Deductive Derivation of Lorentz Transformation for the Special Relativity Theory

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Abstract

Activities of science as discovery of facts, discovery of theories and verification of theory might not be completed yet by the verification of theory's consistence with facts. Theories should be explainable using already known facts and logic human commonly have. This effort is to acquire intellectual properties. This is more meaningful same as science's contribution to society.

Here deductive explanation is tried for inductively verified Special Relativity Theory.

1. Introduction

Lorentz transformation transforms the time-space position of a point from an inertia system view to another inertia system view. Here we think two inertia systems, one is two dimensions S system (time dimension is t and space dimension is x.), another is two dimensions S' system (time dimension is t' and space dimension is x'.) and S' system moves along x-axis with velocity v relatively.

On this environment, in order to derive Lorentz transformation deductively, investigation about following aspects is required.

- Conversion of time dimension
- Definition of time axis of S' system on S system
- Definition of space axis of S' system on S system
- Definition of scale of time and space of S' system

2. Lorentz Transformation

About the transformation, there is a basic system (in this case S system). Another system (in this case S' system) which is for a moving point with velocity v on the basic system, has own frame of reference. Based on these frames of reference, a point can be represented by the S system frame of reference also can be represented by the S' system frame of reference. Lorentz transformation represents this relation using formulas.

Here systems except the basic system may not be orthogonal.

Also if S' system would be recognized as a basic system, its frame of reference becomes same as the S system's and the frame of reference for the S system becomes same as that of system which is moving with velocity $-v$ to the S system.

As we can understand on this, which system is a basic system depends on how we look systems. No system has priority.

3. Time-space graph

Fig.1 is usual time-space graph. Regarding to time-space graph, Lorentz transformation is frame of reference transformation like rotation. If so, unit of all dimensions (axis) of a system should be same. Then time-space graph which indicates Lorentz transformation is a different style of graph from Fig.1. Unit of space and time should be unified. For this requirement, we consider conversion of time dimension.

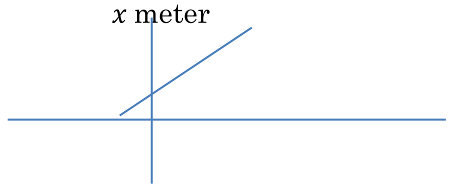

Fig. 1

4. Conversion of time dimension

In order that time-space frame of reference transformation can be done, unit of time need to be same as the unit of space.

Definition

Regarding to any space point, time is passing synchronously. (a)

Time does not exist at the point where space or universe don't exist. (b)

If the expansion speed of universe or space is γ, its front of x dimension is expanding on (1).

$$x = \gamma t \tag{1}$$

So after t_1 from the beginning of universe, front of x dimension reaches to

$$x_1 = \gamma t_1$$

Here infinitesimal short time before t_1 is $t_1 - dt$. At the timing, space point x_1 does not exist. So time also does not exist there.

At time $t_1 - dt$, front space of x dimension expands to $\gamma(t_1 - dt)$.

Based on the above and continuousness requirement, following could be derived.

Time t_1 at point x_1 should not be the result that time at x_1 was counted up because no time was there at $t_1 - dt$.

Time t_1 at point x_1 should be the result that time at $\gamma(t_1 - dt)$ move to x_1 at t_1.

This means time moved with speed γ in space.

When time t passed, time moved space distance γt.

Then if unit of time is same as the unit of space and its value is γt, time dimension and space dimension become isomorphic.

Definition; From time-space graph view, time moves toward time direction also toward space direction with speed γ. (c)

Then required conversion of time dimension for Lorentz transformation is

-to have same unit as space's

-to make its value γt

Here exact definition is required.

Space distance : space dimension difference of two points

Time : For any space point, something passing. Its dimension is called time dimension. And time is time dimension difference of two points

Time distance : space compatible distance converted from time

Time passing move : At any space point, time moves on time-space graph toward time direction according to time passing.

5. Definition of time axis of S' system on S system

In general, time axis of time-space graph is space position zero line.

' time t passed' means 'it is at γt on time axis of time-space graph'.

And at time t or γt time distance, $x' = 0$ point reaches at space point vt.

So $x' = 0$ point moves along the following track (Fig. 2)

$$x = \frac{v}{\gamma}\gamma t = tan\theta \cdot \gamma t \qquad (2)$$

This is space position zero $(x' = 0)$ line or time axis $(\gamma t')$ of S' system.

