

Correlation of $-\cos \theta$ between measurements in a Bell's Inequality experiment simulation calculated using local hidden variables

*Austin J. Fearnley
Manchester
England
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Abstract

This paper shows in Simulation 2 that aggregates of elementary particles' hidden variable unit vector spin axes \mathbf{e} , when projected onto appropriate detector angle vectors, give values which are in exact accordance with values given by quantum mechanics calculations. However, it is found that no method of calculation breaks the Bell Inequality $AB' + BC' \geq AC'$: not the method in Simulation 2 and not the quantum mechanics calculation using projection operators.

Background: Spin in entangled pairs, hidden variables and magnets

Correlation $(\mathbf{a}, \mathbf{b}) = \cos \theta$ is a standard form for the correlation between two vectors differing in direction by angle θ , but nevertheless it is important to derive $-\cos \theta$ for quantum correlations as quantum correlations (Ref. 1).

The left-handed (LH) chiral electron has spin -0.5 while the right-handed (RH) chiral electron has spin $+0.5$. A LH electron can emit a photon with spin -1 and change form to become a RH electron. Spin is conserved by the interaction as the total spin before and after the interaction is -0.5 . Likewise, a RH electron can emit a photon with spin $+1$ and change form to become a LH electron. And similarly for LH and RH positrons. A chiral left-handedness indicates a difference in structure from the right-handed form. This is a different effect from the apparent handedness due only to the observer, which is helicity. A left-handed helicity for observer 1 may simultaneously take on a right-handed appearance for observer 2. Chiral 'left', however, is permanently chiral 'left'.

The spin of an electron is nominally about an axis. The orientation of the axis is always unknown in practice although it can be assumed to be known in a simulation of an experiment. For paired particle creation, say for an electron and positron, arising from an interaction where the incoming total spin was zero, then the outgoing particles have equal and opposite spins, summing to zero angular momentum. The axes are also the same for both particles: that is, treating the axes as vectors without the arrowheads or ignoring the signs of the axis vectors.

If a small, free-standing magnet is brought close to a powerful magnet, then it may precess and radiate away energy until it lines up with the direction of the stronger magnet. Electrons do not behave like this. Instead, if they do undergo an interaction, they emit one photon, as described above, and completely reverse chiral handedness. This can be repeated, for the same magnet orientation, but the electron is now stable and will not flip again. The electron may flip if brought near a differently oriented magnet, but the outcome is that the electron never changes the angle of its axis during the course of the experiment. The vector of the electron spin axis changes from \mathbf{e} to $-\mathbf{e}$ and back to \mathbf{e} in repeated interactions but instead of calling that a change in vector spin direction the electron is given an N or S pole label like the macroscopic magnet, and the label flips from N to S to N in successive interactions while the axis stays constant (ignoring the vector sign).

It needs to be illustrated more clearly what it means for an individual electron to receive a label N or S and to do this requires some discussion of randomness. There is no reason to assume that any one incoming electron has its spin axis pointing in any particular direction, although it is true [making the assumption that these hidden variables exist] that an incoming pair of particles will have the same spin axis and one will be an N and the other an S during their times of flight. In quantum mechanics, on the other hand, the pair of particles share a common entangled state of $0.707 |S\rangle + 0.707 |N\rangle$ until one of them interacts, which is an approach which eschews individual particles' hidden variables.

If two electrons are prepared pointing say north in the 2D space of the laboratory floor, this does not mean that their spin axes are parallel and it also does not mean that their axes are pointing exactly north. Their spin axes are unknown. Say for electron 1 an N label is put at one end of its spin axis and an S label at the other end and ditto for electron 2. To prepare the electrons, the electrons are interrogated each to see if it is the N label or the S label which is nearer the north wall of the laboratory. If the N label is to be nearer the North wall, it may require a switch of the N and S labels to achieve this. If the labels do need to be switched, then an 'interaction' is required to effect the switch. This corresponds to Alice or Bob making a measurement of $+1$. If no switch of labels is required, this is equivalent to Alice or Bob making no measurement. However, in practice, Stern-Gerlach detectors can circumvent this lack of a measurement and instead turn that into an actual interaction, recorded as -1 . So there is an interaction recorded for every particle with no inefficiency by wastage of information.

So now there are two electrons prepared with their labels appropriately adjusted with their N labels nearer to the North wall than their S labels. Say it is now required to simulate preparing the two electrons to point towards the West wall of the laboratory. Without changing their spin axes each particle is tested to flip, or not as necessary, their N and S labels so the two N labels are nearer the West wall than are the S labels. In practice this requires also putting a magnet against the West wall aligned in an East-West direction. As these two electrons are not entangled there is no expectation that both electrons will interact in the

same way. So even though both electrons were originally prepared pointing northwards, there is no knowing which one, if any, was already also pointing west. For two entangled electrons, if one electron needed a switch of labels to point West [or any chosen new direction], the other electron would not need a switch to point West. This is because entangled spin axes are parallel and oppositely labelled.

