Lorentz-invariant theory of gravitation

(summary)

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Annotation

This article is a summary of the non-geometrical Lorentz-invariant theory of gravitation (LIGT) (references and citations here allow to familiarize oneself with known results from the theory of gravitation in more detail). In the framework of the proposed theory the physical meaning of the metric tensor and square of interval in pseudo-Euclidian space was clarified, all the exact solutions of GR were obtained, the violation on the law of conservation of energy-momentum was eliminated, as well as other difficulties have been overcome. A characteristic feature of the proposed theory is that it is built on the basis of the quantum field theory.

Abbreviations:

LIGT - Lorentz-invariant gravitation theory;
EM - electromagnetic;
EMTG - electromagnetic theory of gravitation;
QFT - quantum field theory
SM - Standard Model;
QED – quantum electrodynamics.
GTR or GR – General Theory of Relativity
L-transformation – Lorentz transformation
L-invariant – Lorentz-invariant

1.0. Introduction. Statement of the problem

The modern theory of gravity, which is called General Theory of Relativity (GTR or GR), was verified with sufficient accuracy and adopted as the basis for studying of gravitational phenomena in modern physics.

However, GR has certain disadvantages (see, e.g., (Logunov and Loskutov, 1987; Krogdahl, 2007)): 1) in the general case, the energy and momentum conservation is violated; 2) it can not be quantized; 3) it is based on a geometric basis, unrelated to other existing physical theories.

These deficiencies have resulted in the fact that attempts to improve this theory are being made. However, as the analysis shows (for example, see (Feynman, Morínigo and Wagner, 2002)) - there is no doubt that the GRT contains elements that will be present in any other theory of gravitation.

It is known that general relativity is a classical theory. At the same time, the base of the modern approach to the study of nature is quantum field theory (QFT); and classical physics is considered to derive from the quantum theory.

So the question arises if there is a possibility to improve the gravitation theory on the basis of QFT? In other words, can we get the tested results and eliminate the difficulties of GRT, if we put the quantum field theory in the foundation of the gravitation theory?

Of course, first of all, we should make sure that such an approach, has a prospect. What are the prerequisites that allow us to put QFT in the basis of the gravitation theory? What challenges need to be overcome for the construction of such a theory?

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2.0. Background of the existence of a Lorentz-invariant theory of gravitation based on QFT

1) Modern QFT (e.g., in the form of the Standard Model - SM) describes all the elementary particles and their interactions. Moreover, it was found that all the observable matter in the universe is composed by these elementary particles and the energy of their interaction. Thus the elementary particles and the energy of their interactions are the primary embodiment of the matter. Gravitation is one of the fundamental properties of matter. Therefore elementary particles are, so to speak, the primary "carrier" of gravitation. Therefore, we can assume that there must be a connection between the theory of elementary particles and gravitation theory.

2) QFT is Lorentz-invariant (L-invariant) theory. This, in general, means that it is invariant with respect to the Lorentz group. According to Noether's theorem, this invariance provides all the necessary conservation laws in mechanics .

3) General relativity, as a classical theory, refers to the macroworld, while QFT is the theory of the microworld. Can the first theory be reduced to the second?

It is known that the classical mechanics is a consequence of quantum mechanics. So Newton's equation of motion can be obtained from the Schrodinger equation, and the equation of motion for a charged particle can be obtained by squaring of the Dirac equation. Therefore, it is possible that by using one of the known averaging techniques it is possible to obtain the results of general relativity from QFT. In addition in this case there is hope that gravitation theory can be associated with quantum theory.

4) From experience it follows that energy, momentum, mass, and other mechanical quantities are the same in both theories - general relativity and QFT.

5) To pass on to the actual construction of the theory, it is required to enter the physical characteristics of gravitation theory. One of the key here is, apparently, the source of gravitation.

The source of gravity in general relativity is the pseudo-tensor of energy-momentum , which is the sum of the Lorentz-invariant energy-momentum tensor and some pseudo-tensor (you can read more about it below). This pseudo-tensor is not a Lorentz-invariant tensor, and, in general does not describe the conservation laws of physical quantities in GRT.

The energy-momentum tensor of a classical field theory combines the densities and flux densities of energy and momentum of the fields into one single object. However, the problem of giving a concise definition of this object able to provide the physically correct answer under all circumstances, for an arbitrary Lagrangian field theory on an arbitrary space-time background, has puzzled physicists for decades. (Forger and Römer, 2003).

6) The theory of gravitation, as we know, refers to the gravitational mass / energy (gravitational charge). A mass / energy of the elementary particles is considered to be inertial. The connection of the theory of gravity with QFT here ensures experimental fact of equality between gravitational mass/energy and inertial mass/energy. Thus, if we will understand the nature of the inertial mass of elementary particles, we will also define the nature of the gravitational mass.

We will also analyze the difficulties of the Lorentz-invariant theory when they arise. We will also show how to overcome them.

First of all, we will discuss in more detail the differences of the gravitation sources between GR and L-invariant theory of gravitation based on QFT. The important physical consequences of the theory are associated with these differences; in particular, the compliance with the energy and momentum conservation law.

3.0. The source of gravitation

3.1. The source of gravitation in GRT

Initially Einstein assumed that the source of gravity in the Hilbert-Einstein equations is symmetric energy-momentum tensor $T_{\mu\nu}$ of the Lorentz-invariant mechanics satisfying the law of energy-momentum conservation:

$$\sum_{\nu=0}^{3} \frac{\partial T^{\mu\nu}}{\partial x_\nu} = 0, \tag{3.1}$$

which corresponds to ten integrals of motion of Lorentz-invariant mechanics (Fock, 1964).

The word "relativity" in the title of general relativity suggests that general relativity is a relativistic theory. Einstein assumed that general covariance of equations reflects some general relativity, which includes the special relativity. But the question of whether the GR equation is relativistic in the sense of Lorentz-invariance, is not trivial. It is known that general relativity is considered as relativistic theory, but it is not a L-invariant theory (Katanaev, 2013, pp. 742):

«*Lorentz metric satisfies the Einstein's vacuum equations* . [But] *"in GTR is postulated that space-time metric is not a Lorentz metric, and is found as a solution of Einstein's equations. Thus, the space-time is a pseudo-Riemanian manifold with metric of a special type that satisfies the Einstein equations*."

From this it follows that general relativity is a general covariant theory, but, strictly speaking, it is not a Lorentz-invariant theory (i.e., a theory within the framework of SRT). It is easy to see that this leads to difficulties with the law of conservation of energy in general relativity.

As we know, a generalization, that is, the transition from the Lorenz invariance to the general invariance, is the transition to abstraction of higher level. Such a transition can be achieved only by the introduction of new postulates. In particular, a few rules are formulated for obtaining the general-covariant expressions from Lorentz-invariant expressions .

"*Often this generalization involves only the replacement of partial differentiation by covariant differentiation ("comma-goes-to-semicolon rule"); for example the generalization of the equations of motion is from* $\frac{\partial T^{\mu\nu}}{\partial x_\nu} \equiv T^{\mu\nu}_{,\nu} \to T^{\mu\nu}_{;\nu}$*; this latter [tensor], with semicolons, includes the effects of gravity*". (Lightman, Press et al., 1979).

As the generalization of $T^{\mu\nu}_{,\nu}$ in GR should be the general covariant derivative $T^{\mu\nu}_{;\nu}$, and instead (1.1) we will have (Landau and Lifshits, 1971; Fock, 1964):

$$\nabla_\nu T^{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\nu}\left(\sqrt{-g}T^{\mu\nu}\right) + \Gamma^{\mu}_{\alpha\beta} T^{\alpha\beta} = 0, \tag{3.2}$$

But, it appears that (Landau and Lifshitz, 1971; § 96 "The energy-momentum pseudotensor…") "*in this form, however, this equation does not generally express any conservation law whatever*".

As a way out of this situation, Einstein, and others proposed to introduce an additional term – pseudotensor. It is called like that because it is not a true tensor. Due to $t^{\mu\nu}$, in some coordinate systems a kind of conservation law of energy-momentum can artificially be formulated. Violations of this law arise in other coordinate systems.

On this basis, Einstein even proposed to consider that the violation of the law of energy-momentum conservation is a peculiarity of the gravitational field.

In particular, at least inside the borders of our galaxy, Newton's theory of gravity provides at least 99% of accuracy in the calculation of gravitational problems in comparison with general relativity. The law of conservation of energy and momentum is fully respected here. Therefore, it is easier to assume that general relativity contains a mistake, than to doubt the results of Newtonian mechanics.

Energy-momentum is an important conserved quantity whose definition has been a focus of many investigations in general relativity (GR). Unfortunately, there is still no generally accepted definition of energy and momentum in general relativity. Attempts aimed at finding a quantity for describing distribution of energy-momentum due to matter, non-gravitational and gravitational fields only resulted in various energy-momentum complexes (which are nontensorial under general coordinate transformations) whose physical meaning have been questioned.

Moreover (Forger and Römer, 2003; page 62) "the expression in equation. (3.2) above does not represent a physical energy-momentum tensor for the gravitational field: as is well known, such an object does not exist".

What we have in the Lorentz-invariant (L-invariant) mechanics?

3.2. The source of gravity in Lorentz-invariant mechanics

Since QFT is a Lorentz-invariant (L-invariant) theory, the gravitation theory, built on its basis, will be the L-invariant gravitation theory (LIGT). What can we say about the source of gravity in such a theory?

In general in the Lorentz-invariant mechanics, the elements that make up the energy-momentum tensor $T^{\mu\nu}$ (see above), are used in theory. But in practice they are rarely used in the form of tensor (Fock, 1964, §§ 27-29).

In fact, the energy-momentum tensor, which is included in the formulation of the equations of general relativity, is also not used here as a tensor. The tensorial equation of GR is a short notation of 10 equations. Each of these equations contains only one term of the energy-momentum tensor.

Note that after its division by the square of the speed of light, the tensor components are identical to the mass density and the densities of mass flow. Perhaps, in this regard, Fock called this tensor - the tensor of mass (Fock, 1964, § 31)

Therefore, following to V. Fock (Fock, 1964, §54), "*in formulating Einstein's theory we shall likewise start from the assumption that the mass distribution is insular. This assumption makes it possible to impose definite limiting conditions at infinity as for Newtonian theory, and so makes the mathematical problem a determined one. Theoretically, other assumptions are also admissible*".

(As mass distribution of insular character V. Fock describes *"the case that all the masses of the system studied are concentrated within some finite volume which is separated by very great distances from all other masses not forming part of the system. When these other masses are sufficiently far away One can neglect their influence on the given system of masses, which then may be treated as isolated*.")

The abovementioned allows us, in framework of the Lorentz-invariant problem, to call the source of gravitation, the mass/energy, or simply mass, implying by this term all terms of the energy-momentum tensor of the specific task.

Next, we will analyze the question of what we know about the origin of the inertial mass/ energy of elementary particles as the primary sources of gravity.

