

THURALS & INPOLARS

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Introduction

The aim of this review is firstly, to present again a new family of polar curves (e.g. *thurals* [1]) and secondly, to introduce their so called *inpolars* as main objects of one original geometrical transformation [2]. *Addendum* is completely new with a brief analysis of s-thural.

1. Thurals

Four new, quite original, transcendental curves (let us call them *thurals*) will be presented in this chapter. The very first one (Fig. 1) would be super-spiraling curve given with the polar formula

$$r = a \theta^{b\theta} \quad (1.1)$$

or

$$r = e^{\theta \ln \theta}, \quad (1.1a)$$

assuming $a, b = 1$. The second *thural* (Fig. 2) is formally analogous to the above and defined with the formula

$$r = \theta^{-\theta}. \quad (1.2)$$

The idea for its name comes naturally: *c-curve*. Finally, the last two spirals (Figs. 3. and 4) in this review would be defined with the formulae

$$r = \theta^{1/\theta} \quad (1.3)$$

and

$$r = \theta^{-1/\theta}. \quad (1.4)$$

Loop is what comes in one's mind when one takes a look at both curves, respectively.

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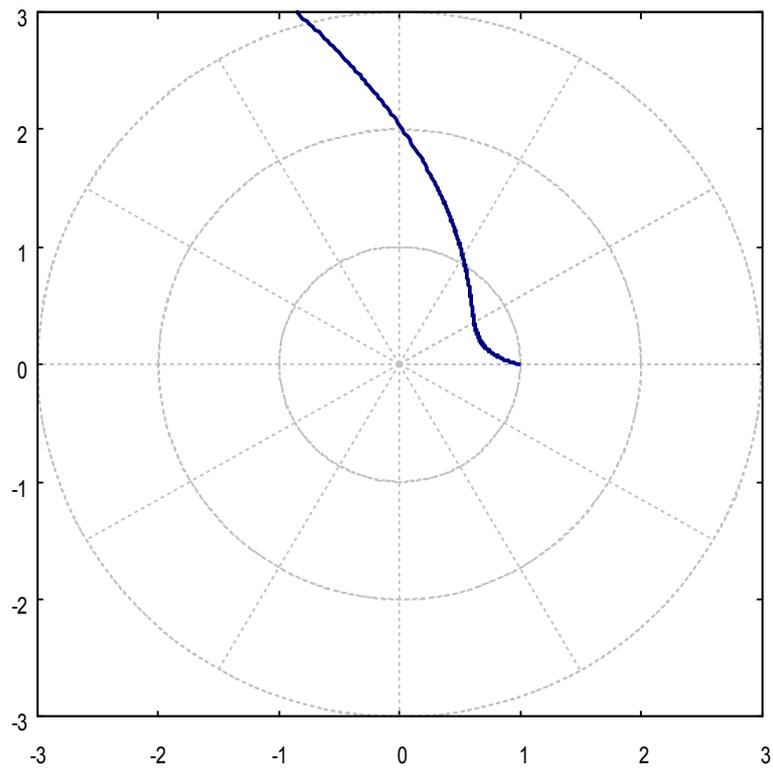


