# Some famous conjectures relative to the consecutive primes

Reza Farhadian

farhadian.reza@yahoo.com

**Abstract**

***In this paper we offer the some details and particulars about some famous conjectures in relative to consecutive primes.***

**Key words:** Consecutive primes, Cramer's conjecture, Firoozbakht's conjecture, Andrica's conjecture, Granville's conjecture, Nicholson's conjecture, Farhadian's conjecture.

**2010 Mathematics Subject Classification :** primary 11A41; secondary 11N05

## 1. Some conjectures about consecutive primes

Given the following conjectures about consecutive primes:

1) **Cramer's conjecture (1936):** This conjecture presented by the Swedish mathematician Harald Cramer in 1936 [4]. By Cramer's conjecture, if $p_n$ denote the *n*th prime, then

$$p_{n+1} - p_n = O(log^2 p_n)$$

In the other word

$$\lim_{n\to\infty} sup \frac{p_{n+1} - p_n}{log^2 p_n} = 1$$

2) **Firoozbakht's conjecture (1982):** This conjecture presented by the Iranian mathematician Farideh Firoozbakht in 1982 [8]. By Firoozbakht's conjecture, if $p_n$ denote the *n*th prime, then

$$p_{n+1} \le p_n^{\frac{n+1}{n}}$$

In fact Firoozbakht's conjecture statement that the sequence $\left\{p_n^{\frac{1}{n}}\right\}_{n\ge1}$ is strictly decreasing. This conjecture is true for all primes up to $4 \times 10^{18}$ [7]. Firoozbakht's conjecture is stronger than Cramer's conjecture [4],[7]. Also if Firoozbakht's conjecture is true, then some other conjectures in the number theory such as Legendre's, Brocard's, Oppermann's, Forgues's conjectures are true [3], [10]. For more than details about Firoozbakht's conjecture, see [3], [6], [12].

3) **Andrica's conjecture (1986):** This conjecture presented by Dorin Andrica in 1986 [1]. By Andrica's conjecture if $p_n$ denote the *n*th prime, then

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 1$$

If Firoozbakht's conjecture is true, then Andrica's conjecture is true [10].

4) **Granville's conjecture (1995):** This conjecture presented by the British mathematician Andrew Granville in 1995 [11]. By Granville's conjecture, if $p_n$ denote the *n*th prime, then

$$p_{n+1} - p_n < R \times log^2 p_n \ , \ R \sim 1.23$$

The Granville's conjecture is weaker than Cramer's conjecture. So if Cramer's conjecture is true, then Granville's conjecture is true [14].

5) **Nicholson's conjecture (2013):** This conjecture presented by John w. Nicholson in 2013[13]. By Nicholson's conjecture, if $p_n$ denote the *n*th prime, then

$$\left(\frac{p_{n+1}}{p_n}\right)^n \leq n \, log \, n \ , \forall n > 4$$

The Nicholson's conjecture is stronger than Firoozbakht's conjecture [10, Theorem 4.4 ].

6) **Farhadian's conjecture (2016):** This conjecture presented by the Iranian statistician and mathematician Reza Farhadian (author this article) in 2016 [9]. By Farhadian's conjecture, if $p_n$ denote the *n*th prime, then for $n > 4$, we have

$$p_n^{\left(\frac{p_{n+1}}{p_n}\right)^n} \leq n^{p_n}$$

If Farhadian's conjecture is true, then Cramer's, Firoozbakht's, Granville's and Nicholson's conjectures are true [9].

## 2. Conjectural upper bound for gap between consecutive primes

Consider the conjectural upper bound for gap between consecutive primes by all conjectures in previous section at the following table.

| Conjecture | Conjectural upper bound for $p_{n+1} - p_n$ |
|---|---|
| Cramer | $O(log^2 p_n)$ |
| Firozbakht | $p_n^{\frac{n+1}{n}} - p_n, \forall n \geq 1$ |
| Andrica | $\left(1 + \sqrt{p_n}\right)^2 - p_n$ |
| Granville | $R \times (\log p_n)^2, \ \ R \sim 1.23$ |
| Nicholson | $p_n (n \log n)^{\frac{1}{n}} - p_n, \forall n > 4$ |
| Farhadian | $p_n \left(p_n \log_{p_n} n\right)^{\frac{1}{n}} - p_n, \forall n > 4$ |