The two entangled particles each approach their respective detectors at one detector pole or the other: aimed with linear momenta at the N or the S poles. The linear momenta of the two particles are independent near the detectors, that is, their trajectories are not always opposite to one another. And that is despite any correlation in trajectories that they might have had on their creation. Changing the linear momenta of electrons has no effect on the absolute spin directions in 3D space as the particles approach near the magnets. A change in spin momentum could only occur at an interaction and then only by changing spin from +0.5 to -0.5 or vice versa in a single interaction. So all pairs of particles always retain exactly opposite spins within the pairs. Also, it doesn't matter which magnet pole a particle approaches: if a particular particle approaches a magnet's N pole with its own N pole facing, it will give a +1 reading. If the same particle had approached the magnet's S pole, it must have done so with its own S pole facing [because an electron never changes its spin alignment except at an interaction] and it would still give a +1 reading. Linear momenta are treated here as beyond the scope of this paper. Although charged particles change linear momenta to enter the earth's atmosphere giving rise to the northern and southern lights, the linear momenta are treated in this paper as irrelevant to the issue. Further, ideally the electrons would be treated as spinors rather than as vectors.

The question of what is the correlation between Alice's and Bob's measurements is addressed in the next section.

Simulation 1: Achieving a correlation of 0.707 for $\theta = 45^\circ$

A computer simulation has been carried out using hidden variables to calculate the correlation between Alice and Bob's A and B measurements. The programming used MS 2013 Excel Visual Basic. The program code is shown in Appendix A and a summary is given below.

PRELIMINARY WORK: set the dimensions of variables, their sizes and types, set counters to zero before use

GENERATE PARTICLE PAIRS IN A LOOP

PICK SPECIFIC VECTORS FOR DETECTORS a AND b AND PARTICLES e

PICK A POLARIZATION VECTOR for detector a (Alice)

PICK A POLARIZATION VECTOR for detector b (Bob)

PICK A POLARIZATION VECTOR for particle pair e.

CALCULATIONS OF A and B BASED ON ONE PARTICLE PAIR

INCREMENT THE A and B COUNTERS FOR THE iTH PAIR OF PARTICLES
 END OF LOOP
 WRITE RESULTS TO SPREADSHEET : 2 x 2 table with means, SDs, correlation and N

In this simulation, only one pair of angle settings of vectors **a** and **b** has been used:

a = (0 ,0, 1) and **b** = (0, 0.707 ,0.707) and the angle between **a** and **b** is 45 degrees. Vector **e** varies from pair to pair, chosen at random within a unit sphere, pointing out from the centre of the sphere and represents the particles.

Outputs from running the program is shown in Appendix A and also below.

When the raw integer measurements A and B are correlated, values of A and B represent the unit magnitudes of fuzzy vectors on a hemisphere, and the correlation is 0.5. When the integer values of A and B are taken as taken as initial values which can be manipulated to obtain fractional projections **a.e** and **b.e** onto exact vectors **a** and **b**, then the correlation is 0.707.

Correlation between fuzzy vectors on a hemisphere, A and B:

mean of A	-0.001264
mean of B	-0.0024
correl of A and B	0.499374804
Total N of pairs	1000000

Correlation between exact vectors, **a** and **b**:

mean of a.e	-5.54003E-05
mean of b.p	-0.000502231
correl of a and b	0.707000432
Total N of pairs	1000000

The correlation of 0.5 is what one would expect from classical correlations while 0.707 is what one would expect from quantum mechanical correlations for entangled particles with detectors angles different by 45°. (Ignore the sign of the correlation as the simulation used cloned particles rather than exactly opposite particles.)

Is it legitimate, however, to use fractional values **a.e** and **b.e**? In a real experiment, non-integer values cannot be measured by Alice and Bob whereas they can be calculated in a simulation. Since the non-integer values are giving the quantum correlation it is worth investigating why this is the case. The difference between the two correlations is that the classical correlation has undergone attenuation or reduction in absolute magnitude. To distinguish between the two correlations, 0.5 is a correlation between **ah** and **bh**, where h simply means hemisphere. The correlation 0.707 is between **av** and **bv** where the v denotes exact vector directions. For the 0.5 correlation, the A=1 measurements represent vectors, with unit magnitude, pointing from the centre of a sphere to anywhere within a particular

hemisphere. The fractional projection magnitudes, for the correlation of 0.707, give the accurate sizes of projections onto exact \mathbf{av} and \mathbf{bv} vectors. Using exact vectors allows greater absolute magnitudes of correlations.

Using 0.707 as the disattenuated value and 0.5 as the raw value of the correlation between Alice's and Bob's measurements, the reliability [Ref.2] of measurement of A (assumed to be the same as for B) can be calculated to be 0.7. So an integer measurement A on a hemisphere has a reliability of 0.7 when used as substitute for an unknown, but preferable, non-integer measurement A on an exact vector, but only in the case where the angle between the detectors is 45 degrees.

