Deductive Derivation of Lorentz Transformation for the Special Relativity Theory

Tsuneaki Takahashi

Abstract

Lorentz transformation of special relativity theory has been derived inductively based on the constancy of light velocity for every inertia systems. The equation predicts possible important results. But many people might feel it is hard to understand these intuitively.

The cause of it may be these have not been derived deductively from basic nature laws for over hundred years. This may mean we don't know any basic nature laws for special relativity theory yet. Here we try to find these.

1. Introduction

Lorentz transformation transforms the time-space position of a point from an inertia system view to another inertia system view. Here we think two inertia systems, one is two dimensions S system (time dimension is t and space dimension is x.), another is two dimensions S' system (time dimension is t' and space dimension is x'.) and S' system moves along x-axis with velocity v relatively.

On this environment, in order to derive Lorentz transformation deductively, investigation about following aspects is required.

- Conversion of time dimension
- Definition of time axis of S' system on S system
- Definition of space axis of S' system on S system
- Definition of scale of time and space of S' system

2. Lorentz Transformation

About the transformation, there is a basic system (in this case S system). Another system (in this case S' system) which is for a moving point with velocity v on the basic system, has own frame of reference. Based on these frames of reference, a point can be represented by the S system frame of reference also can be represented by the S' system frame of reference. Lorentz transformation represents this relation using formulas.

Here systems except the basic system may not be orthogonal.

Also if S' system would be recognized as a basic system, its frame of reference becomes same as the S system's and the frame of reference for the S system becomes same as that of system which is moving with velocity $-v$ to the S system.

As we can understand on this, which system is a basic system depends on how we look systems. No system has priority.

3. Time-space graph

Fig.1 is usual time-space graph. Regarding to time-space graph, Lorentz transformation is frame of reference transformation like rotation, expansion. If so, unit of all dimensions (axis) of a system should be same. Then time-space graph which indicates Lorentz transformation is a different style of graph from Fig.1. Unit of space and time should be unified. For this requirement, we consider conversion of time dimension.

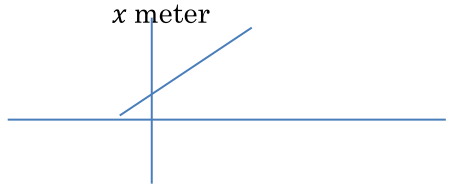

Fig. 1

4. Conversion of time dimension

In order that time-space frame of reference transformation can be done, unit of time need to be same as the unit of space.

Definition

Regarding to any space point, time is passing synchronously. (a)

Time does not exist at the point where space or universe don't exist. (b)

If the expansion speed of universe or space is γ, its front of x dimension is expanding on (1).

$$x = \gamma t \qquad (1)$$

So after t_1 from the beginning of universe, front of x dimension reaches to

$$x_1 = \gamma t_1$$

Here infinitesimal short time before t_1 is $t_1 - dt$. At the timing, space point x_1 does not exist. So time also does not exist there.

At time $t_1 - dt$, front space of x dimension expands to $\gamma(t_1 - dt)$.

Based on the above and continuousness requirement, following could be derived.

Time t_1 at point x_1 should not be the result that time at x_1 was counted up because no time was there at $t_1 - dt$.

Time t_1 at point x_1 should be the result that time at $\gamma(t_1 - dt)$ move to x_1 at t_1.

This means time moved with speed γ in space.

When time t passed, time moved space distance γt.

Then if unit of time is same as the unit of space and its value is γt, time dimension and space dimension become isomorphic.

Definition; From time-space graph view, time moves toward time direction also toward space direction with speed γ. (c)

Then required conversion of time dimension for Lorentz transformation is

-to have same unit as space's

-to make its value γt

Here exact definition is required.

Space distance : space dimension difference of two points

Time : For any space point, something passing. Its dimension is called time dimension. And time is time dimension difference of two points

Time distance : space compatible distance converted from time

Time move : There are following two types time move.

-time passing move : Any space point moves on time-space graph toward time direction according to time passing.

-space time move : Any space point moves on time-space graph toward time direction according to space move. (following (d))

5. Two types of distance

On graph, value of variable is presented by distance from origin. There are two types of the distance. These are time type and space type.

Time type presents time distance. It means time passing.

Space type presents space distance. It means space distance of two space points at a time.

Time axis and space axis should be same type so that time-space graph rotation could be done. About this, because time axis need to represent time passing, so its type cannot be changed. Then space axis should be recognized as time type. So space value x means time $\frac{x}{c}$ passing.

6. Time move

Time passing shows time move with speed of γ on graph. Point (0,0), for example, moves to (γt, 0) after t. This is time passing move. Next definition is space time move.

Definition

When space point moves toward space direction, it also moves same distance toward time direction. (d)

Its speed is not γ. It is v which is the point's speed in space.

When origin (0,0) of S' system moves x_1 to x dimension of S system, the point also moves x_1 to time dimension.