4.0. The mass / energy theories. Classical and contemporary point of view

"*Mass remained an essence - part of the nature of things - for more than two centuries, until J.J. Thomson* (1881), *Abraham* (1903) *and Lorentz* (1904) *sought to interpret the electron mass as electromagnetic self-energy*", (Quigg, 2007).

Theory, created by J.J. Thomson and H. Lorentz (1881 - 1926), lies entirely in the field of classical electromagnetic theory. According to this theory, the inertial mass has electromagnetic origin.

Unfortunately, attempts to apply this theory to quantum theory has not been undertaken. However, until now there was no evidence of that the inertial mass is not fully electromagnetic (Feynman et al, 1964):

"*We only wish to emphasize here the following points:*

1) the electromagnetic theory predicts the existence of an electromagnetic mass, but it also falls on its face in doing so, because it does not produce a consistent theory – and the same is true with the quantum modifications;

2) there is experimental evidence for the existence of electromagnetic mass; and

3) all these masses are roughly the same as the mass of an electron.

So we come back again to the original idea of Lorentz - may be all the mass of an electron is purely electromagnetic, maybe the whole 0.511 MeV *is due to electrodynamics. Is it or isn't it? We haven't got a theory, so we cannot say*."

The modern mass theory is the, so-called, Higgs mechanism of the Standard Model theory (SM) (Quigg, 2007; Dawson, 1999; etc).

The Higgs mechanism, under certain assumptions, allows us to describe the generation of masses of fundamental elementary particles: intermediate bosons, leptons and quarks. But as it is mentioned above (Quigg, 2007), more than 98% of the visible mass in the Universe is composed by the non-fundamental (composite) particles: protons, neutrons and other hadrons.

Thus, the Higgs mechanism can not be used in the gravitation theory.

4.1 The origin of mass/energy in QFT

Starting with quantization of Maxwell's theory of electromagnetism which led to the construction of the QED, physicists have made tremendous progress in understanding the basic forces and particles constituting the physical world.

Modern particle theories, such as the Standard model, are quantum Yang-Mills theories (Houghton, 2005**;** Nielsen, 2007). In a quantum field theory the quanta of the fields are interpreted as particles. In a Yang-Mills theory these fields have an internal symmetry: they are acted on by a space-time dependant non-Abelian group transformations. These transformations are known as local gauge transformations and Yang-Mills theories are also known as non-Abelian gauge theories. Maxwell's equations can be regarded as a Yang-Mills theory with gauge group U(1).

"We have the working renormalizable theory of strong, electromagnetic and weak interactions... This is of course the Yang-Mills theory… Essentially, all that we managed to do is just to generalize quantum electrodynamics (QED). QED was invented around 1929 *and since then has never changed... Now QED is generalized and includes strong and weak interactions along with electromagnetic, quarks and neutrinos, along with electrons*" (Gell-Mann, 1985).

As we know, these theories cover all types of elementary particles and their interactions. They make up the matter of the universe: massless photons and massive leptons, bosons and hadrons.

For us it is important to emphasize that the Yang-Mills equations are nonlinear generalization of Maxwell's theory and can be represented in both classical, and quantum form (see, for example, in details, (Ryder, 1985)).

Therefore it can be argued that the mass/energy of elementary particles, and thus the whole matter, has electromagnetic origin. This answers the question of Feynman in the passage above.

This also implies that *the gravitational mass/energy and gravitational field also have electromagnetic origin*.

Obviously, it follows that the theory of gravitation, in the general case, must be a variant of the nonlinear theory of electromagnetic field. It is understood that the new theory must be L-invariant, since it is based on the electromagnetic theory. Can the L-invariant theory give the same results as general relativity? we will try to answer this question below.

But first, let us note that the base (not the proof!) to assume that the electromagnetic theory of gravity is possible, already exists in the GR. It will be useful to briefly dwell on this.

4.2. Connection between GTR and EM theory

Which consequence of general relativity confirms that the electromagnetic gravitation theory (EMGT) is possible?

First of all, let us underline again that this is not the proof of the existence of EMGT. Nevertheless, it is a reference to the possibility of its existence. We are talking here about the known solution of the linearized equations of general relativity.

Even Einstein himself pointed out some parallels between this approximate solution and the EM formulas (Moeller, 1952). Around the 60s these parallels drew more attention (Forward, 1961; Ruggiero and Tartaglia, 2002; etc.). To date is shown an almost complete agreement of formulas of linearized equations of general relativity and electromagnetic theory (the difference in some numerical coefficients, is not to take into account, a quite understandable fact in terms of features of gravity and QFT).

In the linear approximation, the left side of the equation of general relativity – Ricci's curvature tensor – is equal to the D'Alembert operator (see analysis (Fock, 1964, § 68 and Annex B)). This operator acts on the metric tensor with a first order of approximation. General relativity equation, taking into account the Lorentz gauge, becomes a d'Alembert wave equation with respect to the gravitational 4-potential. This equation up to numerical coefficients coincides with the EM field equations: four d'Alembert equation: one equation for scalar and 3 equations for the components of vector potential.

Moreover, in this case, the fact that these are approximations, rather than exact solutions, is not a reason to reject the connection of GR theory with EM theory. Point is that these approximate solutions were the first solutions of Einstein, which confirmed experimentally the existence of relativistic effects.

However, we emphasize again that this does not prove the existence of EMGT. The electromagnetic representation of these solutions is only their interpretation. We must, independent obtain the exact solutions of general relativity exclusively on the basis of electromagnetic theory and explain the physical meaning of the geometric apparatus of GTR.

5.0. Electromagnetic gravitation theory (EMGT): formulation of the problem

So, we will try to construct a theory of gravity based on EM theories of Yang-Mills. And, probably, it is better for the beginning to choose the simplest of them - the Maxwell-Lorentz theory.

It is clear that to build the EM gravitation theory (EMGT) - does not mean to take the EM theory and use it as a theory of gravity. There are important differences between these theories. The main challenges to overcome are the differences between the electromagnetic and gravitational field: 1) the weakness of the gravitational in comparison with the electromagnetic field, 2) its neutrality, and 3) the absence of repulsion.

In addition, there are some seeming difficulties.

Equations of electromagnetic fields (including a generalization of Yang-Mills) do not contain mass. This seems to make them unsuitable even for describing the Newtonian's theory of gravity. But let us not forget that these equations contain the intensities of EM fields in the force presentation. The square and vector product of these fields are proportional to the densities of energy and momentum, respectively. And in the energy representation, they are directly characterized by energy and momentum per unit of gravitational charge.

In case of self-acting of EM fields, these equations can trigger a mass according to the principle of mass-energy equivalence. Self-action is described by nonlinear equations, which is typical for the Yang-Mills equations. Let us recall, that Higgs mechanism generates the masses due to the self-acting of fields. But there are also other mechanisms of self-acting of the fields.

Another feature of the EM equations is that they describe only the vector bosons (e.g., photons). But this does not contradict with the theory of gravity because, as was recognized in general relativity, the quanta of gravitational field - gravitons - are also bosons.

5.1. The hypothesis of residuality

The idea of an electromagnetic origin of gravitation appeared since formulation of the electromagnetic theory of matter. The first hypothesis of explaining the gravity on the basis of electromagnetic theory was put forward in 1836 by O.Mossotti As shown by further analysis, its justification was not satisfactory.

The electromagnetic origin of the mass of all elementary particles, as well as the weakness of the gravitational field compared to the electromagnetic field, allowed to O.F. Mossotti (Mossotti, 1936) to assume that the gravitational field is a residual electromagnetic field.

"*Wilheim Weber of Gottingen and Friedrich Zollner of Leipzig developed this conception into the idea that all ponderable molecules are associations of positively and negatively charged electrical corpuscles, with the condition that the force of attraction between corpuscles of unlike sign is somewhat greater than the force of repulsion between corpuscles of like sign. If the force between two electric units of like charge at a certain distance is a dynes, and the force between a positive and a negative unit charge at the same distance is y dynes, then, taking account of the fact that a neutral atom contains as much positive as negative electric charge, it was found that* $(\gamma - \alpha)/\alpha$ *need only be a quantity of the order* 10^{-35} *in order to account for gravitation as due to the difference between* α *and* γ *"* (Whittakker, 1953).

"At the meeting of the Amsterdam Academy of Sciences on 31 *March* 1900, *Lorentz communicated a paper entitled "Considerations on Gravitations on Gravitation", in which he reviewed the problem as it appeared at that time*" (Whittaker, 1953).

In other words, according to Mossotti, Weber and Zolneru (Zoellner, We ber, and Mossotti, 1882) *gravitational field - is a residual electromagnetic field.*

Later, scientists come to the conclusion that particles are the field, and therefore, it is necessary to reformulate the hypothesis at the level of electromagnetic fields. As shown by H. Lorentz (and then others), it can be done without entering into conflict with the experimental facts.

"At the meeting of the Amsterdam Academy of Sciences on 31 March 1900, Lorentz communicated a paper entitled "*Considerations on Gravitations on Gravitation*", in which he reviewed the problem as it appeared at that time" (Whittakker, 1953).

In the second half of this article Lorenz examined this concept at the field level and received encouraging results.

But the final theory had not been received yet. And after the occurrence of the general relativity, the interest in the electromagnetic gravitation theory was lost (more information can be found in the book: Vizgin, V.P. (Vizgin, 1981), chapter "Electromagnetic theory of gravitation"; whith sufficient bibliography.

We will not try to finish this solution in framework of QFT, since the solution of this problem requires serious analysis and a lot of time. At the same time, this solution is not important for the conversion of the electromagnetic theory to the gravitation theory. We will proceed differently: we will consider this idea as a postulate (called the Mossotti-Lorentz postulate).

Postulate of Mossotti-Lorentz: *the gravitational field is a residual electromagnetic field, which remained as a result of incomplete compensation of electric and magnetic fields of different polarity.*

It is easy to check, if this postulate contradicts the condition of necessity: can all electromagnetic quantities be rewritten in such a way that they give the correct formula of the gravitation theory (for example, the formula for Newton's force, field, energy, etc.); We must also check the dimensions of all obtained characteristics of the theory of gravitation (gravitational charge, the field intensity, potentials of the field, etc.).

In part, this has been done before by many scientists. A detailed verification within our theory shows that no contradiction arises. Below we show these transformations on an important example.

5.2 Newton's law of gravitation, as a result of EM theory

If we assume that gravity is generated by electric field, but quantitatively, by very small part of it, then Newton's gravitation law:

$$\vec{F}_N = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0}, \qquad (5.1)$$

should take the form of Coulomb's law:

$$\vec{F}_C = k_0 \frac{q \cdot Q}{r^2} \vec{r}^{\,0}, \qquad (5.2)$$

where m and q are the mass and electric charge of the particle, M and Q are the mass and electric charge of the source. In Gauss's units γ_N =6,67×10^{-8} см³/(г·с²). is Newton's gravitational constant, and the coefficient k_0 is $k_0 = 1$.