This leads to the question of why anyone would think that quantum correlations using simple arithmetical manipulation of raw dichotomous, integer values of A and B can give 0.707 for the circumstances in Simulation 1. Quantum correlations seem to be associated with projections onto exact vectors (\mathbf{av} and \mathbf{bv}) and NOT associated with raw integer unit measures connected with fuzzy vectors or vectors on a hemisphere, i.e. fuzzy vectors \mathbf{ah} and \mathbf{bh} .

It should be noted here that whenever a measurement is made, the energy change for the electron spin is always the constant $\hbar/2$. There is no doubt that electron spin change is quantised and likewise there is no doubt that Alice and Bob have no information with which to fractionalise their integer dichotomous measurements in a real experiment.

In the next section, an attempt is made to break Bell's Inequalities using hidden variables in a computer simulation.

Simulation 2: Breaking Bell's Inequalities?

The computer program in Appendix B is an adaptation of the program in Appendix A but amended to cater for the calculations required in a test of Bell's Inequalities. Two outputs are shown in Appendix B and below, one used aggregates of integer, dichotomous values of measurements A, B and C while the other output made use of aggregations of fractional projections derived from the $\mathbf{a.e}$, $\mathbf{b.e}$ and $\mathbf{c.e}$ calculations, where \mathbf{e} is the particle vector.

Details of this particular inequality are given in Susskind's online video lecture [Ref. 3]. An unofficial transcript of the lecture 5 notes is also available online [Ref. 4].

The results of Simulation 2 are shown below.

Using raw aggregates of integer measurements in Simulation 2:

	Count	Proportion
sum of B+ where A is also +	125363	0.125363
sum of C+ where A is also +	249377	0.249377

0.1254 + 0.1254 fails to be less than 0.2494 and so does not break the Bell Inequality:

$$AB' + BC' \geq AC'$$

Using aggregates of fractional projections of integer measurements in Simulation 2:

	Count	Proportion
sum of B+ where A is also +	36604.27625	0.036604276
sum of C+ where A is also +	124758.1586	0.124758159

0.0366 + 0.0366 is less than 0.1248 and so apparently breaks the Bell's Inequality as it gives an exact match with Susskind's quantum mechanical calculations with which he appears to show the breaking of a Bell's Inequality. There is a discrepancy, however, as Susskind calculated the value corresponding to AB' for particle-antiparticle pairs, that is, A & (not B) whereas in Simulation 2 the calculation was also AB' but for cloned particle pairs, so that is a mis-match between Simulation 2 and Susskind's calculations.

Next the details of Susskind's example are given. There are two researchers in the simulation: named First [say measuring particles] and Second [measuring the exactly opposite antiparticles], and there are three angles used for detector magnet settings: denoted $A = 0^\circ$, $B = 45^\circ$ and $C = 90^\circ$.

If a measurement is 1, then that is denoted by the labels A, B and C. If the measurement is -1, then that is denoted by the labels A' , B' and C' . Note that B' being -1 at 45° as measured by Second is equivalent to B being +1 at 45° as measured by First, using counterfactual measurements.

The three sets of compound measurements to be calculated are: AB' , BC' and AC' .

AB' denotes that 'First' measured 1 at angle 0° and 'Second' measured -1 at angle 45° on the particle and antiparticle pair.

And if 'Second' measured -1 at angle 45° on the antiparticle, then 'First' would have measured 1 at angle 45° on the particle.

So this compound measurement is the result of measuring particles at 0° and then their clones at 45° (counterfactual measurements) and seeing how many particles have +1 measurements on both detector settings.

BC' denotes that 'First' measured 1 at angle 45° and 'Second' measured -1 at angle 90° on a particle and antiparticle pair. This compound measurement is equivalent to the first compound measurement AB' but with everything turned around by 45 degrees. The outcome for BC' will therefore be identical to the outcome of AB', so only one of them needs to be simulated.

For the third compound measurement, AC' denotes that 'First' measured 1 at angle 0° and 'Second' measured -1 at angle 90° on a particle and antiparticle pair. And if 'Second' measured -1 at angle 90° then 'First' would have measured +1 at angle 90° counterfactually on cloned particles rather than on the paired partners. So this compound measurement, AC', is the result of measuring the same particles at 0° and then again at 90° (counterfactual measurements) and seeing how many particles have +1 measurements on both detector settings.

A Venn Diagram has been made for the results of these measurements and the areas in the diagram are listed in Table 1.