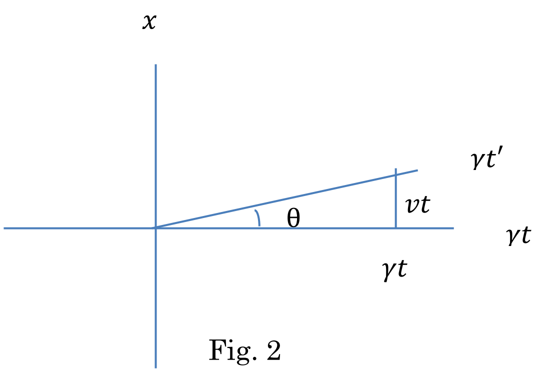

Fig. 2

6. Definition of space x' axis of S' system on S system

On (c), time is moving at every space points. When a moving time reaches at a space point P from origin O, passed time distance at point P is a(Fig.3) and moved space distance for the moving time is x. a and x is equivalent because moving time velocity is same γ for time dimension also for space dimension.

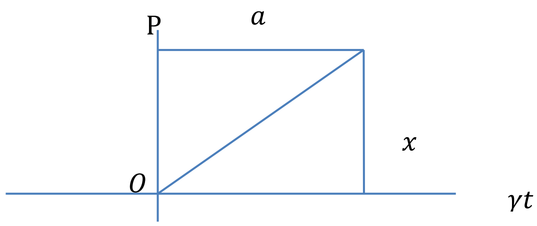

Fig.3

In another words, for the origin(O) of the moving time, moved space distance x for the moving time and passed time distance a at reached point P should be equal.
Following is how origin point(O) recognizes passed time at point P when moving time reaches. (Space point P is distance x away from origin.)
Origin point of S system recognize passed time distance x as on Fig.3

Origin point of S' system moves $\frac{v}{\gamma}x$ until moving time reaches at space point P.(Fig.4)

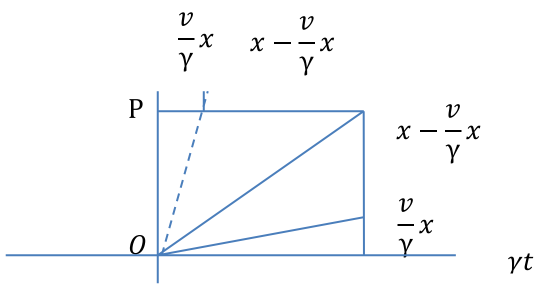

Fig.4

So it recognizes moving time moved $x-\frac{v}{\gamma}x$. Then it recognizes passed time distance is $x-\frac{v}{\gamma}x$ at space point P. This means time distance position on the frame of reference system for S' system is advanced $\frac{v}{\gamma}x$ at space point P. Then time zero for S' system is on following line of S system.

$$\gamma t = \frac{v}{\gamma} x$$

$$x = \frac{\gamma}{v} \gamma t x \qquad (3)$$

This is time zero line or space axis for S' system.

7. Definition of scale of time and space of S' system

As shown in Fig.2,Fig.4, S' system is an oblique frame of reference and S system is an orthogonal frame of reference. This means that a target point is viewed by different type of two frames of reference.

Definition;

When time t pass for space volume V (length l in the case of one dimension space), integration of V for time t is 'experience (exp)'. That is $\exp = \int V(t)dt$

Ten thousand years for Sun is something different from 10 μ sec. for an atom.
Experience is the *something* and its value indicates such huge difference in this case.

In the case of Fig.5, experience of the system S is:

Time t passes for length l or area$\overline{\mathrm{OACB}}$.

In the case of Fig.6, experience of the system S' is:

Time t' passes for length l' or area$\overline{\mathrm{OA'C'B'}}$.

If $l' = l$, $t' = t$,

area$\overline{\mathrm{OA'C'B'}}$ = area$\overline{\mathrm{OACB}} \sin\alpha$ = time $t\sqrt{sin\alpha}$ passes for length $l\sqrt{\sin\alpha}$

This means 'experience' is compressed($0 < \alpha < \frac{\pi}{2}$) on oblique system by $sin\alpha$,

Or this means distance is compressed on oblique system by $\sqrt{sin\alpha}$.

Then distance $of\ S'$ system = $(distance\ of$ S system$)\sqrt{\sin\alpha}$, (4)

