Escape Velocity at the Subatomic Level Leads to Escape Probability

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Abstract

In this paper we look at the escape velocity for subatomic particles. We suggest a new and simple interpretation of what exactly the escape velocity represents at the quantum level. At the quantum level, the escape velocity leads to an escape probability that is likely to be more useful at the subatomic scale than the escape velocity itself. The escape velocity seems to make simple logical sense when studied in light of atomism. Haug [1] has already shown that atomism gives us the same mathematical end results as Einstein’s special relativity. Viewed in terms of general relativity and Newtonian mechanics, the escape velocity seems to be simple to understand. It also seems to explain phenomena at the quantum scale if one maintains an atomist’s point of view. This strengthens our hypothesis that everything consists of indivisible particles and void (empty space). From an atomistic interpretation, our main conclusion is that the standard escape velocity formula likely is the most accurate formula we can generate and it appears to hold all the way down to the Planck scale. An escape velocity of \( v_e > c \) simply indicates that an indivisible particle cannot escape from a fundamental particle (for example an electron) without colliding with the indivisible particles making up the fundamental particle.

To understand this paper in detail, I highly recommend reading the article The Planck Mass Finally Discovered [2] first.

Key words: Escape velocity, escape probability, orbital velocity, quantum, Planck length, Planck mass, atomism, shape of indivisible particles.

1 Introduction

The escape velocity formula is normally found by simply solving the following equation with respect to \( v_e \); see, for example [3]

\[
\frac{1}{2} m v_e^2 - \frac{G M m}{r} = 0
\]  

This gives the traditional Newtonian escape velocity

\[
v_e = \sqrt{\frac{2GM}{r}}
\]  

where \( G \) is Newton’s gravitational constant and \( M \) is the mass we are escaping from. Further, \( r \) is known as the radius we are standing at when we are trying to escape. An identical escape velocity formula can be derived from Einstein’s theory of general relativity [4].

The escape velocity has, to our knowledge, actually never been experimentally confirmed\(^1\). It is assumed to be the minimum velocity needed to leave an object’s gravitational field. A rocket moving out of a gravity field does not actually need to attain the escape velocity to exit the gravity field. A rocket or spaceship can actually escape at any velocity, given a suitable mode of propulsion and sufficient propellant to accelerate with enough force to escape. Still, it is assumed that the escape velocity holds for a “bullet” that not itself have any propulsion to maintain the acceleration.

A particular interesting case is when the radius \( r \) is set equal to

\[
r = r_s = \frac{2GM}{c^2}
\]

At this radius, the escape velocity is \( c \). This has been interpreted as a radius where nothing inside this radius (for a given mass) can escape the gravitational field, not even light. This radius is well-known as the Schwarzschild radius, [5, 6, 7, 8]. At a radius inside this radius, the escape velocity will normally be

\(^1\)We could be wrong here, as we have only completed a limited literature search at this point in time.
higher than $c$, and since nothing can move faster than light, in general this implies that it cannot escape. In some of the literature, this observation has been interpreted as being evidence of black holes. Yet, a few researchers have been very critical of the black hole hypothesis and also of the typical interpretation of the escape velocity. See [10, 9, 11], for example.

Here we will show that the current interpretation of the escape velocity may be incorrect, or at least that it not been fully investigated and understood at the quantum level. We will see that the escape velocity for subatomic particles seems to fit the new atomistic interpretation of matter and energy introduced by [11, 2] perfectly. Haug has reintroduced the old view that the ultimate fundamental particle (making up both mass and energy) has spatial dimension and a diameter and that it is perpetually moving around. He has shown that all of Einstein’s special relativity end results can be derived from atomism. In addition, Haug has shown how atomism is consistent with the equations related to Heisenberg’s uncertainty principle, the Schwarzschild radius, and much more, but often with new and much simpler interpretations than are often given by the established views in physics. In this article we show how the well-known escape velocity makes complete sense under atomism, even at the subatomic level.