We introduce the gravitational charges q_g and Q_g , corresponding to mass m and M (Ivanenko and Sokolov, 1949) , by means of the relations:

$$q \to q_g = \sqrt{\gamma_N}\, m, \;\; Q \to Q_g = \sqrt{\gamma_N}\, M \qquad (5.3)$$

In this case, Newton's law can be rewritten in the form of Coulomb's law:

$$\vec{F}_g = \frac{q_g \cdot Q_g}{r^2} \vec{r}^{\,0} = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0} = \vec{F}_N, \qquad (5.4)$$

Similarly, we can transform all other EM quantities in the gravitational quantities.

If we talk specifically about the weakness or strength of the field, the intensity of the interaction is usually characterized by some bond constant. In the EM theory as such a constant can be considered k_0 in Coulomb Law for vacuum. In the CGSE system units, the charge is selected so that $k_0 = 1$ The gravitational constant γ_N in CGS is approximately γ_N = 6.67 × 10^{-8} cm^3/(g s²). It is characterized by $(\gamma - \alpha)/\alpha$ in the Mossotti et al. approach.

5.3. The relativization of the Newton law

Above we have received a non-relativistic equation of Newton's gravity. But the Lorentz invariance of EM theory promises the existence of a L-invariant version of this equation.

The motion of charged particles in the non-relativistic case is performed under the action of the non-relativistic Coulomb force. This allowed us to obtain the force of Newton. Both of these forces correspond to the stationary source. But how can we give them a relativistic form corresponding to motion of the source?

Obviously, we need to start with the Coulomb force. It turns out that Coulomb force takes the L-invariant form, if we add the Lorentz magnetic force. In this case, the total force acting on the charge, consists of two terms, and called a (complete) Lorentz force.

It is clear that a similar term needs to be added to the force of Newton, to take a relativistic form. What is needed to be done for this?

The most significant here is that in both cases, it is not necessary to add this term artificially, e.g., by a postulate.

A striking unification of electromagnetic theory was published in 1912 by Leigh Page. It had been realized long before by Priestley that form the experimental fact that there is no electric force in the space inside a charged closed hollow conductor, it is possible to deduce the law of the inverse square between electric charges, and so the whole science of electrostatics. It was now shown by Page that if a knowledge of the relativity theory of Poincare and Lorentz is assumed, the effect of electric charges in motion can be deduced from a knowledge of their behavior when *at rest, and thus the existence of magnetic force may be inferred from electrostatics: magnetic force is in fact merely a name introduced in order to describe those terms in the ponderomotive force on an electron which depend on its velocity. In this way Page showed that Ampere's law for the force between current-elements, Faraday's law of the induction of currents and the whole of the Maxwellian electromagnetic theory, can be derived form.* (Whittakker, 1953)

It turns out that in the electromagnetic theory, the additional term to the Coulomb force arises automatically due to the Lorentz transformations. As is known, the magnetic field occurs when the charge moves. During this the Coulomb field remains unchanged. But if we apply the Lorentz transformation to electric charge then automatically due to spatial compression of charges, a magnetic field arises, as well as an additional term of the magnetic force (see. Details (Purcell, 1975)). It is not difficult to transfer this result to gravitation theory.

As is known, GTRactually contains a field which we can name gravito-magnetical. But in our approach, it is a consequence of L-invariance, not of general covariance and the Riemann space-time.

The equations of EM field can be written through the field strengths, but can also be written through the potentials - scalar and vector. These equations, taking into account the Lorenz gauge condition $\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t}+\nabla\vec{A}=0$, are inhomogeneous equations of d'Alembert in the 4-dimensional space-time, which are fully equivalent to Maxwell's equations:

$$\frac{1}{c^2}\cdot\frac{\partial^2 A_\mu}{\partial t^2}-\nabla^2 A_\mu=\frac{4\pi}{c}\rho\upsilon_\mu\,, \tag{5.4}$$

where A_μ is 4-potential.

The equations of Newton's theory of gravity can also be written in these two forms. And the energy form of equation in the stationary state corresponds to the Poisson equation, which is the stationary limit of the GR equation. Obviously, after the transition to the relativistic form of Newton's law, we will obtain an equation which is mathematically identical to the EM field equation and also contain the vector potential.

As we know, the great achievement of general relativity is the presence in its equation of the vector potential.

6.0. Geometry and physics of general relativity and the L-invariant theory of gravitation

6.1. Formulation of the problem in general relativity

The practical side of the Einstein-Hilbert theory is following: "*All the predictions of general relativity follow from*:

1) the solution of the field equations:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\chi T_{\mu\nu}\,, \tag{6.1}$$

where $\chi=\frac{8\pi\gamma_N}{c^4}$, $R_{\mu\nu}=\frac{\Gamma^\lambda_{\mu\nu}}{\partial x^\lambda}-\frac{\Gamma^\nu_{\mu\lambda}}{\partial x^\nu}+\Gamma^\lambda_{\mu\nu}\Gamma^\alpha_{\lambda\alpha}-\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\alpha}$ is the Ricci curvature tensor, $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols, R is the scalar curvature, c is the speed of light in vacuum, $T_{\mu\nu}$ is the stress–energy tensor, and $g_{\mu\nu}$ is the metric tensor of Riemannian space;

2) the law of motion in form of geodesic line equation or the Hamilton-Jacobi equation for a massive body:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}\right)\left(\frac{\partial S}{\partial x^\nu}\right)+m^2c^2=0\,, \tag{6.2}$$

3) the postulat that in the Riemann geometry the metric tensor $g^{\mu\nu}$ is a function of the gravitational field - $g_{\mu\nu}^{GR}$.
4) the requirement that the equations of general relativity in the non-relativistic case is reduced to the equation of Newton's gravity (in the form of Poisson's equation)

The equation (6.1) allows to determine $g_{\mu\nu}$. The non-relativistic limit allows to introduce in this solution the Newton gravitational potential and so allows from $g_{\mu\nu}$ to obtain $g_{\mu\nu}^{GR}$. Putting this value in (6.2) we obtain the solution of given problem of the body motion in the gravitation field of a source.

Since the metric tensor is contained in the square of interval of Riemannian space:

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu , \tag{6.3}$$

it is often said that the purpose of solution of equation (6.1) is to find the interval (6.3).

It is often said that interval in STR is a generalization of interval of Euclidean geometry on pseudo-Euclidean geometry. In turn, the interval in general relativity is a generalization of interval of pseudo-Euclidean geometry on pseudo-Riemannian geometry. But it is easy to make sure, that the introduction of interval in STR and GTR is a postulates rather than a logical conclusion. Indeed, the intervals in STR and GTR are a generalization of interval of Euclidean geometry, but the reason for the introduction of these new intervals is not geometry, but physics.

Obviously, the practical part of the GRT ideas must be entered into any new theory of gravitation. It follows that within the framework of our formulation of the problem (i.e., within the QFT and namely, EM theory) it is desirable to find out the functional meaning of the metric tensor and the interval. Next it is also necessary to obtain the equation of motion, to find out the connection of the metric tensor with the field of gravity. Let us try to implement this program.

6.2. Derivation of the pseudo-Euclidean interval from QFT

The vectors of the Lorentz-invariant (i.e., relativistic) theories necessarily depend on four coordinate: one time coordinate and three space coordinates. Does these theories contain the equations, which have a sum of terms, each of which is associated with one of the four coordinates, like as in the square of the interval?

As we know, in the first time such equations in classical electrodynamics appear, and then in quantum field theory. The wave equations of these theories include a sum of terms, each of which is associated with one of the variables *t, x, y, z*. It would be logical, to seek the cause and the meaning of the appearance of 4-interval in them, instead of introducing them artificially, as did Minkowski.

The well-known relation between the energy, momentum and mass of elementary particles follows from the wave equation of the particles:

$$\varepsilon^2 - c^2 \vec{p}^2 - m^2 c^4 = 0 , \tag{6.4}$$

or in the Cartesian coordinate system:

$$\frac{\varepsilon^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2 c^2 , \tag{6.4'}$$

Since $p_i = m\upsilon_i \gamma_L = m(dx_i/dt)\gamma_L$, a $\varepsilon = mc^2 \gamma_L$, (where $\gamma_L = 1/\sqrt{1-\upsilon^2/c^2}$ and $\gamma_L^{-1} = \sqrt{1-\upsilon^2/c^2}$ are the Lorentz factor and anti-factor, respectively), this relation can be rewritten as:

$$\gamma_L^2 c^2 (dt)^2 - \gamma_L^2 (dx)^2 - \gamma_L^2 (dy)^2 - \gamma_L^2 (dz)^2 = c^2 (dt)^2 , \tag{6.5}$$

Multiplying it by γ_L^{-2} , we get:

$$c^2(dt)^2\gamma_L^{-2} = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \tag{6.6}$$

Taking into account $c^2(dt)^2\gamma_L^{-2} = c^2(dt)^2\left(1-\upsilon^2/c^2\right) = (ds)^2$, the expression (2.7) can be written as square of a 4-interval:

$$(ds)^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \tag{6.7}$$

In the case of generalized curvilinear orthogonal coordinate, this interval takes the form $(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu$: where $g_{\mu\nu}$ is the metric tensor that is not associated with gravity (section 6.4 is dedicated to clarifying its physical meaning).

Obviously, if we go in the opposite direction, we can obtain the equation (6.4') from the square of the interval (6.7). This implies, firstly, that these equations - (6.4) and (6.7) - closely bind the massive elementary particles physics and geometry. From this it follows that (6.7) is not a metric of pseudo-Euclidean geometry, but it is a metric of Euclidean geometry that describes the Lorentz-invariant field equations. The only change in the geometry, which we can observe in this case is the transition from rectilinear to curvilinear geometry.

6.3. The derivation of motion equation in framework of QFT

In addition, another link between the interval (6.4) and the physical equation is detected. As we have shown in chapters 6, using the Schrödinger definition of action ($p_\mu = \partial S/\partial x_\mu$), from the equation (6.4') it is easy obtain Lorentz-invariant Hamilton-Jacobi equation in general view. For this it is enough to write the equation (6.4') in a form, suitable for any of the Euclidean coordinate system:

$$g_{\mu\nu}p^\mu p^\nu = m^2c^2, \tag{6.8}$$

where, we recall, $g_{\mu\nu}$ is the metric tensor of geometrical space, but not of the gravitational space-time of general relativity (in other words, in this case the tensor $g_{\mu\nu}$ does not include the physical characteristics of the field). In this case the Hamilton-Jacobi equation of free particles obtains the form:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}\right)\left(\frac{\partial S}{\partial x^\nu}\right) - m^2c^2 = 0, \tag{6.9}$$

Thus, we conclude that the three equations (6.4), (6.7) and (6.9) are closely bonded to each other and, in fact, follow from one differential equation. From this follows that the interval (6.7) within a relativistic physics is the physical law, and not a geometric relation.

