

Correlation of $-\cos \theta$ between measurements in a Bell's Inequality experiment simulation calculated using local hidden variables

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Abstract

This paper shows that the theoretical correlation between elementary particles' hidden variable unit vector spin axes \mathbf{p} , projected onto Alice's and Bob's respective detector angle unit vectors \mathbf{a} and \mathbf{b} , in a Bell's Inequality experiment, is $-\cos \theta$. This equates with the quantum correlation value and exceeds the "Bell's Inequality" attenuated correlation in absolute magnitude. Further, aggregates of elementary particles' hidden variable unit vector spin axes \mathbf{p} , when projected onto appropriate detector angle vectors, give values which break the Bell's Inequalities in exact accordance with values given by Quantum Mechanics calculations. On the other hand, Bell's attenuated correlations correspond to correlations calculated without fractionalising the raw integer measurements A and B made by Alice and Bob. Also, when aggregating the raw integer measurements, the Bell's Inequalities are not broken.

Background: Spin in entangled pairs, local hidden variables and magnets

Correlation $(\mathbf{a}, \mathbf{b}) = -\cos \theta$ is a standard form for the correlation between two vectors differing in direction by angle θ , but nevertheless it is important to derive it for quantum correlations as quantum correlations are associated with spooky non-real or non-local effects (Ref. 1).

The left-handed (LH) chiral electron has spin - 0.5 while the right-handed (RH) chiral electron has spin + 0.5. A LH electron can emit a photon with spin -1 and change form to become a RH electron. Spin is conserved by the interaction as the total spin before and after the interaction is -0.5. Likewise, a RH electron can emit a photon with spin +1 and change form to become a LH electron. And similarly for LH and RH positrons. A chiral left-handedness indicates a difference in structure from the right-handed form. This is a different effect from the apparent handedness due only to the observer, which is helicity. A left-handed helicity for observer 1 may simultaneously take on a right-handed appearance for observer 2. Chiral 'left' is permanently chiral 'left', although observers observe only a particle's helicity with the chirality being hidden or inferred.

The spin of an electron is nominally about an axis. The orientation of the axis is always unknown in practice although it can be assumed to be known in a simulation of an experiment. For paired particle creation, say for an electron and positron, arising from an interaction where the incoming total spin was zero, then the outgoing particles have equal and opposite spins, summing to zero angular momentum. The axes are also the same for both particles: that is, treating the axes as vectors without the arrowheads or ignoring the signs of the axis vectors.

If a small, free-standing magnet is brought close to a powerful magnet, then it may precess and radiate away energy until it lines up with the direction of the stronger magnet. Electrons do not behave like this. Instead, if they do undergo an interaction, they emit one photon, as described above, and completely reverse chiral handedness. This can be repeated, for the same magnet orientation, but the electron is now stable and will not flip again. The electron may flip if brought near a differently oriented magnet, but the outcome is that the electron never changes the angle of its axis during the course of the experiment. The vector of the electron spin axis changes from \mathbf{p} to $-\mathbf{p}$ and back to \mathbf{p} in repeated interactions but instead of calling that a change in vector spin direction the electron is given an N or S pole label like the macroscopic magnet, and the label flips from N to S to N in successive interactions while the axis stays constant (ignoring the vector sign).

It needs to be illustrated more clearly what it means for an individual electron to receive a label N or S and to do this requires some discussion of randomness. There is no reason to assume that any one incoming electron has its spin axis pointing in any particular direction, although it is true [making the assumption that these local hidden variables exist] that an incoming pair of particles will have the same spin axis and one will be an N and the other an S during their times of flight. In quantum mechanics, on the other hand, the pair of particles share a common entangled state of $0.707 |S\rangle + 0.707 |N\rangle$ until one of them interacts, which is an approach which eschews individual particles' local hidden variables.

If two electrons are prepared pointing say north in the 2D space of the laboratory floor, this does not mean that their spin axes are parallel and it also does not mean that their axes are pointing exactly north. Their spin axes are unknown. Say for electron 1 an N label is put at one end of its spin axis and an S label at the other end. Ditto for electron 2. To prepare the electrons, the electrons are interrogated each to see if it is the N label or the S label which is nearer the north wall of the laboratory. If the N label is to be nearer the North wall, it may require a switch of the N and S labels to achieve this. If the labels do need to be switched, then an 'interaction' is required to effect the switch. This corresponds to Alice or Bob making a measurement of +1. If no switch of labels is required, this is equivalent to Alice or Bob making no measurement. However, in practice, Stern-Gerlach detectors can circumvent this lack of a measurement and instead turn that into an actual interaction, recorded as -1. So there is an interaction recorded for every particle with no inefficiency by wastage of information.