Fig. 1: *s-spiral*, $0 < \theta \leq 4\pi$

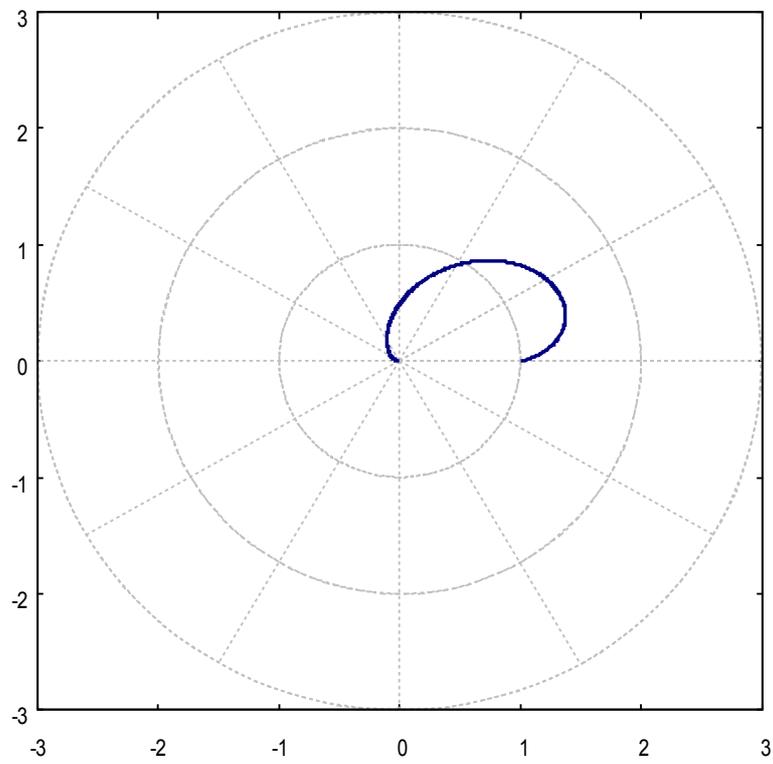


Fig. 2: *c-curve*, $0 < \theta \leq 4\pi$

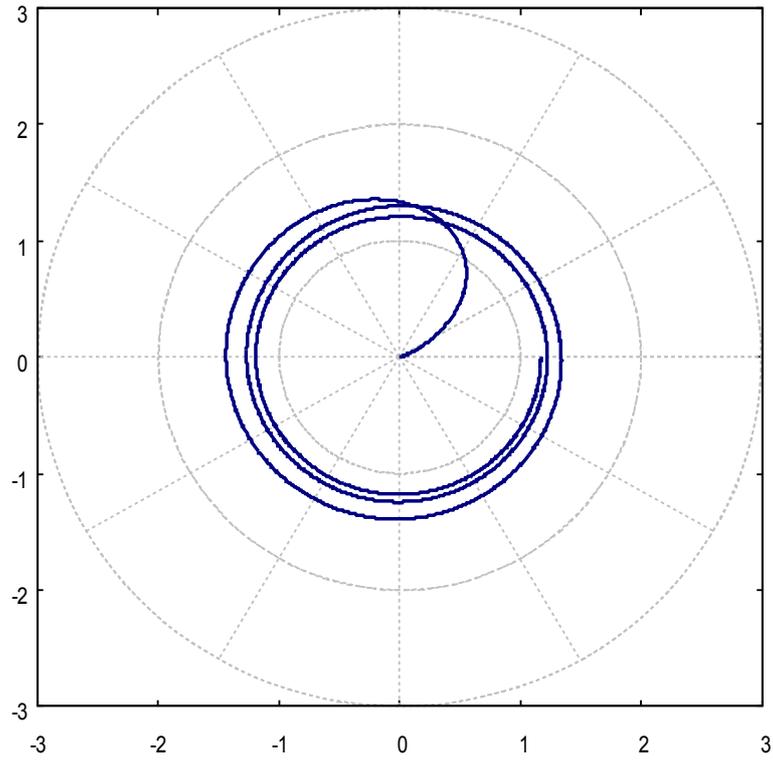


Fig. 3: super-loop, $0 < \theta \leq 6\pi$

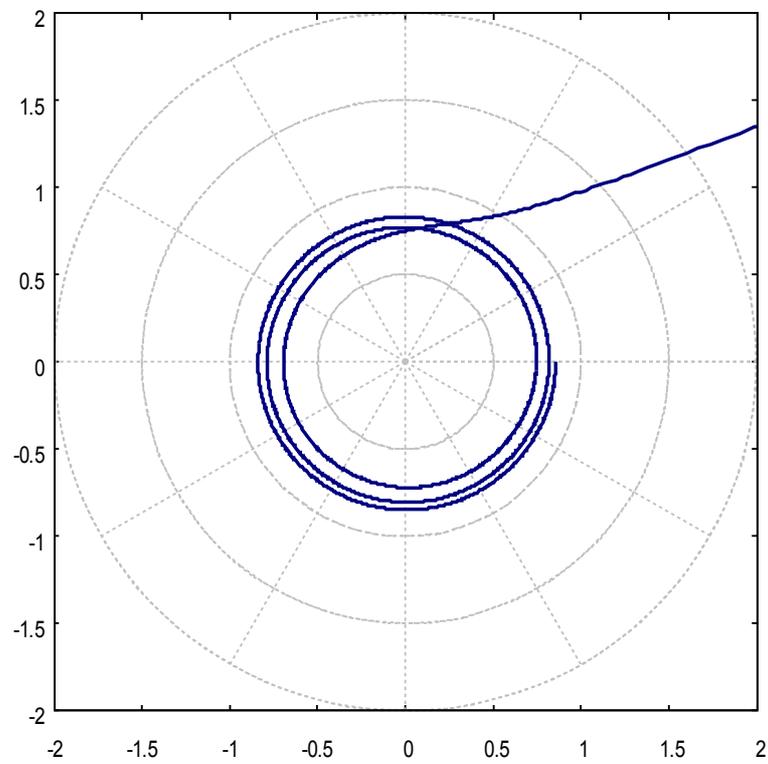


Fig. 4: sub-loop, $0 < \theta \leq 6\pi$

2. Inthurals

An initial, almost naive, question: Does exist (and how looks like) the *inspiral* of the form

$$r^r = \theta, \quad (2.1)$$

has surprisingly forced us to accept a change in our usual geometric intuition of r and θ mutual dependency in general [2]. The positive answer via an *inpolar* transformation leads (among other *inpolars*) towards the inpolar thurals, i.e. *inthurals*, as the consequence (Figs. 5 and 6).

