ON HYPERSPIRAL

\[ r = a e^{\frac{b}{q}} \]

Dragan Turanyanin

1 Alt. e-mail: turanydra@gmail.com, web: wavspace.webs.com

This spiral (given in polar coordinates \( r, \theta \)) can be seen as a missing member of the set of known spirals. Namely, if logarithmic spiral would be generalized in a way

\[ r = a e^{b \theta^q}, \quad q \in \mathbb{Q} \]

(e.g., hyperlog-spirals), then in case \( q = -1 \) follows the above proposed hyperspiral \( r = a e^{\frac{b}{q}} \). The next simplification \( a, b = 1 \) gives
\[ r = e^{\frac{1}{\theta}} \text{ or } \ln r = \frac{1}{\theta}. \]

The spiral has two very distinct parts: the inner part for \( 0 < \theta \) and the outer part for \( \theta > 0 \). The circle \( r = a \) is the asymptotic one. Polar point is the asymptotic point of the spirals' inner part.

Rate of change of \( r(\theta) \) reads

\[ \dot{r} = -\frac{b}{\theta^2} r. \]

Because \( \psi = \arctan \left( \frac{r}{r'} \right) \) defines the angle between radius and tangent in a given point \( (r, \theta) \) of a polar curve, follows

\[ \psi = \arccot \left( \frac{b}{0^2} \right). \]

Second derivative of \( r(\theta) \) reads \( \ddot{r} = \frac{b(b + 2\theta)}{\theta^4} r \). Curvature \( k \) of polar curves is defined as

\[ k = \frac{r^2 + 2r^2 + r\dot{r}}{(r^2 + \dot{r}^2)^{\frac{3}{2}}} \text{, hence} \]

\[ k = r^{-1} \frac{1 + b^2 \theta^{-4} - 2b \theta^{-3}}{(1 + b^2 \theta^{-4})^{\frac{3}{2}}}. \]

The arc length \( s \) of polar curves is defined as \( s = \int_0^{\theta} \sqrt{r^2 + \dot{r}^2} \, d\theta \), thus follows

\[ s = a \int_0^{\theta} e^{\frac{b}{0}} \sqrt{1 + b^2 \theta^{-4}} \, d\theta. \]

Unlike logarithmic spiral this spiral does not possess simple natural, intrinsic equation because there is no exact solution of the above integral. In fact, hyperexp function of the general form \( \text{exp}(1/x) \), does not have its exact prime function at all. This very fact must produce deep geometrical consequences onto hyperspiral as well.

However, this curve does possess a full polar inversion, i.e. regarding the asymptotic circle

\[ r = \frac{a^2}{r(\theta)} = a e^{-\frac{b}{\theta}}. \]
Besides pure geometry, *hyperspiral* may eventually bring new inspiration into areas of science, cosmology, engineering and art.

**Acknowledgment**
Specially thanks to Mr. Svetozar Jovicin for occasional but fruitful discussions.
This longstanding work is dedicated to the idea of spiritual friendship.

**Reference**
Savelov A.A., "Planar curves", Moscow (1960), transl. in Serbian
Bronstein I.N., Semendyaev K.A., Spravochnik po Matematike, (Handbook of Mathematics), Nauka, Moskva, in Russian
Yates R., “Curves And Their Properties”
Famous Curves Index, http://www-history.mcs.st-and.ac.uk/history/Curves/Curves.html
Visual Dictionary of Special Plane Curves,
http://xahlee.info/SpecialPlaneCurves_dir/specialPlaneCurves.html
An index of the included curves and surfaces ,
http://www.math.hmc.edu/~gu/math142/mellon/curves_and_surfaces/all.html
Definitions for associated curves,
http://www-history.mcs.st-and.ac.uk/history/Curves/Definitions2.html#Anallagmatic_curve

Copyright©2012 by Dragan Turanyanin