A Simple Proof of the Collatz-Gormaund Theorem (Collatz Conjecture)

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1 Definitions

We define a function $c(n) \quad (n \in \mathbb{N})$ such that

$$c(n) = \begin{cases} \frac{n}{2} & n \text{ even} \\ 3n+1 & n \text{ odd} \end{cases}$$

Furthermore, we define $c_i(n)$ such that

$$c_i(n) = \underbrace{c(c(c(...c(n)...)))}_{i \text{ times}}$$

We now define a statement $P(n)$, meaning

$$P(n) : \exists i \in \mathbb{N}.C_i(n) = 1$$

Thus, we can state the Collatz Conjecture in the following way

$$\forall n \in \mathbb{N}.P(n)$$

2 Proof

$$c(1) = 1$$

$$\therefore P(1)$$

Now, assume $P(n)\ \forall n < k$ for some $k \in \mathbb{N}$ (Complete Induction)

If k is even:

$$c(k) = \frac{k}{2}$$

$$\frac{k}{2} < k$$

$$P(\frac{k}{2}) \text{ is assumed.}$$

$$\therefore P(k)$$

If k is odd:

$$k = 2m - 1 \text{ for some } m \in \mathbb{N}$$

$$c(2m-1) = 3(2m-1) + 1 = 6m - 2 = 2(3n-1), \text{ which is even.}$$

$$P(k) \text{ has already been shown for even } k$$

$$\therefore P(k)$$

$P(1)$ and $P(n < k) \implies P(k)$

Hence, by complete induction, $P(n)\ \forall n \in \mathbb{N}$

Q.E.D