Below we will consider the physical meaning of the metric tensor in the framework of QFT

6.4. The physical sense of the metric tensor

Recall the transition from Cartesian's system of coordinates to the generalized coordinate system (Korn and Korn, 1968). Let us introduce a new set of coordinates q_1, q_2, q_3, so that among x, y, z and q_1, q_2, q_3 there are some relations:

$$x = x(q_1, q_2, q_3),\ y = y(q_1, q_2, q_3),\ z = z(q_1, q_2, q_3), \tag{6.10}$$

The differentials are then

$$dx = \frac{\partial x}{\partial q_1}dq_1 + \frac{\partial x}{\partial q_2}dq_2 + \frac{\partial x}{\partial q_3}dq_3, \tag{6.11}$$

and the same for dy and dz.

In Cartesian coordinates the measure of distance, or metric, in a given coordinate system is the arc length ds, which is defined by

$$ds^2 = dx^2 + dy^2 + dz^2, \qquad 6.12)$$

In general, taking into account (6.11), from (6.12) we obtain

$$ds^2 = g_{11}dq_1{}^2 + g_{12}dq_1dq_2 + \ldots = \sum_{ij} g_{ij}dq_idq_j\ , \qquad (6.13)$$

where g_{ij} is the metric tensor. Thus in orthogonal system we can write

$$ds^2 = (h_1dq_1)^2 + (h_2dq_2)^2 + (h_3dq_3)^2, \qquad (6.14)$$

where h_i 's are:

$$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}\ , \qquad (6.15)$$

are called Lame coefficients or scale factors, and are 1 for Cartesian coordinates.

Thus, the metric tensor, recorded in coordinates q_i, is a diagonal matrix whose diagonal contains the squares of Lame coefficients:

For example, in the case of spherical coordinates, the bond of spherical coordinates with Cartesian is given by:

$$x = r\sin\theta\cos\varphi,\ y = r\sin\theta\sin\varphi,\ z = r\cos\theta, \qquad (6.16)$$

The Lame coefficients in this case are equal to: $h_r = 1,\ h_\theta = r,\ h_\varphi = r\sin\theta$, and the square of the differential of arc (interval) is:

$$ds^2 = dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2, \qquad (6.17)$$

Since the metric tensor is determined by means of Lame coefficients, let us recall the geometric meaning of the latter:

the Lame coefficients show how many units of length are contained in the unit of length of segment of coordinates of the given point, and used to transform vectors when transition from one system to another takes place.

This means that the metric tensor in Euclidean geometry defines rescaling of three coordinates r, θ, φ, and in the pseudo-Euclidean or pseudo-Riemannian geometry it determines rescaling of four coordinates t, r, θ, φ.

Thus, the elements of the metric tensor $g_{\mu\nu}$ allow the changing of the projections of body trajectory segment on the coordinate axes during the transition from the Cartesian coordinate system to another. In the Cartesian system, all elements equal to one. In other systems, takes place the incommensurability of curve lines relative to the straight, similar to the incommensurability of the diameter of circumference in relation to its length. Therefore, $g_{\mu\nu}$ elements are appeared others than 1.

This analysis raises a question, the answer to which, in practice, determines the calculation of the metric tensor in framework of LITG: what changes of the scales of the coordinates follow from L-invariant transformations? First of all, we are talking about changing the scales of t and r coordinates.

As it is known, the scale changes of t and r in the L-invariant mechanics, are caused by the effects of time dilation and length contraction of Lorentz-Fitzgerald.

From the preceding analysis follows that in accordance with the laws of nature the interval of 4 space-time can be obtained only for the pseudo-Euclidean space-time as an embodiment of the physical law of motion of elementary particles.

Since thereis no other motion law for massive particles, we can assume that the metric tensor of GTR has the same physical meaning.

Thus, we conclude that the equations of Hilbert-Einstein and Hamilton-Jacobi contain $g_{\mu\nu}$ as a factor that takes into account the change in the scales of time and distance, due to L-invariant effects arising from the motion of bodies.

Indeed, it can be shown that the squares of the intervals, corresponding to the exact solutions of the Hilbert-Einstein equations, are defined by L-invariant effects.

6.5. The square of interval of L-invariant gravitation theory in the case of a change of scales of the space coordinates and time

We have shown that MT elements are determined by the Lame coefficients.

The linear arc element in the 3-dimensional mechanics is expressed through Lame's scale factors in the form of linear elements:

$$ds = \sum_{i=1}^{3} h_i dx_i = h_1 dx_1 + h_2 dx_2 + h_3 dx_3 , \qquad (6.18)$$

where $x_i = \vec{r} = (x_1, x_2, x_3)$, $i = 1, 2, 3$. . In a Cartesian coordinate system $x_i = \vec{r} = (x, y, z)$, and all the Lame coefficients equal to one.

In the L-invariant mechanics it is impossible to enter the *line* element of the arc since the physical equation, from which follows the magnitude of the arc, connects the squares of the energy, momentum and mass, and not the first degrees of these values. The exact expression is obtained in the form of the square of length of arc element, which is often referred to simply as an interval. In the 4-geometry it is of the form:

$$(ds)^2 = \sum_{\mu-0}^{3} (h_\mu dx_\mu)^2 = (h_0 dx_0)^2 + (h_1 dx_1)^2 + (h_2 dx_2)^2 + (h_3 dx_3)^2 , \qquad (6.19)$$

or, taking into account that $\lambda_{\mu\mu} = h_\mu h_\mu$, we receive from (6.19) the form:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu} (dx_\mu)^2 = \lambda_{00} (dx_0)^2 + \lambda_{11} (dx_1)^2 + \lambda_{22} (dx_2)^2 + \lambda_{33} (dx_3)^2 , \qquad (6.19')$$

where $\lambda_{\mu\mu}$ is metric tensor in LIGT.

Now let us consider which view takes the square of the interval in the concrete particular case of the change of scales of the space and time coordinate.

6.6. Time dilation and length contraction as a change of the scales of time and space coordinates

Using the definition of the metric tensor in LITG given above, let us calculate it in the simplest case. Consider (Pauli, 1958) Lorentz transformation in the transition from the coordinate system *K* to *K*', which is currently moving at a speed υ along the axis x. In this case only the coordinate x and time t undergo transformations.

The Lorentz effects of length contraction and time dilation are the simplest consequences of the Lorentz transformation formulae, and thus also of the two basic assumptions of SRT.

$$x = \frac{x' - \upsilon t'}{\sqrt{1 - \upsilon^2 / c^2}} , \; y = y' , \; z = z' , \; t = \frac{t' - \frac{\upsilon}{c^2} x'}{\sqrt{1 - \upsilon^2 / c^2}} , \qquad (6.20)$$

The transformation which is the inverse of (6.20) can be obtained by replacing υ by $-\upsilon$:

$$x = \frac{x' + \upsilon t'}{\sqrt{1 - \upsilon^2 / c^2}} , \; y = y' , \; z = z' , \; t = \frac{t' + \frac{\upsilon}{c^2} x'}{\sqrt{1 - \upsilon^2 / c^2}} , \qquad (6.20a)$$

Take a rod lying along the x-axis, at rest in reference system K'. The position coordinates of its ends, x_1' and x_2' are thus independent of t' and $x_2' - x_1' = l_0$ is the rest length of the rod. On the other hand, we might determine the length of the rod in system K' in the following way. We find x_1 and x_2 as functions of t. Then the distance between the two points which coincide simultaneously with the end points of the rod in system K will be called the length l of the rod in the moving system: $x_2(t) - x_1(t) = l$

Since these positions are not taken up simultaneously in system K', it cannot be expected that l equals l_0. In fact, it follows from (6.20) :

$$x_2' = \frac{x_2(t) - \upsilon t'}{\sqrt{1 - \upsilon^2/c^2}}; \quad x_1' = \frac{x_1(t) - \upsilon t'}{\sqrt{1 - \upsilon^2/c^2}}$$

and therefore $l_0 = \dfrac{l}{\sqrt{1 - \upsilon^2/c^2}}$ for infinitesimal time intervals of length dx has form $dx' = \dfrac{dx}{\sqrt{1 - \upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of **coordinates** (denote it as k_x^L) will be equal to:

$$k_x^L = \frac{dx'}{dx} = \frac{1}{\sqrt{1 - \upsilon^2/c^2}} = \gamma_L, \tag{6.21}$$

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{xx} = \frac{dx'}{dx}\frac{dx'}{dx} = (\gamma_L)^2, \tag{6.22}$$

The rod is therefore contracted in the ratio $\sqrt{1 - \upsilon^2/c^2} : 1$, as was already assumed by Lorentz. It therefore follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between two such rods moving relatively to each other, and this relation is in principle observable.

Analogously, the **time** scale is changed by the motion. Let us again consider a clock which is at rest in K'. The time t' which it indicates in x' is its proper time, τ and we can put its coordinate x' equal to zero. It then follows from (6.20a) that $t = \dfrac{\tau}{\sqrt{1 - \upsilon^2/c^2}}$, which for infinitesimal time intervals dt give: $dt = \dfrac{dt'}{\sqrt{1 - \upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of time (denote it as k_t^L) will be equal to:

$$k_t^L = \frac{dt'}{dt} = \sqrt{1 - \upsilon^2/c^2} = \gamma_L^{-1}, \tag{6.23}$$

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{tt} = \frac{dt'}{dt}\frac{dt'}{dt} = (\gamma_L)^{-2}, \tag{6.24}$$

Measured in the time scale of K, therefore, a clock moving with velocity υ will lag behind one at rest in K in the ratio $\sqrt{1 - \upsilon^2/c^2} : 1$. While this consequence' of the Lorentz transformation was already implicitly contained in Lorentz's and Poincare's results, it received its first clear statement only by Einstein.

Then, in framework of LITG the square interval will be as follows:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu}\eta_{\mu\mu}(dx_\mu)^2 = \lambda_{00}\eta_{00}(dx_0)^2 + \lambda_{11}\eta_{11}(dx_1)^2 + \lambda_{22}\eta_{22}(dx_2)^2 + \lambda_{33}\eta_{33}(dx_3)^2, \quad (6.25)$$

where $\eta_{\mu\mu}$ is the geometric metric tensor in LIGT (tensor of pseudo-Euclidian space); $\lambda_{\mu\mu}$ is the physical metric tensor in LIGT.

Using the values $\lambda_{00} = \lambda_{tt} = \gamma_L^{-2} = 1 - \upsilon^2/c^2$ и $\lambda_{11} = \lambda_{xx} = \gamma_L^2 = 1/(1-\upsilon^2/c^2)$ according to (6.22) and (6.24), we obtain in the Cartesian system of coordinates:

$$(ds)^2 = -(\gamma_L)^{-2}(dt)^2 + (\gamma_L)^2(dx)^2 + (dy)^2 + (dz)^2, \quad (6.26)$$

or

$$(ds)^2 = -(1-\upsilon^2/c^2)(dt)^2 + (1/(1-\upsilon^2/c^2))(dx)^2 + (dy)^2 + (dz)^2, \quad (6.26')$$

6.7. Connection of the metric tensor of GR with the gravitational field

Firstly, let us refer to GR for guidance, since a similar problem arises also in the solution of the equations of general relativity (see. (Landau and Lifshitz, 1971, § 100. «The centrally symmetric gravitational field"). Let us skip the details, contained in this book and focus on the question of our interest. Let us start from the point when the square of the interval is specified.