So now there are two electrons prepared with their labels appropriately adjusted with their N labels nearer to the North wall than their S labels. Say it is now required to simulate preparing the two electrons to point towards the West wall of the laboratory. Without changing their spin axes each particle is tested to flip, or not as necessary, their N and S labels so the two N labels are nearer the West wall than are the S labels. In practice this requires also putting a magnet against the West wall aligned in an East-West direction. As these two electrons are not entangled there is no expectation that both electrons will interact in the same way. So even though both electrons were originally prepared pointing northwards, there is no knowing which one, if any, was already also pointing west. For two entangled electrons, if one electron needed a switch of labels to point West [or any chosen new direction], the other electron would not need a switch to point West. This is because entangled spin axes are parallel and oppositely labelled.

The two entangled particles each approach their respective detectors at one detector pole or the other: aimed with linear momenta at the N or the S poles. The linear momenta of the two particles are independent near the detectors, that is, their trajectories are not always opposite to one another. And that is despite any correlation in trajectories that they might have had on their creation. Changing the linear momenta of electrons has no effect on the absolute spin directions in 3D space as the particles approach near the magnets. A change in spin momentum could only occur at an interaction and then only by changing spin from +0.5 to -0.5 or vice versa in a single interaction. So all pairs of particles always retain exactly opposite spins within the pairs. Also, it doesn't matter which magnet pole a particle approaches: if a particular particle approaches a magnet's N pole with its own N pole facing, it will give a +1 reading. If the same particle had approached the magnet's S pole, it must have done so with its own S pole facing [because an electron never changes its spin alignment except at an interaction] and it would still give a +1 reading. Linear momenta are treated here as beyond the scope of this paper. Although charged particles change linear momenta to enter the earth's atmosphere giving rise to the northern and southern lights, the linear momenta are treated in this paper as irrelevant to the issue. Further, ideally the electrons would be treated as spinors rather than as vectors.

The question of what is the correlation between Alice's and Bob's measurements is addressed in the next section.

Achieving a correlation of 0.707 for $\theta = 45^\circ$: Simulation 1

A computer simulation has been carried out using local, realistic, hidden variables to calculate the correlation between Alice and Bob's A and B measurements. The programming used MS 2013 Excel Visual Basic. The program code is shown in Appendix A and a summary is given below.

PRELIMINARY WORK: set the dimensions of variables, their sizes and types, set counters to zero before use

GENERATE PARTICLE PAIRS IN A LOOP

PICK SPECIFIC VECTORS FOR DETECTORS **a** AND **b** AND PARTICLES **e**

PICK A POLARIZATION VECTOR for detector **a** (Alice)

PICK A POLARIZATION VECTOR for detector **b** (Bob)

PICK A POLARIZATION VECTOR for particle pair **e**.

CALCULATIONS OF **A** and **B** BASED ON ONE PARTICLE PAIR

INCREMENT THE **A** and **B** COUNTERS FOR THE **i**TH PAIR OF PARTICLES

END OF LOOP

WRITE RESULTS TO SPREADSHEET : 2 x 2 table with means, SDs, correlation and N

In this simulation, only one pair of angle settings of vectors **a** and **b** has been used:

a = (0 ,0, 1) and **b** = (0, 0.707 ,0.707) and the angle between **a** and **b** is 45 degrees. Vector **e** varies from pair to pair, chosen at random within a unit sphere, pointing out from the centre of the sphere.

Two outputs from running the program are also shown in Appendix A and below.

When the raw integer measurements **A** and **B** are correlated, values of **A** and **B** represent the unit magnitudes of fuzzy vectors on a hemisphere, and the correlation is 0.5. When the integer values of **A** and **B** are taken as taken as initial values which can be manipulated to obtain fractional projections onto exact vectors **a** and **b**, then the correlation is 0.707.

Correlation between fuzzy vectors on a hemisphere

mean of A	-0.001264
mean of B	-0.0024
correl of A and B	0.499374804
Total N of pairs	1000000

Correlation between exact vectors, **a** and **b**

mean of A	-5.54003E-05
mean of B	-0.000502231
correl of a and b	0.707000432
Total N of pairs	1000000

The correlation of 0.5 is what one would expect from classical correlations while 0.707 is what one would expect from quantum mechanical correlations for entangled particles with detectors angles different by 45°. (Ignore the sign of the correlation as the simulation used cloned particles rather than exactly opposite particles.)

Is it legitimate, however, to use non-integer values derived from integer measurements **A** and **B**? In a real experiment, non-integer values cannot be measured by Alice and Bob whereas they can be calculated in a simulation. Since the non-integer values are giving the quantum

correlation it is worth investigating why this is the case. The difference between the two correlations is that the classical correlation has undergone attenuation or reduction in absolute magnitude. To distinguish between the two correlations, 0.5 is a correlation between \mathbf{a}_h and \mathbf{b}_h , where h simply means hemisphere. The correlation 0.707 is between \mathbf{a}_v and \mathbf{b}_v where the v denotes exact vector directions. For the 0.5 correlation, the $A=1$ measurements represent vectors, with unit magnitude, pointing from the centre of a sphere to anywhere within a particular hemisphere. The fractional projection magnitudes, for the correlation of 0.707, give the accurate sizes of projections onto an exact \mathbf{a}_v vector. Using exact vectors allows greater correlations.