7. Definition of time axis of S' system on S system

In general, time axis of time-space graph is space position zero line.

' time t elapses' means 'it is at γt on time axis of time-space graph'.

And at time t or γt point, $x' = 0$ point reaches at space point vt.

So $x' = 0$ point moves along the following track (Fig. 2)

$$x = \frac{v}{\gamma}\gamma t = tan\theta \cdot \gamma t \tag{2}$$

This is space position zero $(x' = 0)$ line or time axis $(\gamma t')$ of S' system.

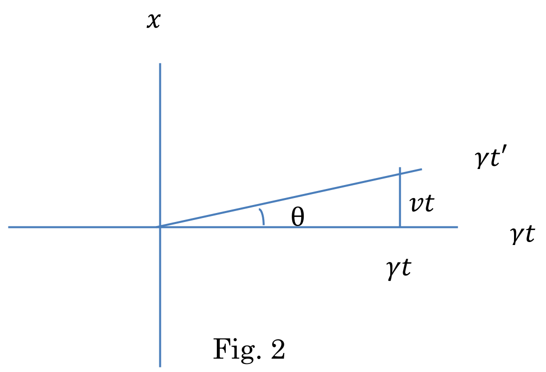

Fig. 2

8. Definition of space x' axis of S' system on S system

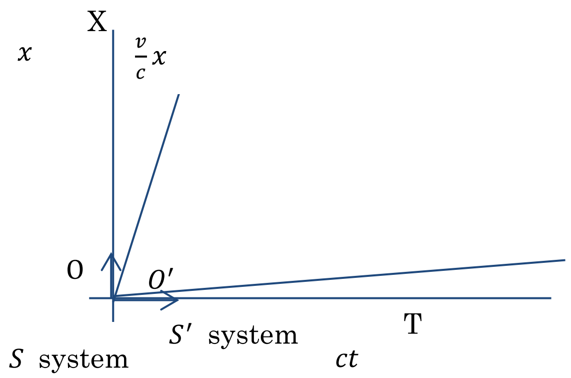

Fig. 3

About Fig. 3, origin of S system is O, origin of S' system is O'. Space point X is away x from origin O and O'.

Same as for time axis, it takes time distance x for time to move from origin to space point X. While the timing, origin O' moves $\frac{v}{\gamma}x$ toward space direction also time direction. This relation is

$$\gamma t = \frac{v}{\gamma}x$$

$$x = \frac{\gamma}{v}\gamma t. \quad (3)$$

This is time zero line or space axis for S' system.

On (2)(3), frame of reference for S system and S' system is as Fig. 4

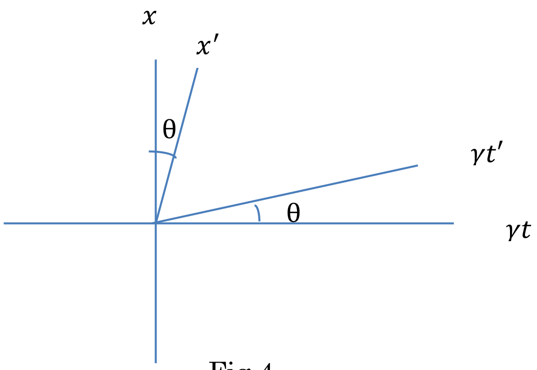

Fig.4

The difference of time position for each system is on difference of view. These indicate same event though its value is different.

9. Definition of scale of time and space of S' system

As shown in Fig.4, S' system is an oblique frame of reference and S system is an orthogonal frame of reference. This means that a target point is viewed by different type of two frames of reference.

Definition;

When time t passes for space volume V (length l in the case of one dimension space), integration of V for time t is 'experience (exp)'. That is $\exp = \int V(t)dt$

Ten thousand years for Sun is something different from 10 μ sec. for an atom. Experience is the *something* and its value indicates such huge difference in this case.

In the case of Fig.5, experience of the system S is:

Time t passes for length l or area$\overline{\mathrm{OACB}}$.

In the case of Fig.6, experience of the system S' is:

Time t' passes for length l' or area$\overline{\mathrm{OA'C'B'}}$.

If $l' = l$, $t' = t$,

area$\overline{\mathrm{OA'C'B'}}$ = area$\overline{\mathrm{OACB}} \sin\alpha$ = time $t\sqrt{sin\alpha}$ passes for length $l\sqrt{\sin\alpha}$

This means 'experience' is compressed$(0<\alpha<\frac{\pi}{2})$ on oblique system by $sin\alpha$,

Or this means distance is compressed on oblique system by $\sqrt{sin\alpha}$.