Here, in the square of the interval, the designation h corresponds to MT element λ_{xx}, and l corresponds to the element λ_{tt}, and it is assumed that $h = -e^{\lambda}$, as well $l = c^2 e^{\nu}$, where λ and ν are some functions of r and t.

After substituting these exponents into the vacuum equation of general relativity $R^{\mu\nu} = 0$ and its integration, we get that $e^{-\lambda} = e^{\nu} = 1 + const/r$.

At that, the constant const remains unknown (i.e., it is not defined by the solution of the equation of general relativity). According to Schwarzschild solution, this “constant” can be easily expressed through a mass body, requiring that at large distances, where the field is weak, the law of Newton was opperating.

From the results of the preliminary analysis (see (Landau and Lifshitz, 1971), §87 «Motion of a particle in a gravitational field" and §8 «Principle of least action."), $g_{00} \equiv g_{tt} = 1 - 2\varphi_N/c^2$, where φ_N is the Newtonian potential. In compliance with this, we assume that $const = 2\varphi_N$. However, by tracing the sequence of this analysis, we can easily confirm that the "derivation" of expression $g_{00} = 1 - 2\varphi/c^2$ is not connected with general relativity. This means that the derivation of Schwarzschild’s solution is also found by trial and error method.

However, this solution is proved to be correct, and shows us that the non-relativistic theory of gravitation of Newton is the basis of relativistic theory of gravitation for the Schwarzschild problem. In other words, the Newton solution is the zero approximation of the problem, and the relativistic theory should only add minor changes to this result in accordance with perturbation theory.

6.8. Global and local Lorentz transformation

It is interesting that the requirement to relate the velocity of the body with the gravitational field in framework of LIGT also follows from the fact of the equivalence of gravitational and inertial mass.

As we know, the usual Lorentz transformations in classical relativistic mechanics are global; that is they take place for all spatial and temporal space-time points.

Based on the factor of the equivalence of gravitational and inertial mass, Einstein and others showed, that Lorentz's transformation of gravitation theory must be carried out locally, that is, independently at each point of space-time. Thus, the question arises, what are these local L-transformation?

Let us use the analogy with phase invariance in quantum field theory, better known as "gauge invariance". The phase invariance is the invariance of wave equations regarding the internal rotation of the particle field. As it is known, in quantum field theory exist both global, and local phase (gauge) invariance. Moreover, all successes in QFT after the creation of QED are linked to the transition from the first to the second.

Global invariance is discovered through the addition of a constant complex value $i\alpha$ to the phase of the wave function of the particle, where α is independent of place and time: $\alpha = const$. Physical quantities, calculated according to the new wave functions are the same as they were before the introduction of this quantity (this fact means invariance).

Later, local invariance was introduced to QFT, i.e., the invariance that is true only at every point of space and time, but not for the entire space-time as a whole. It is characterized by a variable alpha: $\alpha = \alpha(x, y, z, t)$.

In the L-invariant theory we are talking about the Lorentz transformations at the inertial motion. This is a global transformation: in relation to the spatial coordinates $x \to x'(\upsilon_0)$ and similarly for the time $t \to t'(\upsilon_0)$, where υ_0 is the particle speed constant. According to Einstein's equivalence principle, the gravitational field is equivalent to the non-inertial motion of the particle (movement with a variable speed). Therefore, we must introduce the Lorentz transformations for non-inertial motion, by taking into account the local interaction of the field with the particle. The obvious way to do this is described further.

We introduce local transformation through transition to infinitely small distances and times: $dx \to dx'(\upsilon)$ and $dt \to dt'(\upsilon)$, in which $\upsilon = \upsilon(x, y, z, t)$ is the variable speed of particle motion. This transformation does not violate any laws of physics, but makes them local in space and time. Their correctness is verified by results, which are confirmed by experiment. In this case, their use allows us to calculate the elements of the MT, identical to those that we have from the solutions of general relativity (this is the topic discussed in the following sections).

Note that according to Poincare and Sommerfeld, Lorentz transformations describe invariance in relation to rotations in 4-dimensional space. This brings them close to phase transformations. In addition, they can be recorded through hyperbolic (i.e., exponential) functions. The question of how far these analogies go, requires a separate analysis.

6.9. The connection of motion velocity of the body in the gravitation field with characteristics of the gravitation field in the framework of LIGT

In Section 6.6 we noted that for the bond of the metric tensor with gravitational field, the speed of a body in the Lorentz transformation must depend on the gravitational field. In the previous section we came to the conclusion that the local L-invariance requires this rate to be variable. We show that both these requirements can be met within LIGT.

Let us consider the equivalence principle of gravitational and inertial masses. Begin with the Newtonian law of motion of a particle with inertial mass m_{in} in a gravitational field of source with a mass M:

$$m_{in}\frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{m_{gr}M}{r^2}\vec{r}^{\,0} \ , \qquad (6.27)$$

where m_{gr} is gravitational mass. Since $m_{in} = m_{gr} = m$, then dividing (1.1) by m we obtain in the case of gravitation the movement equation of the form:

$$\frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} \ , \qquad (6.27')$$

where acceleration is on the left and the Newton force per unit mass is on the right.

It is easy to see that *this equation is the mathematical expression of Einstein's principle of equivalence of gravitational and inertial forces of Einstein*. As we know, on the basis of this principle, Einstein concluded that space should be heterogeneous and gravity must be described as a curved space-time of Riemann.

The question arises *whether it is possible to give another explanation to this principle.*

It appears, that based on the same mathematics, we can actually find this connection. As is known, the equation (6.27') can be represented in the energy form. For this let us rewrite the Newton's motion law in the form:

$$d\vec{\upsilon} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} dt\,, \qquad (6.28)$$

Multiplying the left and right hand side of equation (6.28) on the speed $\vec{\upsilon}$, and taking into account that $\vec{\upsilon} = d\vec{r}/dt$ and $d\vec{r}/r^2 = -d(1/r)$, we have from (6.28) after integration:

$$\frac{\upsilon^2}{2} + \gamma_N \frac{M}{r} = const\,, \qquad (6.29)$$

where $\upsilon^2/2 = \varepsilon_\kappa/m$ is the kinetic energy of the moving particle per unit mass, and $\gamma_N M/r = \varepsilon_{pot}/m$ is the potential energy of a particle per unit mass at a given point of the gravitational field.

Thus, taking into account the postulate of equivalence and the expression for the potential of the gravitational field $\varphi_N = \gamma_N M/r$, we obtain from (1.1), the relationship between the velocity of the particle and potential of the gravitational field at the position of the particle:

$$\upsilon^2 = 2\varphi_N + const\,, \qquad (6.30)$$

If at the initial moment a particle was at rest, and the motion is only carried out via the potential energy outlay, then during the whole period of motion *const* = 0. For example, this occurs when the reference frame, that is related to the observer, falls freely to the center of gravity source along the radius (*radial infall*) from infinity, where it had a zero velocity. In this case, we have:

$$\upsilon^2 = 2\varphi_N = \frac{2\gamma_N M}{r}\,, \qquad (6.31)$$

Thus, as a mathematical consequence of Newton's theory of gravity, we have received another interpretation of the fact of the equality of inertial and gravitational mass. Following the example of Einstein's equivalence principle, it can be expressed as follows: *the potential of the gravitational field is equivalent to the square of the velocity of the motion of particles in this field.*

In addition, (see chapter 4) the electromagnetic basis of gravitational equations allows one to write the vector potential of the gravitational field through the scalar potential $\vec{A} = (\upsilon/c)\varphi$.

Expression (6.31) was obtained on the basis of non-relativistic energy conservation law (6.29). Obviously, to obtain the post-Newtonian corrections to (6.31), it is necessary to transition to the relativistic law of conservation (see more details in the full version of the theory).

6.10. Derivation of the Schwarzschild metric

It is easy to see that by substituting (6.31) in the L-invariant square of the interval (6.26’), obtained in paragraph 6.6, we obtain the interval of Schwarzschild-Droste:

$$ds^2 = \left(1 - 2\varphi/c^2\right)c^2 dt^2 - \frac{dr^2}{\left(1 - 2\varphi/c^2\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right), \quad (6.32)$$

Thus, we have, indeed, received the first tested and most important result of GTR, only in framework of the L-invariant gravitation theory.

7.0. The Kepler problem

The Kepler problem is the problem of motion of a body of litle mass in a centrally symmetric gravitational field of a stationary source of great mass.

7.1. Direct solution of the Kepler problem in the framework of LIGT

As the motion equation of LITG we use the Hamilton-Jacobi equation. As we have shown above, the equation of motion of Hamilton-Jacobi has a one-to-one connection with the square of the interval (square of arc element of trajectory) in framework of LITG. Therefore, as we will show below, it is not necessarily to find an appropriate interval to write the corresponding Hamilton-Jacobi equation for particle motion in gravitation field.

According to our results *all features of the motion of matter in the gravitational field owed their origin to effects associated with the Lorentz transformations*.

Two of the most important effects from the point of view of mechanics that arise due to the Lorentz transformations, are the Lorentzian time dilation and contraction of lengths:

$$d\tilde{t} = dt\sqrt{1-\beta^2}\ , \quad d\tilde{r} = \frac{dr}{\sqrt{1-\beta^2}}\ , \tag{7.1}$$

where, as shown previously, $\beta^2 = \upsilon^2/c^2 = r_S/r$, and r_S is the Schwarzschild radius.

7.2. The equation of motion of a particle in a gravitational field with the Lorentz time dilation and length contraction

We will use the Hamilton-Jacobi equation (7.3) in form:

$$\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - c^2\left(\frac{\partial S}{\partial \tilde{r}}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^2\ , \tag{7.11}$$

Substituting in (7.11) $d\tilde{t} = dt\sqrt{1-\beta^2}$ and $d\tilde{r} = dr\Big/\sqrt{1-\beta^2}$, we obtain:

$$\frac{1}{1-\beta^2}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(1-\beta^2\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^4\ , \tag{7.12}$$

Taking into account that in our theory $1-\beta^2 = 1 - r_s/r$, we obtain from (7.12) the well-known Hamilton-Jacobi equation for general relativity in the case of the Schwarzschild-Droste metric (Schwarzschild, 1916; Droste, 1917):

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^4, \tag{7.13}$$

As is known (Landau and Lifshitz, 1971) the term $\frac{1}{1-r_s/r}\left(\frac{\partial S'}{\partial t}\right)^2$ (which contains the Lorentz time dilation effect) in the classical approximation leads to the equation of motion with Newton's gravitational energy. From this it follows that the precession of the orbit ensure the introduction of an additional term $c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2$.

