A Simple Proof of the Collatz-Gormaund Theorem (Collatz Conjecture)

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1 Definitions

We define a function \( c(n) \) \((n \in \mathbb{N})\) such that

\[
c(n) = \begin{cases} 
\frac{n}{2} & \text{n even} \\
3n + 1 & \text{n odd}
\end{cases}
\]

Furthermore, we define \( c_i(n) \) such that

\[
c_i(n) = c(c(...c(n)...))) \quad \text{\(i\) times}
\]

We now define a statement \( P(n) \), meaning

\[
P(n) : \exists i \in \mathbb{N}.C_i(n) = 1
\]

Thus, we can state the Collatz Conjecture in the following way

\[
\forall n \in \mathbb{N}.P(n)
\]

2 Proof

\[
c(1) = 1
\]

\[
\therefore P(1)
\]

Now, assume \( P(n) \forall n < k \) for some \( k \in \mathbb{N} \) (Complete Induction)

If \( k \) is even:

\[
c(k) = \frac{k}{2}
\]

\[
\frac{k}{2} < k
\]

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\( P\left( \frac{k}{2} \right) \) is assumed.

\[ \therefore P(k) \]

If \( k \) is odd:

\[ k = 2m - 1 \text{ for some } m \in \mathbb{N} \]

\[ c(2m - 1) = 3(2m - 1) + 1 = 6m - 2 = 2(3n - 1), \text{ which is even.} \]

\( P(k) \) has already been shown for even \( k \)

\[ \therefore P(k) \]

\( P(1) \) and \( P(n < k) \implies P(k) \)

Hence, by complete induction, \( P(n) \forall n \in \mathbb{N} \)

\( Q.E.D \)