxed{\frac{x}{10^i - 1} = -\frac{x}{2} - \frac{x}{2} i \cdot cot\left(\frac{1}{2}\ln(10)\right)} => \boxed{Re\left(\frac{x}{10^i - 1}\right) = -\frac{x}{2}}$$

$$\frac{x}{10^i + i} = \frac{x}{10^i + i} \cdot \frac{10^{-i} - i}{10^{-i} - i}$$

$$= \frac{x(10^{-i} - i)}{2 + i \cdot 10^{-i} - i \cdot 10^i}$$

$$= \frac{x(e^{-i \cdot \ln(10)} - i)}{2 + i \cdot e^{-i \cdot \ln(10)} - i \cdot e^{i \cdot \ln(10)}}$$

$$= \frac{x\left[\left(\cos\left(ln(10)\right) - i \cdot \sin\left(ln(10)\right)\right) - i\right]}{2 + 2 \cdot \frac{i \cdot e^{-i \cdot \ln(10)} - i \cdot e^{i \cdot \ln(10)}}{2}}$$

$$= \frac{x \cdot \cos\left(ln(10)\right) - x \cdot i \cdot \sin\left(ln(10)\right) - x \cdot i}{2 + 2 \cdot \sin(\ln(10))}$$

$$= \frac{x \cdot \cos\left(ln(10)\right) - x \cdot i\left(\sin\left(ln(10)\right) + 1\right)}{2(\sin(\ln(10)) + 1)}$$

$$= \frac{x}{2} \cdot \frac{\cos\left(ln(10)\right)}{\sin(\ln(10)) + 1} - \frac{x\left(\sin\left(ln(10)\right) + 1\right)}{2(\sin(\ln(10)) + 1)} i$$

$$\boxed{\frac{x}{10^i + i} = \frac{x}{2} \cdot \frac{\cos\left(ln(10)\right)}{\sin(\ln(10)) + 1} - \frac{x}{2} i} => \boxed{Im\left(\frac{x}{10^i + i}\right) = -\frac{x}{2}}$$

$$\frac{x}{10^i - i} = \frac{x}{10^i - i} \cdot \frac{10^{-i} + i}{10^{-i} + i}$$

$$= \frac{x(10^{-i} + i)}{2 - i \cdot 10^{-i} + i \cdot 10^i}$$

$$= \frac{x(10^{-i} + i)}{-2\left(\frac{i \cdot 10^{-i} - i \cdot 10^i}{2} - 1\right)}$$

$$= \frac{x(e^{-i \cdot \ln(10)} + i)}{-2\left(\frac{i \cdot e^{-i \cdot \ln(10)} - i \cdot e^{i \cdot \ln(10)}}{2} - 1\right)}$$

$$= \frac{x\left[\left(\cos\left(ln(10)\right) - i \cdot \sin\left(ln(10)\right)\right) + i\right]}{-2(sin(\ln(10)) - 1)}$$

$$= \frac{x\left[\left(\cos\left(ln(10)\right) - i \cdot \sin\left(ln(10)\right)\right) + i\right]}{-2(sin(\ln(10)) - 1)}$$

$$= \frac{x \cdot \cos\left(ln(10)\right) - x \cdot i \cdot \sin\left(ln(10)\right) + x \cdot i}{2 - 2 \cdot \sin(\ln(10))}$$

$$= \frac{x \cdot \cos\left(ln(10)\right) - x \cdot i\left(\sin\left(ln(10)\right) - 1\right)}{2\left(1 - \sin(\ln(10))\right)}$$

$$= \frac{x}{2} \cdot \frac{\cos\left(ln(10)\right)}{1 - \sin(\ln(10))} - \frac{x}{2} i \cdot \frac{\sin\left(ln(10)\right) - 1}{1 - \sin(\ln(10))}$$

$$\boxed{\frac{x}{10^i - i} = \frac{x}{2} \cdot \frac{\cos\left(ln(10)\right)}{1 - \sin(\ln(10))} + \frac{x}{2} i} => \boxed{Im\left(\frac{x}{10^i - i}\right) = \frac{x}{2}}$$

$$\frac{10^{xi}}{10^{xi} + 1} = \frac{10^{xi}}{10^{xi} + 1} \cdot \frac{10^{-xi} + 1}{10^{-xi} + 1}$$

$$= \frac{10^{xi} + 1}{10^{xi} + 10^{-xi} + 2}$$

$$= \frac{e^{\text{ix}\ln(10)} + 1}{2\left(\frac{e^{\text{ix}\ln(10)} + e^{-\text{ix}\ln(10)}}{2} + 1\right)}$$

$$= \frac{\cos\left(x\ ln(10)\right) + i \cdot \sin\left(x\ ln(10)\right) + 1}{2\left(\cos\left(x\ ln(10)\right) + 1\right)}$$

$$= \frac{\cos\left(x\ ln(10)\right) + 1}{2\left(\cos\left(x\ ln(10)\right) + 1\right)} + \frac{i}{2} \cdot \frac{\sin\left(x\ ln(10)\right)}{\cos\left(x\ ln(10)\right) + 1}$$

$$\boxed{\frac{10^{xi}}{10^{xi} + 1} = \frac{1}{2} + \frac{1}{2} i \cdot tan\left(\frac{1}{2}\text{x} \cdot \ln(10)\right)} => \boxed{Re\left(\frac{10^{xi}}{10^{xi} + 1}\right) = \frac{1}{2}} <=> \boxed{x \cdot Re\left(\frac{10^{xi}}{10^{xi} + 1}\right) = \frac{x}{2}}$$