As is known, the Kepler problem solution, based on this equation, gives an additional term in the energy, which is missing in Newton's theory:

$$U(r) = -\frac{\gamma_N m M_s}{r} + \frac{M^2}{2mr^2} - \frac{\gamma_N M_s M^2}{c^2 m r^3}, \tag{7.14}$$

which is responsible for the precession of the orbit of a body, rotating around a spherically symmetric stationary center. From the above analysis it follows that the appearance of this term is provided by Lorentz effect of the length contraction.

As is well known (Landau and Lifshitz, 1971), the solutions of this equation disclose three well-known effects of general relativity, well confirmed by experiment: the precession of Mercury's orbit, the curvature of the trajectory of a ray of light in the gravitational field of a centrally symmetric source and the gravitational frequency shift of EM waves.

8.0. The Kerr metric calculation in the framework of LIGT

Here, we will use the approach for obtaining the corresponding square of the interval, which is described above for calculating of the Schwarzschild metric.

In this chapter in the framework of LIGT we consider the problems arising in the description of the test particle motion in a gravitational field of a moving source. We will show that the solution for the moving body is connected with the solution for the fixed body on the basis of the Lorentz transformations.

Perhaps the only book, in which the authors, to obtain the results of GTR, have used such approach, is the review of the problems of gravitation in book (Vladimirov et al, 1987). In this book, along with the Schwarzschild solution, by means of this method are obtained the solutions of Lense-Thirring and Kerr for the metric around a rotating body, and solutions of Reissner-Nordstrom and Kerr-Newman, when this source has an electric charge.

We will present below the Kerr solution only.

8.1. Gravitational fields around rotating source

To begin with it is worth discussing some general properties of rotation (Vladimirov et al, 1987).

In order to describe the rotation of a rigid body an angular velocity Ω is introduced in addition to the conventional (linear) velocity V of a point of the body, because the angular velocity is constant for a rigid rotation, whereas the linear velocity of any point of the body is proportional to the distance between the point and the axis of rotation.

The relationship between angular and linear velocities in cylindrical coordinates is

$$V = \Omega\rho, \tag{8.1}$$

and in spherical coordinates (here $\rho = r\sin\theta$).

$$V = \Omega r\sin\theta, \tag{8.2}$$

However, a body can rotate not as a rigid one (for example, Jupiter's atmosphere rotates with different angular velocities at different latitudes as a result having different periods of rotation). The rotation period is related to angular velocity thus: $T = 2\pi/\Omega$. Hence the angular velocity may depend on position (coordinates) of point.

A reference frame may be rotating, too; though a rigid body rotation is even less natural for such a system than a rotation with different angular velocities at different points. Also, if a reference frame extended to infinity could rotate as a rigid body, that is, with a constant angular velocity Ω, then a linear velocity at a finite distance from its axis (on a cylinder $\rho = c/\Omega$) would reach the velocity of light c, and outside of this "light cylinder" would surpass it. Obviously, this kind of reference frame is impossible to simulate for any material bodies, therefore, the angular velocity of any realistic reference frame must change with distance from the axis. The slowdown must not be less than inversely proportional to that distance. But there should be a domain, well within the light cylinder, where the reference frame would rotate as a rigid body.

A rotating physical body possesses an angular momentum L as a conserved characteristic, which in certain respects is related to energy and momentum, which are also subject to the conservation laws.

In Newton's theory, mass (or energy, divided by the velocity of light squared) is the source of a gravitational field, while linear and angular momenta have no such a role. In the GR, however, a gravitational field is generated by a combination of distributions of energy, and linear and angular momenta, and the stress, too.

Let us examine the angular momentum of an infinitely thin ring (which has, however, a finite mass M_r) rotating around its axis. This angular momentum is a vector which is directed along the axis of rotation and has an absolute value of

$$L = M_r VR = M_r \Omega R^2 \ , \qquad (8.3)$$

where M_r is the mass of the ring, V is its linear velocity, Ω is its angular velocity, and R is the radius of the ring.

8.1.1. *The satellite motion around rotational Earth*

In the real case, we have to evaluate the effect of rotation of the Earth to the satellite and to show that it is associated with the angular moment of the Earth.

Here we will use the work of R. Forward (Forward, 1961), who, following to the work of Moeller (Moeller, 1952), presented an analogy between electromagnetism and gravitation, which allows calculation of various gravitational forces by considering the equivalent electromagnetic problem.

When the analogy is carried out and all the constants are evaluated, we obtain an isomorphism between the gravitational and the electromagnetic quantities.

First we need to know the gravi-rotational field of the earth. From Smythe (Smythe, 1950) we find an expression for the external magnetic field produced by a ring current i at a latitude $\theta = \alpha$ on a spherical shell of radius R. By transforming the magnetic quantities in gravitational quantities, we obtain an expression for the gravi-rotational field of a rotating massive ring with mass current i_m:

$$P_\theta = \frac{-\eta\, i_m \sin\alpha}{2R} \sum_{n=1}^{\infty} \frac{1}{(n+1)} \left(\frac{R}{r}\right)^{n+2} \cdot P_n^1(\cos\alpha) P_n^1(\cos\theta),$$

Since it is assumed that superposition is valid, we can construct the gravi-rotational field of a solid spinning body by integrating over the volume:

$$P_\theta = \frac{-\eta\, \Omega \sin\theta}{8\pi\, r^2} \int_V \left[\mu(\alpha, R) R^2 \sin^2\alpha\right] R^2 \sin\alpha \, d\alpha \, d\phi \, dR + higher\ multipoles,$$

Since $r\sin\theta$ is the distance from the axis of rotation to the mass element, we see that the integral is merely the **moment of inerti**a I of the body (Earth). Thus, in general the rotational field of any rotation body is approximately:

$$P_\theta = \frac{-\eta\, I\Omega \sin\theta}{8\pi\, r^2} = \frac{-\eta L \sin\theta}{8\pi\, r^2},$$

Similarly, it can be show that:

$$P_r = \frac{-\eta\, I\Omega \cos\theta}{4\pi\, r^2} = \frac{-\eta L \cos\theta}{4\pi\, r^2},$$

But the Forward approach does not allow to compare the results of his calculation with metrics Lense-Thirring and Kerr. That is why we will try to obtain a metric which describes the gravitational field around the rotating ring using a technique like the above technique.

8.2. The Kerr metric in framework of LIGT

Now, let us try to obtain a metric which describes the gravitational field around the rotating ring (Vladimirov et al, 1987). It is understood that solution for any body, which is symmetrical relative to the axis of rotation, can be obtained by integrating of solution for the ring by the volume of this body.

We begin with the Euclidean space-time in which we introduce spherical coordinates in a non-rotating frame; we assume the basis is, thus relative to it the flat space-time metric will be

$$(ds')^2 = (cdt)^2 - (d\vec{r})^2 - r^2(d\theta)^2 - r^2\sin^2\theta(d\varphi)^2, \qquad (8.4)$$

A transition to a non-uniformly rotating reference frame is done by locally applying Lorentz transformations so that every point has its own speed of motion directed towards an increasing angle φ. The absolute value of this velocity is a function V which depends, generally speaking, on the coordinates r and θ: $V = V(r,\theta)$.

Such a local Lorentz transformation is not equivalent to the transformation of the coordinates in the domain studied (in practice this domain is the whole of space) but is limited only to the transformation of the basis at each point.

Thus, we have:

$$\begin{aligned} d\tilde{s}_0 &= \left(ds'_0 - \frac{V}{c}ds'_3\right)\Big/\sqrt{1-V^2/c^2},\ d\tilde{s}_1 = ds'_1 \\ d\tilde{s}_2 &= ds'_2,\ d\tilde{s}_3 = \left(ds'_3 - \frac{V}{c}ds_0'\right)\Big/\sqrt{1-V^2/c^2} \end{aligned}, \qquad (8.5)$$

Now let the box with the observer be released from infinity. In this case we can write a new basis in which time has slowed down, and the lengths in radial direction have shortened. This is equivalent to the substitution of the ds'_0 in (1.5) by the basis linear elements from (1.16)

$$\begin{aligned} ds''_0 &= d\tilde{s}_0\sqrt{1-\upsilon^2/c^2},\ ds''_1 = d\tilde{s}_1\Big/\sqrt{1-\upsilon^2/c^2}, \\ ds''_2 &= d\tilde{s}_2,\ ds''_3 = d\tilde{s}_3 \end{aligned} \qquad (8.6)$$

Thus, we have assumed that the observer makes his measurements in the rotating frame and notices the relativistic changes in his observations.

Now let us do the reverse transformation to the nonrotating reference frame by applying Lorentz transformations (inverse to (8.5)) to the basis (8.6):

$$\begin{aligned} ds_0 &= \left(ds''_0 + \frac{V}{c}ds''_3\right)\Big/\sqrt{1-V^2/c^2}, ds_1 = ds''_1, \\ ds_2 &= ds''_2, ds_3 = \left(ds''_3 + \frac{V}{c}ds''_0\right)\Big/\sqrt{1-V^2/c^2} \end{aligned}, \qquad (8.7)$$

We now insert into (8.7) the basis ds''_μ, which is expressed in terms of the $d\tilde{s}_\mu$ from (8.6), and then write this expression in terms of the ds'_μ from (8.5). We postulate, as we did previously, that the resulting basis (8.7) remains orthonormalized. A few manipulations yield:

$$\begin{aligned} ds^2 = &\left(1-\frac{\upsilon^2}{c^2-V^2}\right)c^2dt^2 - \frac{dr^2}{\left(1-\dfrac{\upsilon^2}{c^2}\right)} - r^2d\theta^2 - \left(1-\frac{\upsilon^2V^2}{c^2-V^2}\right)r^2\sin^2\theta\, d\varphi^2 + \\ &+ \frac{2V\upsilon^2}{c^2-V^2}r\sin^2\theta\, d\varphi dt \end{aligned}, \qquad (8.8)$$

But at the beginning, for simplicity we have made the assumption about a coordinate system as the normal spherical coordinate system, which is, of course, not suitable for a rotating body because its gravitational field should have the symmetry of an oblate spheroid.

To do this, at first we must pass to the ellipsoidal coordinates, and secondly, use Newtonian potential source (ring). If in accordance with what has been said we minimally modify the formula (1.22) without discarding any terms (of the type V^2/c^2 in (1.18), we can directly come to the exact Kerr metric.