Then distance $of\ S'$ system $= (distance\ of$ S system$)\sqrt{\sin\alpha}$, (4)

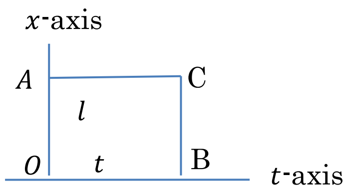

Fig. 5

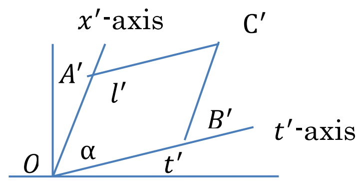

Fig. 6

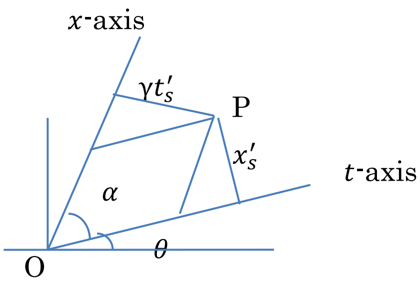

Fig. 7

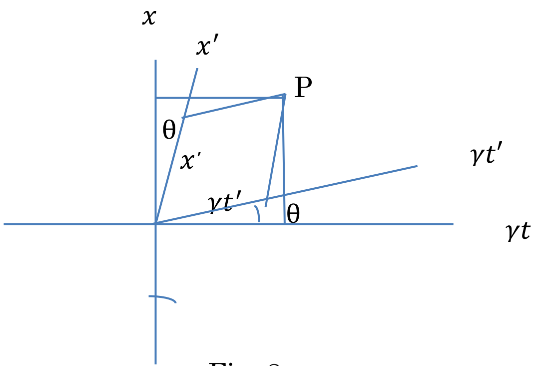

Fig. 8

10. Description of a time-space point

A point P on Fig.4 is described as Fig.7.

Relation of position value is

$$\gamma t_s' = \gamma t cos\theta - x sin\theta \quad (5)$$

$$x_s' = -\gamma t sin\theta + x cos\theta \quad (6)$$

These t_s', x_s' are shortest time distance and shortest space distance as on Fig.7.

On Fig.8, these t', x' are space position constant time distance and time constant space distance.

As time dimension and space dimension, the latter distance t', x' are used.

These relation are

$$\gamma t' = \frac{\gamma t_s'}{\sin\alpha}$$

$$x' = \frac{x_s'}{\sin\alpha}$$

Then

$$\gamma t' = \frac{(\gamma t cos\theta - x sin\theta)}{\sin\alpha} \quad (7)$$

$$x' = \frac{(-\gamma t sin\theta + x cos\theta)}{\sin\alpha} \quad (8)$$

Scale of oblique (4) is applied to these.

$$\gamma t' = \frac{\gamma t cos\theta - x sin\theta}{\sin\alpha}\sqrt{\sin\alpha} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{\sin\alpha}} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{\sin(\frac{\pi}{2} - 2\theta)}} = \frac{\gamma t cos\theta - x sin\theta}{\sqrt{cos^2\theta - sin^2\theta}} \quad (9)$$

$$x' = \frac{-\gamma t sin\theta + x cos\theta}{\sin\alpha}\sqrt{\sin\alpha} = \frac{-\gamma t sin\theta + x cos\theta}{\sqrt{\sin\alpha}} = \frac{-\gamma t sin\theta + x\cos\theta}{\sqrt{cos^2\theta - sin^2\theta}} \quad (10)$$

Here, from (2)

$$cos\theta = \frac{\gamma}{\sqrt{\gamma^2 + v^2}}, \quad sin\theta = \frac{v}{\sqrt{\gamma^2 + v^2}}$$

Then (9) (10) are

$$\gamma t' = \frac{\gamma t - \frac{v}{\gamma} x}{\sqrt{1 - \frac{v^2}{\gamma^2}}} \quad (11)$$

$$\mathrm{x}' = \frac{-vt + x}{\sqrt{1 - \frac{v^2}{\gamma^2}}} \quad (12)$$

On the constancy of light velocity c for every inertia systems [1] and (11) (12)

$$\frac{dx\prime}{dt\prime} = \frac{dx}{dt} = \mathrm{c}$$

$$\frac{dx\prime}{dt\prime} = \frac{-vdt+dx}{dt-\frac{v}{\gamma^2}dx} = c$$

Based on these,

$$\gamma = c \tag{13}$$

Here the previous definition (b) is revised as following.

Definition; Time point moves toward time direction also toward space direction with speed c.

(c)

Then (11) (12) are

$$ct' = \frac{ct-\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{14}$$

$$x' = \frac{-vt+x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

This is Lorentz transformation itself.

11. Conclusion

Lorentz transformation which has been derived inductively and proven is derived deductively using some recognitions for following aspects. This means the recognitions and related definitions would be correct concept of nature.

- Conversion of time dimension
- Definition of time axis of S' system on S system
- Definition of space axis of S' system on S system
- Definition of scale of time and space of S' system

Reference

[1] Peter Gabriel Bergmann, *Introduction to the Theory of Relativity*, (Dover Publication, INC 1976),p19

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