Table 1 **Data based on a Venn diagram data for a simulation of breaking a Bell Inequality [Ref. 4]**

<p>A Venn Diagram is constructed out of three overlapping circles A, B and C. Seven distinct non-overlapping areas can be labelled:</p> <p style="margin-left: 40px;">abc</p> <p style="margin-left: 40px;">ab (which excludes abc above)</p> <p style="margin-left: 40px;">ac (which excludes abc above)</p> <p style="margin-left: 40px;">bc (which excludes abc above)</p> <p style="margin-left: 40px;">a (which excludes all the above)</p> <p style="margin-left: 40px;">b (which excludes all the above)</p> <p style="margin-left: 40px;">c (which excludes all the above)</p> <p style="margin-left: 40px;">where:</p> <p style="margin-left: 40px;">circle A contains $a + ab + ac + abc$</p> <p style="margin-left: 40px;">circle B contains $b + ab + bc + abc$</p> <p style="margin-left: 40px;">circle C contains $c + ac + bc + abc$</p>

AB' corresponds to the area on the Venn Diagram given by $(a + ab + ac + abc) - (ab + abc)$
 $= a + ac$. This is the area within circle A which is not also contained in circle B.

BC' corresponds to the area on the Venn Diagram given by $(b + ab + bc + abc) - (bc + abc)$
 $= b + ab$. This is the area within circle B which is not also contained in circle C.

AC' corresponds to the area on the Venn Diagram given by $(a + ab + ac + abc) - (ac + abc)$

= $a + ab$. This is the area within circle A which is not also contained in circle C.

So an inequality: $a + ac + b + ab \geq a + ab$ can be made as the RHS quantity is subsumed within the LHS.

This gives $AB' + BC' \geq AC'$, and as AB' is equivalent to BC' then the inequality to break becomes: $2 AB' \geq AC'$.

Susskind used quantum mechanical calculations, using Projection Operators, to show that these three areas were:

$0.073 + 0.073$ is less than 0.25 and the inequality is broken.

In Simulation 2 shown in Appendix B the Inequality appears to be broken as $0.0366 + 0.0366$ is less than 0.1248 . Multiplying by a scale factor of 2 gives Susskind's results exactly but only in terms of numerical value as Simulation 2 is calculating AB' for cloned particles which is equivalent to calculating AB for particle-antiparticle pairs. Susskind on the other hand claimed to have calculated AB' on particle-antiparticle pairs

The close correspondence of Simulation 2 results and Susskind's calculations shows Simulation 2 using hidden variables is on the correct lines but first, an explanation of why Simulation 2 results differ from those of Susskind by a scale factor of 2. Take one set of measurements on Alice's detector for a bunch of randomly generated particles, where half of the particles register 1 and the other half register -1. In other words the proportion of +1 measurements is 0.5. For integer unit measurements on a hemisphere, the aggregate of all the +1 vectors is a simple count in the ah hemisphere, giving the proportion 0.5. But when the measurement made is a sum of projections of unit vectors onto an exact vector axis av , then the average loading on the av axis will be only half of the count made for the integer case as the average particle vector in the ah hemisphere only projects half way along the av vector, for the particles with $A = 1$. That accounts for the scale factor of 2. Simulation 2 proportions for breaking the inequality need to be scaled up by a factor of 2 to attain a unitary value for the total number of particles as a proportion.

In the Simulation 2 computer output for raw integer sums, the result for AC is 0.25 but for sums of fractional projections, this output value is halved, to 0.125 , which further demonstrates that a scaling factor of $x2$ is required to preserve a unitary total proportion.

Simulation 2 does not, therefore, break Bell's inequality and, further, Susskind's QM results also do not appear to break the inequality. First, the target proportion 0.073 found by Susskind using quantum mechanical calculations was intended to relate to a proportion for $A = 1$ and $B = -1$ for particle-antiparticle pairs whereas Simulation 2 found the same value 0.073 for $A = 1$ and $B = -1$ for cloned particle pairs, which is the incorrect type of particle pair for the $-\cos \theta$ quantum correlation (see Table 2(i)).

A comparison of the results for the two types of pairs is shown in Table 2. However if the value of 0.073 is inserted in Table 2 (ii) in the position where it must go for particle-antiparticle pairs then **0.073** must fit into the cell corresponding to A=1 and B=1. My re-positioning of Susskind's result does not break the Bell Inequality as the inequality $AB' + BC' \geq AC'$ requires the calculated value to be for cell (A, B) = (1, -1), not cell (1, 1).

Next, the correlations for Tables 2 are checked.

The formula for the correlation of a 2 x 2 table is:

$$\frac{(N_{++}) + (N_{--}) - (N_{+-}) - (N_{-+})}{(N_{++}) + (N_{--}) + (N_{+-}) + (N_{-+})}$$

Which gives

$$\frac{0.427 + 0.427 - 0.073 - 0.073}{0.427 + 0.427 + 0.073 + 0.073} = 0.707 \quad \text{for Table 2(i) using cloned pairs}$$

and

$$\frac{0.073 + 0.073 - 0.427 - 0.427}{0.073 + 0.073 + 0.427 + 0.427} = -0.707 \quad \text{for Table 2(ii) using particle-antiparticle pairs.}$$

This shows that Table 2 (ii) has been filled in correctly for Susskind's example, and as the quantum correlation for particle-antiparticle pairs is -0.707 for $\theta = 45^\circ$, and the value of 0.073 corresponds to the proportion for which A=1 and B=1 and this is not the appropriate cell for the breaking of the Bell Inequality.