Here the Newton's potential φ_N represents a solution of the Laplace equation, though under the new symmetry, that is rotational and not spherical. Therefore it is now worth considering oblate spheroidal coordinates in flat space. These coordinates, ρ, θ, and φ are defined as

$$x + iy = (\rho + ia)e^{i\varphi}\sin\varphi, z = \rho\cos\theta$$
$$\frac{x^2+y^2}{\varphi^2+a^2} + \frac{z^2}{\rho^2} = 1, r = \sqrt{x^2+y^2+z^2} \quad , \tag{8.9}$$

We know that $\Delta(1/r) = 0$ when $r \neq 0$, and this equality holds under any translation of coordinates. Let this translation be purely imaginary and directed along the z axis, i.e., $x \to x, y \to y,$ and $z \to z - ia/c$. Then we easily find that $cr \to (c^2r^2 - a^2 - 2iacz)^{1/2} = c\rho - ia\cos\theta$. From here the expression for Newton's potential follows,

$$\varphi_N = -\frac{\gamma_N M_s}{c^2 r} \to \varphi_N = \frac{\gamma_N M_s}{c}\operatorname{Re}\frac{1}{c\rho - ia\cos\theta} = \frac{\gamma_N M_s \rho}{c^2\rho^2 + a^2\cos^2\theta}, \tag{8.10}$$

since the Laplace equation is satisfied simultaneously by both the real and imaginary parts of the potential. Hence we can get with the help of $\upsilon^2 = 2\gamma_N M_s/r = 2\varphi_N$:

$$\frac{\upsilon^2}{c^2 - V^2} = \frac{\gamma_N M_s \rho}{c^2\rho^2 + a^2\cos^2\theta} \quad , \tag{8.11}$$

We determine the velocity V using a model of rotating ring of some radius ρ_0 for the source of the Kerr field, this ring being stationary relative to the rotating reference frame (8.6).

On the one hand, $V = \Omega(x^2+y^2)^{1/2} = (\rho^2 + a^2/c^2)^{1/2}\Omega\sin\theta$ corresponds to the relation (8.2). On the other hand, it is clear that the reference frame cannot rotate as a rigid body, otherwise the frame wouldn"t be extensible beyond the light cylinder as we dropped our box from infinity. Therefore the angular velocity Ω has also to be a function of position.

The ring lies naturally in the equatorial plane, so that its angular momentum is

$$L = mV\sqrt{(\rho^2 + a^2/c^2)}, \quad \left(\rho = \rho_0, \theta = \frac{\pi}{2}\right)$$

We now introduce an important hypothesis which establishes a connection between the angular momentum and the Kerr parameter *a*, which is also a characteristic for spheroidal coordinates (8.9), namely we put $a = L/M_s$. These last three statements yield

$$\Omega(\rho = \rho_0, \quad \theta = \pi/2 = c^2 a/(c^2\rho_0^2 + a^2))$$

If we now add a second hypothesis, that the field is independent of the choice of the ring radius (depending only on its angular momentum), then naturally we can get for Ω:

$$\Omega = c^2 a/(c^2\rho^2 + a^2)$$

and finally

$$V = ca(c^2\rho^2 + a^2)^{-1/2}\sin\theta, \tag{8.12}$$

It only remains for us to choose the expression for a basis ds'_μ which would correspond to the assumed rotational symmetry (i.e., to the oblique spheroidal coordinates). We may substitute the coordinates x, y and z from (8.9) into the pseudo-Euclidean squared interval,

$ds^2 = cdt^2 - dx^2 - dy^2 - dz^2$, hence getting a quadratic form with a non-diagonal term. This term, which contains $d\rho d\varphi$, can be excluded by a simple change of the azimuth angle: $d\varphi \to d\varphi + ca\left(c^2\rho^2 + a^2\right)^{-1} d\rho$ thus leading to a diagonal quadratic form. If now the square roots of the separate summands are taken, we get the final form of the initial basis ds'_μ:

$$\begin{aligned} &ds'_0 = c^2 dt, \quad ds'_1 = \sqrt{\left(c^2\rho^2 + a^2\cos^2\theta\right)/\left(c^2\rho^2 + a^2\right)}\, d\rho, \\ &ds'_2 = \sqrt{\rho^2 + a^2 \cos\theta / c^2}\, d\theta, \\ &ds'_3 = \sqrt{\rho^2 + a^2/c^2}\, \sin\theta d\varphi, \end{aligned} \quad , \tag{8.13}$$

A mere substitution of these expressions into (8.8) yields the standard form of the Kerr metric in terms of the Boyer-Lindquist coordinates,

$$\begin{aligned} ds^2 &= \left(1 - \frac{r_s r}{\rho^2}\right) c^2 dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 - \\ &- \left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\, d\varphi^2 + \frac{2 r_s r a}{\rho^2}\sin^2\theta\, d\varphi dt \end{aligned} \quad , \tag{8.14}$$

where we have introduced the notation

$$\Delta = r^2 - r_s r + a^2\,, \quad \rho^2 = r^2 + a^2\cos^2\theta\,, \quad a = L/M_S\,, \tag{8.15}$$

The resulting metric is a solution of Einstein's gravitational field equations, and the method does give some hint as to how to understand the Kerr metric and its sources, and it lets us look at the structure of the latter.

If we assume in the calculations that $(V/c)^2 << 1$, thus dropping the corresponding terms in (8.5) and (8.7). This is the assumption of slow rotation (more exactly, of the smallness of L, the angular momentum of the source) and it leads to $V = a\sin\theta/r$ instead of (8.12). Thus instead of the Kerr metric (8.14) we will get the approximate the Lense-Thirring metric:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt\,, \tag{8.16}$$

(we have written in it r instead of ρ and taken into account the approximate sense of the expressions).

9.0.The cosmological solutions in the framework of LIGT

9.1. Cosmological solutions of GR

In addition to the non-cosmological solutions the solutions exist that are interpreted as cosmological, that is such which related to the entire Universe.

At the moment, as a tested solution is considered the solution, obtained by means of the postulates of the homogeneity and anisotropy of Universe, jointly with the results of general relativity and thermodynamics.

Basic cosmological solutions of general relativity (for three types of curvature of space-time Universe) were obtained by Friedman (1922). Their derivation is reported in numerous textbooks, lectures and monographs; See, for example. (Bogorodsky, 1971; Dullemond et al. 2011, Ch. 4.).

9.2. The Robertson-Walker Universe metric in framework of LIGT

Since Newton's equation is a first approximation of the equations of gravitation LIGT, you can expect that the results of Friedman's (at least to a first approximation) can be derived from Newton's theory of gravitation.

Such solutions were indeed found in 1934 (Milne, 1934; McCrea and Milne, 1934). Moreover, it appears that these solutions are the same as the solutions of Fridman. Later they were refined (Milne, 1948; Krogdahl, 2004).

"*A Lorentz-invariant cosmology based on E. A. Milne's Kinematic Relativity is shown to be capable of describing and accounting for all relativistic features of a world model without space-time curvature. It further implies the non-existence of black holes and the cosmological constant. The controversy over the value of the Hubble constant is resolved as is the recent conclusion that the universe's expansion is accelerating. "Dark matter" and "dark energy" are possibly identified and accounted for as well*" (Krogdahl, 2004).

A modern formulation of this solution in Russian can be found, for example, in the presentation of the expert in the field of general relativity, academician Ya.B.Zeldovich; see Appendix I to the book (Weinberg, 2000), p. 190, titled "The classical non-relativistic cosmology" , who note here:

"All the calculations could have been made not only in the nineteenth century, but also in the eighteenth century".

The lecture 2 from the modern cosmology course ((Dullemond et al. 2011, Ch. 2) is dedicated to this subject.

Of the issues, identified in the statement of the problem, remains only the question of quantization of gravitation theory in framework of LIGT.

10.0. On the quantum theory of gravity

Numerous attempts to quantize general relativity, which are continued for almost a century, have not led to a positive result. In framework of GR the quantization is only possible in the linearized theory. But also it has its own difficulties.

Is it possible to build a quantum theory of gravity within the L-invariant gravitation theory?

Since the L-invariant gravitation theory is tied to the theory of electromagnetic field, the problem of quantum gravity is placed differently here than in general relativity.

Recall that, according to GR, the source (charge) of the gravitational field is the mass/energy. Its peculiarity lies in the fact that it has almost a sufficiently strong field only if its value is much larger than the mass-energy of the elementary particles (so even a small grain of sand contains up to 10^{19} electrons and nucleons). Let's call this gravitational charge "effective". Therefore, because of its value, it can be difficult to characterize by means of the quantum parameters of an elementary particle.

In addition, gravitational charge may have angular momentum (let us say, spin), but its quantization also does not make sense because of the magnitude of the effective charge. Therefore, from this point of view, we can not attribute the gravitational charge either to bosons or to fermions. At the same time it has the property of bosons: the superposition of individual masses-energies is possible and creates a new gravitational charge as the sum of mass-energy. In addition, as part of the GEM the gravitational radiation field is considered as composed of bosons - gravitons: particles with spin 2.

All this corresponds to the consequences of LIGT. Since we can conditionally say that the basis of LIGT is EM theory of the "massive photon" (see chapter 2), then we can assume that it can be the basis of the quantum theory of gravity. To some extent this is true. But such a theory is almost meaningless because of the size of the effective gravitational charge.

However, these quantum equations can be used because they coincide with the classical ones as it is the case for all bosons:

"Something similar can happen with neutral particles. *When we have the wave function of a single photon, it is the amplitude to find a photon somewhere. Although we haven't ever written it down there is an equation for the photon wave function analogous to the Schrödinger equation for the electron.* ***The photon equation is just the same as Maxwell's equations for the electromagnetic field, and the wave function is the same as the vector potential A.*** *The wave* *function turns out to be just the vector potential. The quantum physics is the same thing as the classical physics because photons are noninteracting. Bose particles and many of them can be in the same state — as you know, they like to be in the same state. The moment that you have billions in the same state (that is, in the same electromagnetic wave), you can measure the wave function, which is the vector potential, directly.*

Now the trouble with the electron is that you cannot put more than one in the same state" (Feynman, Leighton and Sands, 1964a).

This means that, having classical equations of gravity, we, in fact, already use quantum equations of gravity.

According to our approach, the words of Feynman in bold, can be attributed to gravitation after some adjustments:

The graviton equation is just the same as Maxwell's equations for the gravitation field, and the wave function is the same as the vector potential A.

Therefore, it is obvious that the quantization of gravity has no practical value: we already use, so to say, quantum theory, without introducing the requirement of quantization of energy-momentum to it..

From a formal point of view we can accept the existence of the graviton in LITG. Moreover, it can be assumed that the graviton should have spin 2, not 1 as a photon, since neutral waves can only be radiated by a system of quadrupole gravitational charges (see. Ivanenko and Sokolov, 1949, §56).

What can be said in the framework of LIGT about the existence of graviton and its spin value? If we will consider the linear approximation of GR (e.g., GEM) as reliable enough, then, because the graviton is a boson, it makes no sense to speak about its spin; in this case it is enough to speak about the classical gravitational waves.