2 Orbital Velocity from Subatomic Particles

Let us start with the standard orbital velocity for an object with the mass of an electron. We will concentrate on the case where the radius is set equal to the reduced Compton wavelength of the electron. Do not confuse this scenario with the orbital velocity of an electron around a proton, which is not the topic of this article\(^2\). The electron’s mass is given by $m_e \approx 9.1094 \times 10^{-31}$ kg and the reduced Compton wavelength of an electron is $\lambda_c \approx 3.86 \times 10^{-13}$ meter. Further, Newton’s gravitational constant is $G \approx 6.674 \times 10^{-11}$ N m$^2$/kg$^2$. We can now calculate the orbital velocity from the electron mass:

$$v_o = \sqrt{\frac{G m_e}{\lambda_c}}$$

$$v_o = \sqrt{\frac{6.674 \times 10^{-11} \times 9.1094 \times 10^{-31}}{3.86 \times 10^{-13}}} \approx 1.25473 \times 10^{-14} \text{ m/s}$$

That is equal to $1.25473 \times 10^{-14}$ meter per second, which is equal to $4.52 \times 10^{-14}$ km/s. So, as expected, this represents an incredibly low orbital velocity, due to a very low mass. There is no surprise so far, as an electron’s mass is a very low and the gravitational field, even at such a short distance, is also very low. Therefore, it should require a low velocity to orbit such a low mass. Still, we pose the question, Why is the orbital velocity for an electron exactly this number? Some physicists would possibly even question whether or not we can calculate the orbital velocity for such small masses at such short distances. After all, Newton and Einstein’s gravitational theories do not necessary hold at the subatomic level. However, as we will see, the orbital velocity given here seems to make sense under atomism, even for subatomic particles like the electron.

Assume for a moment that an electron consists of two small sphere-shaped indivisible particle. These indivisible particles each have a diameter equal to the Planck length and they are moving back and forth along the reduced Compton wavelength of the electron at the speed of light; see [2] for a detailed introduction on looking at matter in this way. When the two indivisible particles are at a maximum distance from each other, they are twice the reduced Compton wavelength away. And when they are closest together, then they are $l_p$ apart. This makes the average distance between them equal to the reduced Compton wavelength. Next let us assume that an indivisible particle with diameter $l_p$ is placed inside the electron and is now moving out of the electron. To allow as much time as possible to escape the electron, we will place this indivisible particle at a position in the very center of the electron; see Figure 1.

The indivisibles making up the electron are shown in red and the indivisible particle that is trying to escape from the electron is marked in blue. To escape the electron, the blue indivisible particle (that has a diameter of $l_p$ itself) needs to move a distance of $l_p$ without getting hit by the two indivisibles making up the electron. How much time does the indivisible have to escape from the electron? Again, the indivisibles making up the electron are moving at the speed of light and they are taking the following time to move along the reduced Compton wavelength

$$t = \frac{2\lambda_c}{2c} = \frac{\lambda_e}{c} = 1.28809 \times 10^{-21}$$

\(^2\)The proton is considered a composite particle and we have too little knowledge about the proton to explain its orbital and escape velocities from this approach at the present time.
One can look at this as the time it takes for the sliding doors (of the electron) to close.\footnote{Bear in mind that the speed of an indivisible particle relative to another indivisible particle moving towards it is \(2c\), as observed from the laboratory frame. This is not in contrast with special relativity theory, but is actually well-known according to special relativity; it has been described as the closing speed or the mutual speed.}

For an indivisible particle to escape from the electron, it would need, at a minimum, to move at a speed of

\[
v = \frac{l_p}{\bar{\lambda}} = \frac{l_p}{\lambda c} = 1.25473 \times 10^{-14} \text{ m/s (5)}
\]

This is exactly the same rate as the orbital velocity calculated from the standard formula. In other words, the orbital velocity calculated by the traditional orbital velocity formula can, at the subatomic level, be interpreted as the minimum velocity necessary to escape the fundamental particle in question. When the fundamental particle consist of two indivisible particles moving at the speed of light and the particle passing through this fundamental particle has a diameter of \(l_p\), it makes full logical sense and we get exactly the same value as is produced by the standard orbital velocity formula. An identical mathematical result \(v_o = \frac{l_p}{\lambda c}\) has recently been derived by \cite{12} in a slightly different way as well.

Any velocity slower than this and one would not be able to escape before colliding with the sliding “doors” of the electron, that is to say, with the indivisible particles making up the electron. The same principle holds true for any subatomic “fundamental” particle. The escape velocity formula makes complete sense at a subatomic level when we follow the insights on matter and energy given by Haug’s atomism.