Table 2 Quantum mechanics correlations, based on counts of fractional loadings on exact vectors a and b

(i) Cloned particles (for both A and B)

(ii) Particles & antiparticles (for A and B)

Simulation 2

Susskind's calculations

	A = 1	A = - 1	
B = 1	0.427	0.073	0.5
B = - 1	0.073	0.427	0.5
	0.5	0.5	1.0

Correlation between exact vectors **a** and **b** = 0.707

	A = 1	A = - 1	
B = 1	0.073?	0.427	0.5
B = - 1	0.427	0.073	0.5
	0.5	0.5	1.0

Correlation between exact vectors **a** and **b** = - 0.707

As a back-of-an-envelope check, the quantum mechanics calculation used by Susskind for 0.073 had as its final line the expression $0.25 * (1 - 0.5 * \sqrt{2})$. A similar expression can be derived using simple algebra and the constraints on a 2x2 symmetrical table, given the target correlation.

By symmetry,

$$N(+, +) = N(-, -),$$

$$N(+, -) = N(-, +), \text{ and}$$

$$N(+, +) + N(+, -) = 0.5$$

$$\text{Correlation in Table 2(ii)} = -0.707 =$$

$$\frac{(N_{++}) + (N_{--}) - (N_{+-}) - (N_{-+})}{(N_{++}) + (N_{--}) + (N_{+-}) + (N_{-+})} = \frac{(N_{++}) - (N_{+-})}{(N_{++}) + (N_{+-})} = \frac{(N_{++}) - \{0.5 - (N_{++})\}}{+0.5} = -0.707 = -0.5 * \sqrt{2}$$

$$\Rightarrow 2 * (N_{++}) - 0.5 = -0.5 * 0.5 * \sqrt{2}$$

$$\Rightarrow (N_{++}) = 0.25 * (1 - 0.5 * \sqrt{2})$$

$\Rightarrow (N_{++}) = 0.073$ just as in the quantum mechanics calculation but for the (+ +) cell not the (+ -) cell in Table 2(ii). This check of the result is a simple calculation based on the properties of a symmetric 2x2 table of proportions and does not rely in any way on hidden variables. This means that for a correlation of -0.707, any calculation of 0.073 must be for the (+ +) cell and it does not appear possible, therefore, that the Bell Inequality has been broken by anything in this case: not by Simulation 2 and not by quantum mechanics.

The proportions in Table 2 are based on fractional loadings of hidden, exact vectors of particles onto the exact vectors **a** and **b** of the detectors. Correlations in real experiments are based on the fuzzy vectors on a hemisphere, A and B, as the fractional loadings are unknown. The fuzzy correlations and proportions based on A and B are given in Table 3 and it is shown that the fuzzy correlations coefficients are attenuated to 0.5 in absolute magnitude.

Table 3 Attenuated correlations, based on integer counts of As and Bs

(i) Cloned particles (for both A and B)

	A = 1	A = - 1	
B = 1	0.375	0.125	0.5
B = - 1	0.125	0.375	0.5
	0.5	0.5	1.0

Correlation between fuzzy vectors
A and B = 0.5

(ii) Particles & antiparticles (for A and B)

	A = 1	A = - 1	
B = 1	0.125	0.375	0.5
B = - 1	0.375	0.125	0.5
	0.5	0.5	1.0

Correlation between fuzzy vectors
A and B = -0.5

Summary

Two simulations have been carried out in this paper. Simulation 1 gives the quantum mechanical correlation of 0.707 between $\mathbf{a}\cdot\mathbf{e}$ and $\mathbf{b}\cdot\mathbf{e}$ when Alice's and Bob's detectors are aligned at an angle of 45° to one another. To do this, the exact unit vectors (\mathbf{e}) of particle underlying the unit A and B measurements were generated. That allowed fractional projections of the unit vectors onto the exact detector vectors (\mathbf{a} and \mathbf{b}) to be calculated using their dot product. This result corresponded with the quantum mechanical value of the correlation, and also corresponds with the normal statistical definition of the correlation between two exact vectors to be $\cos \theta$. $\cos 45^\circ$ is approximately 0.707.

Simulation 2 used a similar method to Simulation 1 in that exact unit particle vectors were generated and fractional projections onto exact detector vectors were calculated. In this case, correlations were not calculated but instead fractional projections were accumulated to correspond to areas in a Venn Diagram which could be mapped into a Bell's Inequality calculation. The accumulated areas gave the exact values which were generated by quantum mechanical calculations [Refs. 3 and 4] but not for the same cells of Table 2(ii). It appears that the Bell Inequality ($AB' + BC' \geq AC'$) in this case was not broken in Simulation 2, and not by Susskind's calculations, and is not breakable by quantum mechanics when $\mathbf{a}=0^\circ$, $\mathbf{b}=45^\circ$, $\mathbf{c}=90^\circ$ and $\theta = 45^\circ$.