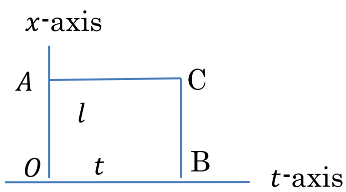

Fig. 5

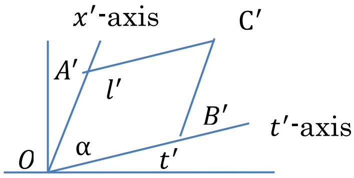

Fig. 6

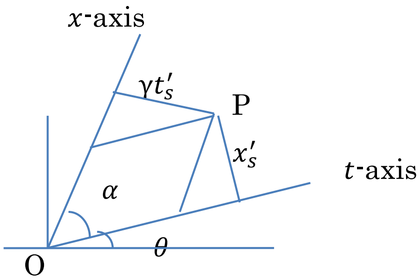

Fig. 7

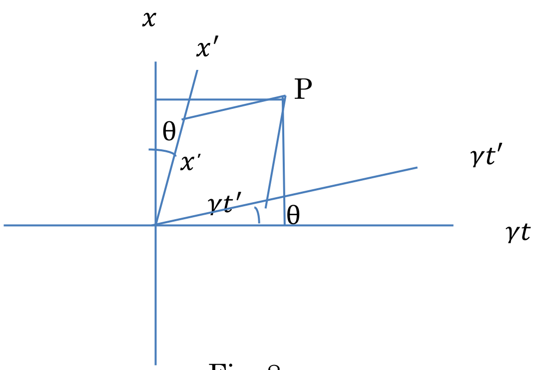

Fig. 8

8. Description of a time-space point

A point P is described as Fig.7.

Relation of position value is

$$\gamma t'_s = \gamma t cos\theta - x sin\theta \tag{5}$$

$$x'_s = -\gamma t sin\theta + x cos\theta \tag{6}$$

These t'_s, x'_s are shortest time distance and shortest space distance as on Fig.7.

On Fig.8, these t', x' are space position constant time distance and time constant space distance.

As time dimension and space dimension, the latter distance t', x' are used.

These relation are

$$\gamma t' = \frac{\gamma t_s'}{\sin\alpha}$$

$$x' = \frac{x_s'}{\sin\alpha}$$

Then

$$\gamma t' = \frac{(\gamma t cos\theta - x sin\theta)}{\sin\alpha} \tag{7}$$

$$x' = \frac{(-\gamma t sin\theta + x cos\theta)}{\sin\alpha} \tag{8}$$

Scale of oblique (4) is applied to these.

$$\gamma t' = \frac{\gamma t cos\theta - x sin\theta}{\sin\alpha}\sqrt{\sin\alpha} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{\sin\alpha}} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{\sin(\frac{\pi}{2} - 2\,\theta)}} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{cos^2\theta - sin^2\theta}} \tag{9}$$

$$x' = \frac{-\gamma t sin\theta + x cos\theta}{\sin\alpha}\sqrt{\sin\alpha} = \frac{-\gamma t sin\theta + x cos\theta}{\sqrt{\sin\alpha}} = \frac{-\gamma t sin\theta + x\cos\theta}{\sqrt{cos^2\theta - sin^2\theta}} \tag{10}$$

Here, from (2)

$$cos\theta = \frac{\gamma}{\sqrt{\gamma^2 + v^2}}, \quad sin\theta = \frac{v}{\sqrt{\gamma^2 + v^2}}$$

Then (9) (10) are

$$\gamma t' = \frac{\gamma t - \frac{v}{\gamma}x}{\sqrt{1 - \frac{v^2}{\gamma^2}}} \tag{11}$$

$$\mathrm{x}' = \frac{-vt + x}{\sqrt{1 - \frac{v^2}{\gamma^2}}} \tag{12}$$

On the constancy of light velocity c for every inertia systems [1] and (11) (12)

$$\frac{dx\prime}{dt\prime} = \frac{dx}{dt} = \mathrm{c}$$

$$\frac{dx\prime}{dt\prime} = \frac{-vdt + dx}{dt - \frac{v}{\gamma^2}dx} = c$$

Based on these,

$$\gamma = c \tag{13}$$

Here the previous definition (c) is revised as following.

Definition; Time moves toward time direction also toward space direction with speed c. (c)

Then (11) (12) are

$$ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{14}$$

$$x' = \frac{-vt + x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

This is Lorentz transformation itself.

9. Conclusion

On (c) 'Time moves toward time direction also toward space direction with speed c.',

moved distance of time = passed distance of time

moved distance of time = moved distance of light

Then all different inertial systems view a light having same speed because it is ratio of moved distance of light for each system and passed time for each system.

But we never recognize this situation intuitively. We know this situation by an experiment.

On the contrary formulation(Lorentz transformation) has been derived on the experiment(Michelson-Morley experiment) inductively. This report provides some definitions and recognitions to derive the formulation deductively.

If the formulation can be derived correctly, it means the recognitions and related definitions provided regarding to following areas would be correct concept of nature.

- Conversion of time dimension
- Definition of time axis of S' system on S system
- Definition of space axis of S' system on S system
- Definition of scale of time and space of S' system

Reference

[1] Peter Gabriel Bergmann, *Introduction to the Theory of Relativity*, (Dover Publication, INC 1976),p19

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