However, according to Haug’s atomist particle model, an indivisible particle will always travel at the speed of light. This means that an indivisible particle positioned as in Figure 1 always would be able to escape the electron, because its velocity is enormous compared to the minimum pass velocity. However, one would typically not know where the indivisible particles making up the electron are situated along the reduced Compton wavelength when they are trying to escape from the electron. At the subatomic level, it makes more sense to focus on the escape probability rather than the minimum escape velocity. After all, the indivisible particle is always traveling at speed \(c\) and not at the minimum escape velocity. The escape probability is given by

\[
p_e = 1 - \frac{l_p}{\bar{\lambda}}
\]

where \(\bar{\lambda}\) is the reduced Compton wavelength of the “fundamental” particle one indivisible particle try to escape from. Thus, the probability that an indivisible half Planck mass particle (Uniton) can escape from an electron is very high. For an electron, this probability is
This also means the probability that an indivisible trying to escape an electron will get hit is given by
\[ p_n = 1 - p_e = \frac{p_e}{\lambda} \approx 4.18532 \times 10^{-23} \] (8)

3 Escape Velocity

The escape velocity given by Einstein’s general relativity or Newtonian mechanics is equal to \( \sqrt{2} \) times the orbital velocity, that is
\[ v_e = \sqrt{\frac{2GM}{r}} \] (9)

For an electron, the escape velocity is given by
\[ v_e = \sqrt{\frac{2Gm_e}{\lambda}} \]
\[ v_e = \sqrt{\frac{2 \times 6.674 \times 10^{-11} \times 9.1094 \times 10^{-31}}{3.86 \times 10^{-13}}} \approx 1.7745 \times 10^{-14} \text{ m/s} \]

What does this velocity truly represent at the subatomic level? At a subatomic level, the difference between the orbital velocity and the escape velocity could be interpreted simply as the angle at which the particle is escaping relative to the reduced Compton wavelength axis of the electron (or any other subatomic particle). When it comes to escape velocity, this is when the indivisible particle is crossing at a 45° degree angle relative to the axis of the fundamental particle it is escaping from, as illustrated in Figure 2.

![Figure 2: Illustration of the escape velocity for a subatomic particle from an atomist point of view.](image)

Using the Pythagorean theorem, we can find the distance the indivisible particle has to cross in order to escape the “sliding doors” must be
\[ d^2 = l_p^2 + l_p^2 \]
\[ d = \sqrt{l_p^2 + l_p^2} \]
\[ d = l_p \sqrt{2} \] (10)
The time the sliding door is open in the electron is given by 
\[ t = \frac{\hbar}{2mc} = \frac{\hbar}{c} \]
and this means that the minimum escape velocity, when escaping at a 45° degree angle, is
\[ v_e = \sqrt{\frac{2Gm_e}{\lambda} - \frac{d}{t}} = \frac{\sqrt{l_p^2 + l_p^2}}{\lambda} = \sqrt{2}\frac{l_p}{\lambda_e}c \approx 1.7745 \times 10^{-14} \text{ m/s} \quad (11) \]

This formula can be derived directly from the gravitational potential as shown in Appendix A, and an identical result has recently been given by [12], without any discussion on the precise interpretation. Again, the indivisible particle is always moving at the speed of light, so at the subatomic level it makes more sense to look at an escape probability than a minimum escape velocity. The escape probability at a subatomic level for an electron is given by
\[ p_e = 1 - \sqrt{2}\frac{l_p}{\lambda_e} \approx 0.99999999999999999999994081068 \quad (12) \]
and the probability that an indivisible particle will collide with the electron while trying to escape from it is
\[ p_h = 1 - p_e = \sqrt{2}\frac{l_p}{\lambda_e} \approx 5.91893 \times 10^{-23} \quad (13) \]

4 Any Angle Generalized Minimum Escape Velocity

We can generalize the approach above to hold for an escape attempt at any angle. The generalized minimum escape velocity is then given by
\[ v = \frac{l_p}{\sin(\theta)\lambda}c \quad (14) \]

where \( \theta \) is the angle at which the indivisible particle is traveling relative to the particle axis of the particle it is traveling through. If \( \theta = 45° \), we have the traditional escape velocity formula, and if \( \theta = 90° \), we have the traditional orbital velocity formula.

Similar the generalized escape probability is given by
\[ p_e = 1 - \frac{l_p}{\sin(\theta)\lambda} \quad (15) \]

Further, the corresponding hit probability when trying to escape a fundamental particle is
\[ p_h = 1 - p_e = \frac{l_p}{\sin(\theta)\lambda} \quad (16) \]

5 Escape Velocity for an Indivisible Particle

When a particle is moving close to or even at the speed of light, as we assume that the indivisible particle does, then we should also assume that its energy is \( E = mv^2 \), rather than the kinetic energy \( \frac{1}{2}mv^2 \). This gives us an escape velocity of (see Appendix B)
\[ m_v e_i = -GMm_i \quad (17) \]

\[ v_e = \sqrt{\frac{GM}{r}} \quad (18) \]