Simple back-of-an-envelope calculations in this paper, performed as a check and which have no connection whatsoever with hidden variables, bypass Simulation 2 calculations and Susskind's calculations and clearly show that for a symmetric, 2x2 table of proportions to have a correlation of -0.707, the value in the (+ +) cell MUST be 0.073. This is where the value of 0.073 is placed according to Simulation 2, but this is not in a cell which breaks Bell's inequality. It is mystifying why the value of 0.073 could be thought to break Bell's Inequality ($AB' + BC' \geq AC'$) when it cannot possibly be placed in the appropriate cell to do so, that is, cell (+ -).

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Austin Fearnley

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Manchester
England

APPENDIX A Excel Visual Basic program to calculate the correlation between Alice's and Bob's measurements; and an example of the output

Sub AJFModel1()

```

'-----
' Excel visual basic program to try to produce results for Bell curves: for theta = 45 deg
'-----
' Produces correlation 0.707 which exceeds sawtooth value of 0.5.
'-----
' Integer values of A and B taken as absolute magnitudes of fuzzy vectors on a hemisphere
' centred on vectors a and b ... gives sawtooth Bell curve
'-----
' Integer values of A and B fractionally projected onto exact vectors a and b ... gives disattenuated quantum correlation
'-----
'   Remove the following two lines from the body of the program to give the quantum value
'   a = Sgn(a) 'REMOVE THIS LINE TO GET qm CORRELATION 1ST OF 2
'   b = Sgn(b) 'REMOVE THIS LINE TO GET qm CORRELATION 2ND OF 2
'-----

'PRELIMINARY WORK:

' set the dimensions of variables, their sizes and types
Dim ithPair As Long 'ithPair is a long integer used as the index for the ith pair of particles
Dim TotalNoOfPairs As Long 'a long integer used to store the total number of pairs to be generated in the run of the program
Dim x, y, z, xa, ya, za, xb, yb, zb, xe, ye, ze, length As Double 'extra precise real numbers
Dim a, b, sa, sb, ssa, ssb, sab, sn, meanA, meanB, sda, sdb, corrAB, total As Double 'extra precise real numbers

' "Randomize" initializes the random-number generator, to avoid using the same set of random numbers in every run
Randomize

'define constants to be used in the program
TotalNoOfPairs = 100000 'this sets the total number of pairs of particles which are to be generated
DELTA0 = 0.000000000000001 'used to help to avoid dividing by zero
DELTA1 = 0.9999999999999999 'used to help to avoid dividing by zero
Pi = 3.1415926535 'pi radians is equivalent to an 180 degree angle

' set accumulator counters for means, sds, N and correlation to zero before use
sa = 0
sb = 0
ssa = 0
ssb = 0
sab = 0
sn = 0

'END OF PRELIMINARY WORK
'-----

'MAIN PART OF PROGRAM

' generate particle pairs in a For/Next loop
For ithPair = 1 To TotalNoOfPairs

' PICK SPECIFIC VECTORS FOR DETECTORS A AND B AND PARTICLES, E

' PICK A POLARIZATION VECTOR for detector a (Alice)
' Alice's unit vector has components xa, ya and za

        xa = 0
        ya = 0
        za = 1

```

' PICK A POLARIZATION VECTOR for detector b (Bob)
 ' Bob's unit vector has components xb, yb and zb

xb = 0
 yb = Sqr(2)/2
 zb = Sqr(2)/2

' PICK A POLARIZATION VECTOR for particle pair e.
 ' Note that the two particles have the same vector, e, so that we are aiming for result +cos (theta) not -cos(theta) *****

' The particles' vector has components xe, ye and ze.

'pick uniform random numbers x between -1 and +1
 3

x = Rnd * 2 - 1
 y = Rnd * 2 - 1
 z = Rnd * 2 - 1

length = x * x + y * y + z * z

If length < DELTA0 Then GoTo 3
 If length > DELTA1 Then GoTo 3

xe = x
 ye = y
 ze = z

length = Sqr(length) 'the partide vector is not unity, but lies within a units sphere

'CALCULATIONS BASED ON ONE PARTICLE PAIR

'calculation of a angle between stations a and b

x = xa * xb + ya * yb + za * zb
 If Abs(x) > DELTA1 Then GoTo 3
 theta = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1) ' excel's formula to give the arccos function
 'need now to turn theta (decimal in radians) into IntegerTheta (an integer angle in degrees)