Moreover, quantizing all waves does not make any sense. As we know, it does not make sense to quantize all the waves. In particular, the quantization of low-energy (long) EM waves does not make sense. The energy of gravitational waves in many orders of magnitude is lower, than of EM waves. As it is impossible to prove the existence of the graviton experimentally, it hardly makes any sense to discuss further.

Above we demonstrated that all proven solutions of general relativity can be obtained on the basis of L-invariant gravitation theory. Thus our task can be considered exhausted. In detail the theory is expounded in the books (Kyriakos, 2016a) and (Kyriakos, 2016b). The first variant of the theory provides more detailed information about the classical and quantum field theory and is intended for all specialists, including the undergraduate. The second book contains the abridged version of the theory, and is intended for professionals in the area of the gravitational field theory.

But the question arises, what connection can LIGT have with GR. In fact, it is often found that two theories which give identical results are often in a certain connection with each other. It seems to be impossible, to obtain general relativity from LIGT: they significantly differ by their initial postulates. But we can show that in the base of the cumbersome mathematics of general relativity lie the simple maths of LIGT.

11.0. GRT as a hidden Lorentz-invariant gravitation theory

Let us enumerate the distinguishing features of GTR compared to LITG.

1) GR is a relativistic theory, but it is not a Lorentz-invariant theory.

2) GTR is a general-covariant theory.

3) GR is constructed on a geometrical basis, while in the rest of physics, geometry does not play any role.

Does these features have a physical sense? Characteristic are the Fock remarks about the first two points (Fock, 1964):

(p.7): “*However, when Einstein created his theory of gravitation, he put forward the term " general relativity " which confused everything. This term was adopted in the sense of " general covariance ", i.e. in the sense of the covariance of equations with respect to arbitrary transformations of coordinates accompanied by transformations of the* $g_{\mu\nu}$. *But we have seen that this kind of covariance has nothing to do with the uniformity of space, while in one way or another relativity is connected with uniformity. This means that " general relativity " has nothing to do with " relativity as such ". At the same time the latter received the name " special " relativity, which purports to indicate that it is a special case of "general" relativity*.".

p. 7: “*Remembering that even in Newtonian mechanics one deals with the generally covariant Lagrange equations of the second kind, one would also have to say that Newtonian mechanics contain in itself "general" relativity*.".

p. 120:. “*We refer to Lagrange's equations (of the second kind) which describe the motion of a system of mass points in generalized coordinates and also their generalization for continuous media. While they state nothing physically new as compared to equations in Cartesian coordinates, Lagrange's equations nevertheless play an important part both in practical applications and in theoretical investigations. In the Theory of Relativity general tensor analysis has a similar purpose.*”

It is clear that these two first features of general relativity have no physical meaning. Analysis of the third aspect - the geometric representation of general relativity - leads to the same conclusion, and is the cause of difficulties of general relativity , as we will show below.

We will try to show that the verifiable results of general relativity do not depend on all these features. Strangely enough, in practice GTR demonstrates that the cumbersome mathematics of general relativity is equivalent to the simple math of LIGT.

Let us analyze the solution of problems in GR in terms of the features of the mathematical apparatus of GR, which are associated with the geometric approach and the general covariance of the theory.

The geometric foundation of general relativity is the Riemann curvature tensor.

" *The Riemann tensor consists of* $4 \times 4 \times 4 \times 4 = 256$ *components. Planes* xy *and* yx *etc. coincide, and planes* xx, *etc. are degenerated into a line and do not contribute to the mismatching* δA. *Therefore many components of the Riemann tensor either vanish or are expressed through each other. As a result, the four-dimensional Riemann tensor contains 14 independent components ...*

On the other hand, in differential geometry tensor of 4×4 *can be determined, which also contains information about curvature. It is called the Ricci tensor and is obtained from the Riemann tensor by means of the defined summation of its components.*" Etc .. (Beskin, 2009)

The physical sense of the Riemann curvature tensor and the Ricci curvature tensor is unknown. It is known that it contains all possible relationships of the coordinate differentials, in recording, which fit for any possible coordinate systems. These relationships may be formulated in the form of a generalized metric tensor (MT).

The invariance of the transition from one coordinate system to another is taken as some general principle of relativity. This principle is called the principle of general covariance.

Let us now trace the accurate solution within the framework of GR through the example of the most important Schwarzschild solution (see book (Landau and Lifshitz, 1971, § 100)).

Firstly, the original equation of general relativity is reduced to the equation:

$$R^{\mu\nu} = 0 , \tag{8.1}$$

The Ricci tensor $R^{\mu\nu}$ (as, in fact also the Riemann tensor) can be expressed through Christoffel symbols, which represent complex mathematical differential expressions, containing the elements of the metric tensor and its derivatives.

To pass on to the physically meaningful quantities, the pseudo-Euclidean spherical rectangular coordinates are chosen. Next the symmetry of the problem is analysed (i.e., to simplify the task, conservation laws are used, contemplated in it according to Newton's mechanics).

According to the results of the analysis, a conclusion is reached about the general form of four of the 16 elements of the metric tensor. It turns out that two of them are given by the geometry of the coordinate system. And only the other two elements are functions of the radius-vector and time of movement of the body. As suggested, they define the relativistic corrections of the Newtonian theory.

Further, by substituting these 4 functions in 40 Christoffel symbols, 40 quantities should be calculated. Of these, only 11 are meaningful, while the rest equal zero or coincide with each other.

These values are substituted in Ricci's tensor, and from the equation (8.1), we obtain a system of 3 ordinary differential equations after considerable simplifications.

The solution of this system is obtained in the form of two components of the metric tensor, which include an undefined constant. It is suggested that this constant is connected to the gravitational field. As a characteristic of the gravitational field, this constant is interpreted using Newton's theory. Only then do these elements receive a connection with the gravitational field.

Let us analyze these results.

As it was noted by V. Fock (see above), Newton's mechanics (as well as any other theory) can be written in the general-covariance form. This is done, for example, in the books of J.L.Synge (Synge, 1960), and others. This form of course, does not give any new results of the theory, because it is just an other mathematical notation.

It is important to note that in this form, for the sake of brevity all expressions are written, using the rule of summation over repeated indices Einstein. Let us compare the normal recording with the general covariant.

For example, the basic invariant form of SRT and GRT - the square of the interval of the particle trajectory - in the Cartesian coordinate system is written as:

$$(ds)^2 = (c\,dt)^2 - (d\vec{r})^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 , \tag{8.2}$$

In the curvelinear rectangular coordinates this square of the interval has the form:

$$(ds)^2 = g_{00}(dx_0)^2 - g_{11}(dx_1)^2 - g_{22}(dx_2)^2 - g_{33}(dx_3)^2 , \tag{8.3}$$

which in both cases has four summands (in the case of rotating bodies, one or two more terms are added).

In general relativity the transition of equation (8.2) to the general-covariant form is postulated:

$$(ds)^2 = \sum_{\mu,\nu} g_{\mu\nu} dx^\mu dx^\nu , \tag{8.4}$$

or, using the rule of Einstein's summation:

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu , \tag{8.5}$$

Comparing the expressions (8.3) and (8.5), we can conclude that in general relativity the mathematics is much simpler than the mathematics of STR (see (8.2), (8.3)). But this is a completely erroneous conclusion. We simply used new hieroglyphs, in which additional information is encrypted. Indeed, in the decrypted form (8.4) appears as the sum of 16 terms

$$\begin{aligned}(ds)^2 = {} & g_{00}dx^0dx^0 + g_{01}dx^0dx^1 + g_{02}dx^0dx^2 + g_{03}dx^0dx^3 + \\ & + g_{10}dx^1dx^0 + g_{11}dx^1dx^1 + g_{12}dx^1dx^2 + g_{13}dx^1dx^3 + \\ & + g_{20}dx^2dx^0 + g_{21}dx^2dx^1 + g_{22}dx^2dx^2 + g_{23}dx^2dx^3 + \\ & + g_{30}dx^3dx^0 + g_{31}dx^3dx^1 + g_{32}dx^3dx^2 + g_{33}dx^3dx^3\end{aligned} , \tag{8.6}$$

The most remarkable thing here is that, the Schwarzschild solution allocates in Cartesian coordinate system just four of these 16 terms, conditionally speaking, "diagonal" elements ("conditionally", since the expression (8.6) is not a tensor):

$$g_{00}dx^0dx^0,\ \ g_{11}dx^1dx^1,\ g_{22}dx^2dx^2,\ g_{33}dx^3dx^3, \tag{8.7}$$

which in a spherical coordinate system are displayed only by two elements:

$$g_{00}dx^0dx^0 \text{ и } g_{rr}dx^rdx^r, \tag{8.8}$$

Each of these two elements $g_{00}dx^0dx^0$ and $g_{rr}dx^rdx^r$ have each of them identical indexes 00, rr, which correspond to the transition from one coordinate system **to an identical** coordinate system. In general, these two systems can move relatively to each other. *But this movement is not defined in the framework of general relativity. Therefore, additional postulates are required to enter it , as we saw in the analysis of the Schwarzschild solution.*

It is easy to understand that in framework of SRT this corresponds to the transition, which takes place at the Lorentz-invariant transformations. Moreover, these elements are identical to those that we have received in framework of LIGT directly from the Lorentz transformations.

Similarly, each expression in the book of J.L Synge can be translated into a form of explicit sums. In this case, we will obtain a huge number of physically meaningless terms, compared to what we have in traditional mechanics. These terms owe their existence to the generalized coordinates: rectangular, oblique, and any others.

Obviously, such a redundancy of results has nothing to do with physics, but only with mathematical form. In the transition to the usual rectangular coordinates, the majority of terms of the general-covariant recording, taking into account mutual perpendicular coordinates, will be zeros or will be coincide with each other.

The only useful quality of general covariance is thatt, along with many meaningless terms, it contains a small number of terms, which are the basis for solving the problem.

Thus, the geometric formulation of general relativity drastically complicates the theory, hiding and masking simple results that can be obtained in the framework of special relativity.

This conclusion is particularly emphasized by the fact that Newton’s theory of gravitation can also be dressed in a meaningless from the physical point of view, geometric form.

11.1. Riemannian’s form of Newton's theory of gravitation

The geometrization of classic conservative theories is known for almost one and a half century. It is based on the variational principle and was proposed by Carl Gustav Jacobi in 1837 (Jacobi, 1837; 1884). The results of this article are set out in many courses of mechanics; for example, see (Buchholz, 1972, p. 271-277; Encyclpedia of matematics; Polac, 1959). Therefore, we present only the conclusion of the analysis of the book of Buchholz:

“*Thus, the motion of a holonomic system under the action of potential forces can always be considered as an inertial motion in the Riemann space, the metric of which is determined by a fundamental metric form*”.

It is clear that the Riemann form adds nothing to Newton's theory of gravitation. This is a mathematical "form" that does not contain new physics. And as we have shown above, the exemption from this mathematical form in general relativity leads to a simple Lorentz-invariant theory, which provides in a simple way all GRT results without resorting to geometric representations.

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