That is to say, the escape velocity for an indivisible particle is the same as the orbital velocity. Sato and Sato [13] have recently pointed out (with somewhat different arguments) that the escape velocity of a photon must actually be identical to the orbital velocity formula. That is the well-known escape velocity formula divided by \( \sqrt{2} \). Since the indivisible particle in our theory is the light particle itself, this makes much more sense under atomism. From a quantum perspective, this is the minimum velocity needed to escape a particle with mass \( M \). This can be rewritten as
\[ v_{e,i} = \sqrt{\frac{GM}{\lambda}} = \frac{l_p}{\lambda}c \quad (19) \]

where \( \lambda \) is the reduced Compton wavelength of the mass we need to escape from. Further, the escape probability for an indivisible particle is
6 The Shape of the Indivisible Particles and the Need for Planck Scale Quantum Corrections

Modern physics does not really explain what matter and energy consist of at the depth of reality. Therefore, despite their great success, they seem to offer a limited understanding of space-time, in particular down to the Planck scale. As Richard Feynman once stated, “It is important to realize that in physics today, we have no knowledge what energy is.”.

On the other hand, under atomism we assume that matter consists of indivisible particles moving back and forth in the void. Thus, in our theory we have very precise definitions of the subatomic world, all the way down to the Planck scale. The indivisible particle has a radius of half a Planck length and a diameter equal to the Planck length. The reduced Compton wavelength is the average distance from the center of one indivisible particle to the center of the other indivisible particle making up the mass. Taking an electron as an example, on average the distance between the two indivisible particles making up the electron is $\lambda_e$, and the closest distance is naturally $l_p$, when the two indivisible particles are laying side by side counter-striking and creating a Planck mass, and the furthest away is $2\lambda_e$. When two indivisible particles are $l_p$ apart (center to center), there is no space for another indivisible particle inside the particle and it is therefore meaningless to talk about an escape velocity. Even when the two indivisible particles are $2l_p$ apart (center to center), there is only a void distance of $l_p$ between them, and no indivisible particle can escape from this, because the indivisible particles making up the mass we are trying to escape from are closing the $l_p$ “gap” at the speed of light.

To get closer to understanding any quantum corrections that may be needed, let us first start with the simplified assumption that the indivisible particles are cube-shaped rather than sphere-shaped. In this case, the exact escape velocity would be

$$v_{e,i,c} = \sqrt{\frac{GM}{\lambda} \left(\lambda - l_p\right)} = \frac{l_p c}{\lambda - l_p}$$

and the corresponding escape probability would be

$$p_{e,i,c} = 1 - \frac{l_p}{\lambda - l_p}$$

Further, the minimum reduced Compton wavelength that an indivisible particle could escape from would then be $\bar{\lambda} = 2l_p$. Figure 3 illustrates the escape velocity in relation to cube-shaped indivisible particles.

![Cubed shaped escape velocity](image)

**Figure 3:** Illustration of escape velocity for a cube-shaped indivisible particle.

Table 1 shows the escape velocity for cube-shaped indivisible particles. The cube-shaped formula gives quite a large error compared to the standard formula. The standard formula is basically much closer to what we get for sphere-shaped particles.

\[\text{The Planck mass only last for a Planck second and is created in an electron with a frequency of } \frac{c}{\bar{\lambda}}, \text{which again gives us the electron mass.}\]
Table 1: The table shows the exact escape velocity predicted by cube-shaped indivisible particles versus what is predicted from Einstein and Newton (sphere-shaped). We see that the errors in assuming cube-shaped indivisible particles are quite substantial when we approach the Planck scale. We have no reason to think that the ultimate particle is cube-shaped; in fact, we expect it to be sphere-shaped.

The shortest reduced Compton wavelength we can have in a mass that a cube-shaped indivisible particle can escape from without collision (getting caught by the sliding doors) is 1/\(\lambda\). When the two indivisible particles making up this mass are maximum distance away from each other, they are 4/\(\lambda\) away from each other. This is the distance center to center between them. However, the void distance between them is only 3/\(\lambda\). In one Planck second, 2/\(\lambda\) is closed. Be aware that we have to handle mutual velocities (also known as closing speed) here; in short, this is related to the speed of a particle relative to another particle as observed from the laboratory frame (a third frame). See [14], for example.