'calculation of A

' a dot e = xa * xe + ya * ye + za * ze
 ' A = a dot e where a and e need to be unit vectors
 'here, a is a unit vector, but e has size = length, so
 a = (xa * xe + ya * ye + za * ze) / length ' length is magnititude of partide vector
 a = Sgn(a) 'REMOVE THIS LINE TO GET qm CORRELATION 1ST OF 2

'calculation of B

b = (xb * xe + yb * ye + zb * ze) / length
 b = Sgn(b) 'REMOVE THIS LINE TO GET qm CORRELATION 2ND OF 2

'INCREMENT the accumulators

sa = sa + a
 sb = sb + b
 ssa = ssa + a * a
 ssb = ssb + b * b
 sab = sab + a * b
 sn = sn + 1

10 Next ithPair ' jump to top of FOR/NEXT loop for generation of the next pair of particles

theta = theta * 180 / Pi

'END OF MAIN PART OF PROGRAM

'-----

' WRITE RESULTS TO SPREADSHEET : means, SDs, correlation and number of pairs of particles

'calculate means and correls etc

```
meanA = sa / sn
meanB = sb / sn
sdA = ssa / sn - meanA * meanA
sdA = Sqr(sdA)
sdB = ssb / sn - meanB * meanB
sdB = Sqr(sdB)
corrAB = sab / sn - meanA * meanB
corrAB = corrAB / (sdA * sdB)
total = sn
```

' set which row and column will be used as starting point to write results to spreadsheet

Range("a1").Select

```
ActiveCell.Offset(0, 1).Formula = "theta ="
ActiveCell.Offset(11, 2).Formula = "mean of A"
ActiveCell.Offset(12, 2).Formula = "mean of B"
ActiveCell.Offset(13, 2).Formula = "SD of A"
ActiveCell.Offset(14, 2).Formula = "SD of B"
ActiveCell.Offset(15, 2).Formula = "correl of A and B"
ActiveCell.Offset(16, 2).Formula = "Total N of pairs "
```

```
ActiveCell.Offset(0, 2).Formula = theta
```

```
' ActiveCell.Offset(9, 5).Formula = total 'total number of pairs of particles
```

```
ActiveCell.Offset(11, 3).Formula = meanA
ActiveCell.Offset(12, 3).Formula = meanB
ActiveCell.Offset(13, 3).Formula = sdA
ActiveCell.Offset(14, 3).Formula = sdB
ActiveCell.Offset(15, 3).Formula = corrAB
ActiveCell.Offset(16, 3).Formula = total
```

' NOW USER CAN GO TO RESULTS SPREADSHEET TO INSPECT 2X2 TABLE AND CORRELATION

'-----

End

End Sub

Example of output from the Excel Visual Basic program in Appendix A

theta = 45

Integer values of A and B taken as absolute magnitudes of **fuzzy** vectors on a hemisphere centred on vectors **a** and **b**

i.e. correlation between fuzzy vectors on a hemisphere **ah** and **bh**

mean of A	0.00024
mean of B	-0.00043
SD of A	0.999999971
SD of B	0.999999908
correl of A and B	0.499454164
Total N of pairs	1000000

Integer values of A and B fractionally projected onto **exact** vectors **a** and **b**

i.e. correlation between exact vectors **av** and **bv**

mean of A	0.000911218
mean of B	0.001414591
SD of A	0.577522836
SD of B	0.57779221
correl of fractional projections of A and B	0.707258632
Total N of pairs	1000000

APPENDIX B Excel Visual Basic program to reproduce Susskind's version of a Bell Inequality; and an example of the output

```
Sub AJFModel2()
```

```
'-----
' Excel visual basic program to break Bells Inequality
'-----
' a = Sgn(a)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 1 OF 3
' b = Sgn(b)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 2 OF 3
' c = Sgn(b)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 3 OF 3
'-----

'PRELIMINARY WORK:

' set the dimensions of variables, their sizes and types
Dim ithPair As Long ' ithPair is a long integer used as the index for the ith pair of particles
Dim TotalNoOfPairs As Long ' a long integer used to store the total number of pairs to be generated in the run of the program
Dim x, y, z, xa, ya, za, xb, yb, zb, xe, ye, ze, length As Double ' extra precise real numbers
Dim xc, yc, zc As Double ' extra precise real numbers
Dim a, b, c, sa, sb, sc, sn, total As Double ' extra precise real numbers

' "Randomize" initializes the random-number generator, to avoid using the same set of random numbers in every run
Randomize

'define constants to be used in the program
TotalNoOfPairs = 1000000 ' this sets the total number of pairs of particles which are to be generated
DELTA0 = 0.000000000000001 ' used to help to avoid dividing by zero
DELTA1 = 0.9999999999999999 ' used to help to avoid dividing by zero
Pi = 3.1415926535 ' pi radians is equivalent to an 180 degree angle

' set accumulator counters for means, sds, N and correlation to zero before use
sa = 0
sb = 0
sc = 0
sn = 0

'END OF PRELIMINARY WORK

'-----

'MAIN PART OF PROGRAM

' generate partide pairs in a For/Next loop
For ithPair = 1 To TotalNoOfPairs

' PICK SPECIFIC VECTORS FOR DETECTORS A AND B AND PARTICLES, E

' PICK A POLARIZATION VECTOR for detector a (Alice)
' Alice's unit vector has components xa, ya and za

    xa = 0
    ya = 0
    za = 1

' PICK A POLARIZATION VECTOR for detector b (Bob)
' Bob's unit vector has components xb, yb and zb

    xb = 0
    yb = -Sqr(2) / 2
    zb = -Sqr(2) / 2
```