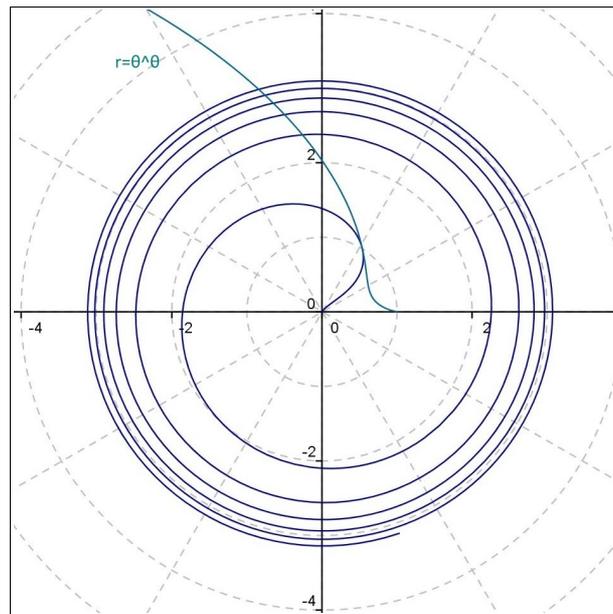


Fig. 5: *s-inthural* $r^r = \theta$

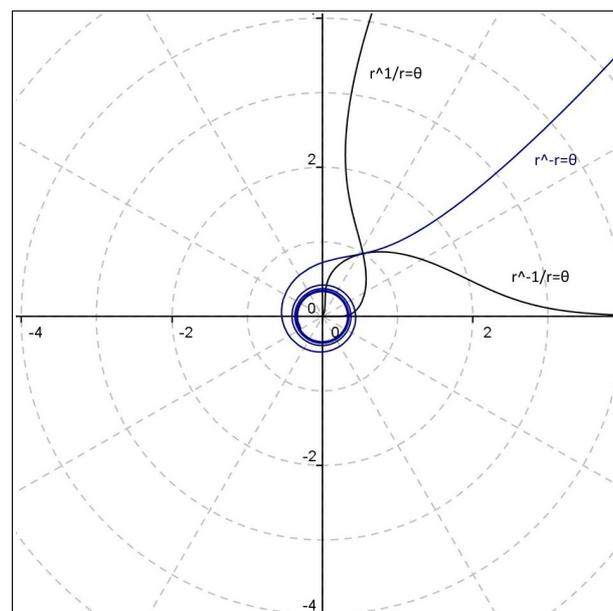


Fig. 6: *The rest three inthurals*

Acknowledgments

And the heaven departed as a *scroll* when it is *rolled together*...

Revelation 6:14

References

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Addendum

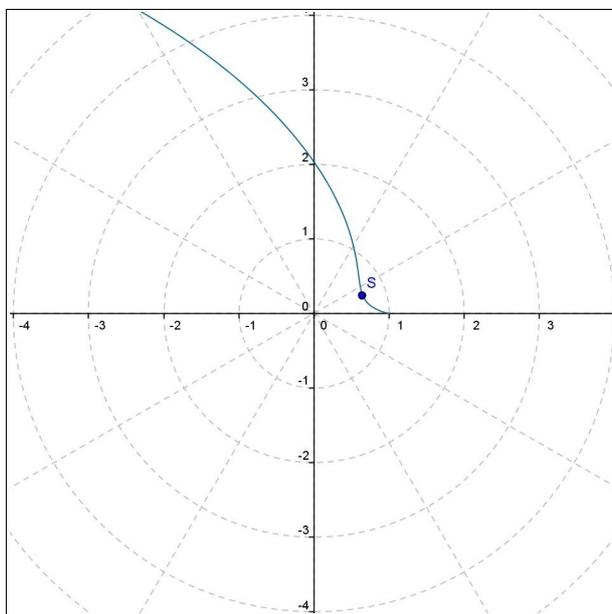


Fig. 7: *s-thural's S point*

The point S is the prominent point of the *s-thural* (Fig. 7). It can be seen as analogous to the minimum of the Cartesian function $y=x^x$. Its polar coordinates we deduce by examining the standard condition $r'=0$, thus

$$r'=(\theta \ln \theta)' e^{\theta \ln \theta}=(\ln \theta+1) e^{\theta \ln \theta}=0 .$$

The condition is satisfied with $\ln \theta = -1$, hence $\theta = 1/e$ in radians. Writing in degrees S coordinates reads (0.692, 21.078).

Finally, it is interesting to notice the curves' S tangent¹ as well as the polar line through S.

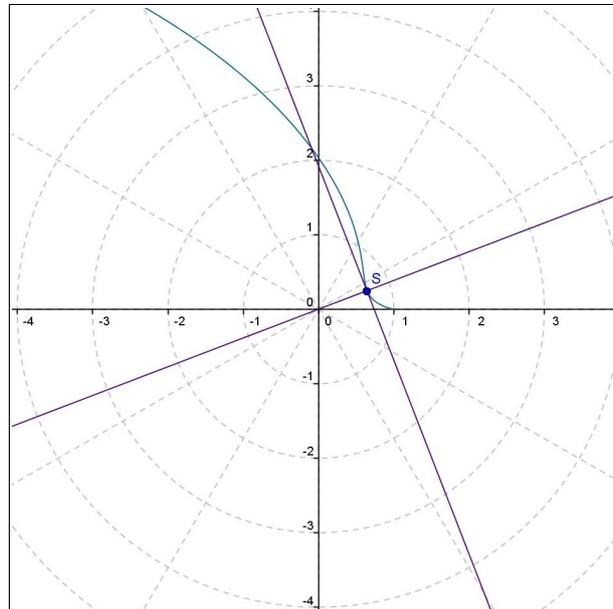


Fig. 8: S tangent and normal

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¹In [GeoGebra](#) solution reads `Tangent[S, curve]`