In the much more realistic case of sphere-shaped indivisible particles, the best formula we have come up with so far is

\[
v_{e,i,s} \approx \frac{11}{12} \frac{l_p}{\lambda} c = \sqrt{\frac{GM}{\lambda} \frac{112}{144}}
\]  

Figure 4: Illustration of escape velocity for a sphere-shaped indivisible particle.
Formula 23 was simply figured out by playing with identical circle shapes that I moved around at the same velocity combined with simple logic to figure out the approximate minimum velocity needed for a spherical indivisible particle to escape at Planck scale distances. The intuition is that even when the door opening is less than \( l_p \), the indivisible particle trying to escape can still be moving part of itself out of the “door” opening. This because both the sliding “doors” are circular (sphere-shaped) and the indivisible particle attempting to escape is also sphere-shaped, as illustrated in figure 4. If these quantum corrections should hold later on, it would mean that the standard escape formulas of Einstein and Newton (actually the orbital formulas) are always off (for light) by about 9%. However, this mostly only has an impact on conclusions based on distances close to the Planck length. The reason for this is that the indivisible particles never move at the escape velocity (which is just the minimum velocity needed to have a chance to escape), but instead they are always moving at the speed of light. Even so, this has implications on escape probabilities that could be relevant for moving deeper into physics. The theory presented here, in particular our Planck scale quantum correction approximations, should be studied further.

The corresponding modified escape probability is

\[
pe,i,s \approx 1 - \frac{11}{12} \frac{l_p}{\lambda}\tag{24}
\]

This special case with quantum corrections is only valid if the indivisible particle is situated exactly at the middle of the fundamental particle; this is extremely unlikely in practice. If even it lies slightly off from this position, then the standard formula: \( ve,i = \sqrt{\frac{GM}{r}} = \frac{l_p}{\lambda} \) would be a better approximation. We conclude that the Einstein and Newton escape velocities likely are the most accurate formulas we can come up with in atomism for the escape velocity at the Planck scale. However, the interpretation is quite different than the one given in modern physics. The escape velocity is the minimum velocity needed for an indivisible particle placed at the very center of the “fundamental” particle to travel at in order to avoid collision with the indivisible particles making up the “fundamental” particle from which it is trying to escape.

7 Conclusion

We have looked at the escape velocity at the subatomic level and evaluated the well-known formulas in the light of an atomist interpretation. It seems that the escape velocity at a subatomic scale can be interpreted as the minimum velocity needed to escape the particle that contains it. However, since the indivisible particle always moves at the speed of light, it makes more sense to look at escape probabilities at the subatomic scale. Atomism seems to have a very simple explanatory model for escape velocities and probabilities. The escape velocity is simply the minimum velocity an ideally placed indivisible particle inside the fundamental particle needs to move at in order to avoid collision with the indivisible particles that make up the fundamental particle.

Appendix A

The escape velocity in the subatomic world can be derived in terms of Haug’s discovery that the Newton gravitational constant likely is a universal composite constant; thus it can be written as

\[
\begin{align*}
\frac{1}{2}mv^2 - \frac{GMm}{r} &= 0 \\
v^2 - \frac{2GM}{r} &= 0 \\
v^2 - \frac{2\frac{c^2}{\lambda^2} \frac{l_p}{\lambda}}{r} &= 0 \\
v^2 - \frac{2c^2 \frac{l_p^2}{\lambda^2}}{r} &= 0 \\
v &= c\sqrt{\frac{2\frac{l_p^2}{\lambda^2}}{r}}
\end{align*}\tag{25}
\]

\(^5\)As the Planck length here would be hypothetical represented by the diameter of the circle played with, it is not hard to work out good approximations even by moving some circles.

\(^6\)More precisely the orbital velocity formula, when dealing with a light particle escaping from a mass, or alternatively the standard Einstein and Newton escape velocity formula \( ve,i = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{l_p}{\lambda}} \) if dealing with an escape at a 45° angle.
and when we in the quantum world assume $r = \lambda$, we get

$$v = \sqrt{\frac{2GM}{\lambda}} = \sqrt{\frac{\ell_p}{\lambda}} c.$$  

(26)

Appendix B

For light, the escape velocity is given by

$$m_i v^2 - \frac{GM m_i}{r} = 0$$
$$v^2 - \frac{GM}{r} = 0$$
$$v^2 - \frac{l_p^2 c^2}{r} = 0$$
$$v^2 - \frac{c^4 l_p^2}{r} = 0$$

and when we assume $r = \lambda$, we get

$$v = c \sqrt{\frac{l_p^2}{\lambda r}}$$  

(27)

When this $v$ is lower than $c$ this simply means that it is the minimum velocity the indivisible particle would have to travel at in order to escape the mass it is inside. However, an indivisible particle always travels at a speed of $c$, so it makes more sense to talk about escape probabilities than escape velocity, at least when it comes to indivisible particles.

References