**Table.** Conjectural upper bound for *n*th prime gaps.

Now, by plotting the charts of above conjectural upper bounds, we have

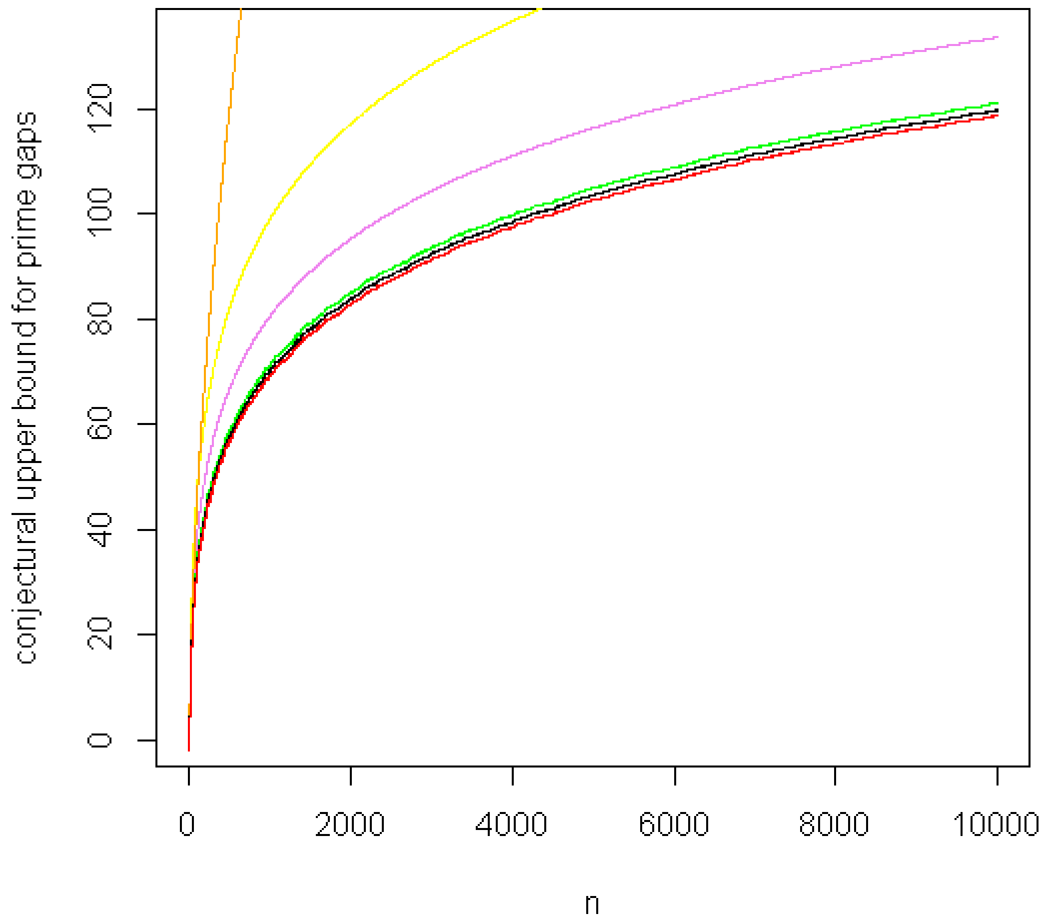


**Figure.** Graph of upper bound for gap between consecutive primes by Andrica's, *Granville's* , *Cramer's*, *Firoozbakht's*, *Nicholson's* and *Farhadian's* conjectures for first 10000 primes, without the actual real data of prime gaps.

As shown in above figure, the crump of upper bound for gap between consecutive primes by the Farhadian's conjecture is lower than other conjectures.Therefore the Farhadian's conjecture is sharper than other conjectures. Hence, by details from section 1, and also above figure, we have

$$\text{Farhadian's conjecture} \Rightarrow \text{Nicholson's conjecture} \Rightarrow \text{Firoozbakht's conjecture} \Rightarrow \begin{cases} \text{Cramer's conjecture} \\ \text{Andrica's conjecture} \\ \text{Granville's conjecture} \end{cases}$$

## References

[1] D. Andrica, *Note on a conjecture in prime number theory*, Studia Univ. Babes-Bolyai Math, 31(1986), 44-48.

[2] J. Barkley Rosser & L. Schoenfeld, *Approximates Formulas for Some Functions of Prime Numbers*, Illinois Journal Math. 6 (1962), 64-94.

[3] Octavian Cira, *Smarandache's Conjecture on Consecutive Primes*, International J.Math. Combin. Vol.4(2014), 69-91.

[4] H. Cram´er, *On the order of magnitude of the difference between consec-utive prime numbers*. Acta Arith. 2 (1936), 23-46.

[5] Pierre Dusart , *Estimate of some functions over primes without R.H*, arXiv:1002.0442v1, 2010.

[6] Alexei Kourbatov, *Upper Bounds for Prime Gaps Related to Firoozbakht's Conjecture,* Journal of Integer Sequences, Vol. 18 (2015).

[7] Alexei Kourbatov, *Verification of the Firoozbakht conjecture for primes up to four quintillion*, Int. Math. Forum,10 (2015), 283–288.

[8] P. Ribenboim, *The Little Book of Bigger Primes*, New York, Springer, 2004.

[9] C. Rivera, ed., Conjecture 78: $p_n^{\left(\frac{p_{n+1}}{p_n}\right)^n} \leq n^{p_n}$ , 2016. Available at http://www.primepuzzles.net/conjectures/.

[10] L.A. Ferreira, *some consequences of the Firoozbakht's conjecture*, arXiv:1604.03496v1,2016.

[11] A. Granville, *Harald Cramer and the distribution of prime numbers*, Scandanavian Actuarial J. 1 (1995), 12-28.

[12] Zhi-Wei Sun, *On a sequence involving sum of primes*, Bull. Aust. Math. Soc, 88(2013), no.2, 197-205.

[13] https://oeis.org/A182514

[14] https://en.wikipedia.org/wiki/firoozbakht%27s_%20conjecture