' PICK A POLARIZATION VECTOR for detector c (Charlie)
 ' Charlie's unit vector has components xc, yc and zc

xc = 0
 yc = 1
 zc = 0

' PICK A POLARIZATION VECTOR for particle pair e.
 ' Note that the two particles have the same vector, e, so that we are aiming for result +cos(theta) not -cos(theta) *****

' The particles' vector has components xe, ye and ze.

'pick uniform random numbers x between -1 and +1
 3

x = Rnd * 2 - 1
 y = Rnd * 2 - 1
 z = Rnd * 2 - 1

length = x * x + y * y + z * z

If length < DELTA0 Then GoTo 3
 If length > DELTA1 Then GoTo 3

xe = x
 ye = y
 ze = z

length = Sqr(length) 'the particle vector is not unity, but lies within a unit sphere

'CALCULATIONS BASED ON ONE PARTICLE PAIR

'calculation of angle between stations a and b

x = xa * xb + ya * yb + za * zb
 If Abs(x) > DELTA1 Then GoTo 3
 theta = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1) ' excel's formula to give the arccos function
 'need now to turn theta (decimal in radians) into IntegerTheta (an integer angle in degrees)

'calculation of A

' a dot e = xa * xe + ya * ye + za * ze
 ' A = a dot e where a and e need to be unit vectors
 'here, a is a unit vector, but e has size = length, so
 a = (xa * xe + ya * ye + za * ze) / length ' length is magnitude of particle vector
 'a = Sgn(a) 'REMOVE THIS LINE TO GET QM CORRELATIONS 1ST OF 3

If a < 0 Then GoTo 10

'calculation of B

b = (xb * xe + yb * ye + zb * ze) / length
 'b = Sgn(b) 'REMOVE THIS LINE TO GET QM CORRELATIONS 2ND OF 3

'INCREMENT the accumulators

sa = sa + a
 If b < 0 Then GoTo 110
 sb = sb + b

110 'calculation of C

c = (xc * xe + yc * ye + zc * ze) / length
 'c = Sgn(c) 'REMOVE THIS LINE TO GET QM CORRELATIONS 3RD OF 3

If c < 0 Then GoTo 10
 sc = sc + c

10 Next ithPair ' jump to top of FOR/NEXT loop for generation of the next pair of particles

theta = theta * 180 / Pi

'END OF MAIN PART OF PROGRAM

'-----

' WRITE RESULTS TO SPREADSHEET :

' set which row and column will be used as starting point to write results to spreadsheet
Range("a1").Select

ActiveCell.Offset(11, 2).Formula = "sum of A+"
ActiveCell.Offset(12, 2).Formula = "sum of B+ where A is also +"
ActiveCell.Offset(13, 2).Formula = "sum of C+ where A is also +"

ActiveCell.Offset(16, 2).Formula = "Total N of pairs"

ActiveCell.Offset(11, 3).Formula = sa
ActiveCell.Offset(12, 3).Formula = sb
ActiveCell.Offset(13, 3).Formula = sc

ActiveCell.Offset(16, 3).Formula = TotalNoOfPairs
ActiveCell.Offset(11, 5).Formula = sa / TotalNoOfPairs
ActiveCell.Offset(12, 5).Formula = sb / TotalNoOfPairs
ActiveCell.Offset(13, 5).Formula = sc / TotalNoOfPairs
ActiveCell.Offset(9, 3).Formula = "sum"
ActiveCell.Offset(9, 5).Formula = "Proportion"

' NOW USER CAN GO TO RESULTS SPREADSHEET TO INSPECT 2X2 TABLE AND CORRELATION

'-----

End
End Sub

Example of output from the Excel Visual Basic program in Appendix B

Integer values of A, B and C taken as integer magnitudes of fuzzy vectors on a hemisphere centred on vectors **a**, **b** and **c**

i.e. sums of fuzzy vectors on a hemisphere

sum of A+	500143	0.500143
sum of B+ where A is also +	125318	0.125318
sum of C+ where A is also +	249611	0.249611

Integer values of A, B and C fractionally projected onto exact vectors **a**, **b** and **c**

i.e. sums of exact vectors with fractional magnitudes

sum of A+	250942.5605	0.25094256
sum of B+ where A is also +	36604.27625	0.036604276
sum of C+ where A is also +	124758.1586	0.124758159