Using 0.707 as the disattenuated value and 0.5 as the raw value of the correlation between Alice's and Bob's measurements, the reliability [Ref.2] of measurement of A (assumed to be the same as for B) can be calculated to be 0.7. So an integer measurement A on a hemisphere has a reliability of 0.7 when used as substitute for an unknown, but preferable, non-integer measurement A on an exact vector, but only in the case where the angle between the detectors is 45 degrees.

This leads to the question of why anyone would think that quantum correlations using simple arithmetical manipulation of raw dichotomous, integer values of A and B can give 0.707 for the circumstances in Simulation 1. Quantum correlations seem to be associated with projections onto exact vectors (\mathbf{a}_v and \mathbf{b}_v) and NOT associated with raw integer unit measures connected with fuzzy vectors or vectors on a hemisphere, i.e. fuzzy vectors \mathbf{a}_h and \mathbf{b}_h .

It should be noted here that whenever a measurement is made, the energy change for the electron spin is always the constant $\hbar/2$. There is no doubt that electron spin change is quantised and likewise there is no doubt that Alice and Bob have no information with which to fractionalise their integer dichotomous measurements in a real experiment. Moreover, quantum mechanical calculations are famed for their statistical accuracy and this paper is not querying that well-established fact.

In the next section, a successful attempt is made to break Bell's Inequalities using local, realistic, hidden variables in a computer simulation.

Breaking Bell's Inequalities: Simulation 2

The computer program in Appendix B is an adaptation of the program in Appendix A but amended to cater for the calculations required in a test of Bell's Inequalities. Two outputs are shown in Appendix B and below, one used aggregates of integer, dichotomous values of measurements A, B and C while the other output made use of aggregations of non-integer projections derived from the measurements of A, B and C.

In a manner very similar to that for Appendix A, the use of raw aggregates of integer measurements did not break the Bell Inequality whereas the aggregates of non-integer projections did break Bell's Inequalities, and further gave an exact match with quantum mechanical calculations.

Details of this particular inequality are based on a Susskind's online video lecture [Ref. 3]. One person's transcript of the lecture 5 notes are also available online [Ref. 4].

Using raw aggregates of integer measurements in Simulation 2:

	Count	Proportion
sum of B+ where A is also +	125363	0.125363
sum of C+ where A is also +	249377	0.249377

$0.1254 + 0.1254$ fails to be less than 0.2494 and so does **not** break a Bell Inequality.

Using aggregates of non-integer projections of integer measurements in Simulation 2:

	Count	Proportion
sum of B+ where A is also +	36604.27625	0.036604276
sum of C+ where A is also +	124758.1586	0.124758159

$0.0366 + 0.0366$ is less than 0.1248 and so **does** break a Bell's Inequality.

Not only does this latter output of the simulation break a Bell's Inequality, it gives a very good estimate of the actual quantum mechanical calculation.

To show this needs more details of the simulation, which is based on a Susskind's online video lecture [Ref. 3 and 4].

There are two researchers in the simulation: named First [say measuring particles] and Second [measuring the exactly opposite antiparticles], and there are three angles used for detector magnet settings: denoted $A = 0^\circ$, $B = 45^\circ$ and $C = 90^\circ$.

If a measurement is 1, then that is denoted by the labels A, B and C. If the measurement is -1, then that is denoted by the labels A', B' and C'. Note that B' being -1 at 45° is equivalent to B being +1 at 45° using counterfactual measurements.

The three sets of compound measurements to be calculated are: AB', BC' and AC'.

AB' denotes that 'First' measured 1 at angle 0° and 'Second' measured -1 at angle 45° on the pair partner particles.

And if 'Second' measured -1 at angle 45° then 'First' would have measured 1 at angle 45° .

So this compound measurement is the result of measuring particles at 0° and then their clones at 45° (counterfactual measurements) and seeing how many particles have +1 measurements on both detector settings.

BC' denotes that 'First' measured 1 at angle 45° and 'Second' counterfactually measured -1 at angle 90° .

This is equivalent to measuring 1 at angle 45° and measuring +1 at angle 90° counterfactually on the same or cloned particles. This compound measurement is equivalent to the first compound measurement AB' but with everything spun around by 45 degrees. The outcome for BC' will therefore be identical to the outcome of AB', so only one of them needs to be simulated.

For the third compound measurement, AC' denotes that 'First' measured 1 at angle 0° and 'Second' measured -1 at angle 90° on the pair partner particles.

And if 'Second' measured -1 at angle 90° then 'First' would have measured +1 at angle 90° counterfactually on cloned particles rather than on the paired partners.

So this compound measurement, AC', is the result of measuring the same particles at 0° and then again at 90° (counterfactual measurements) and seeing how many particles have +1 measurements on both detector settings.

A Venn Diagram has been made for the results of these measurements and the areas in the diagram are listed in Table 1.

Table 1 **Data based on a Venn diagram data for a simulation of breaking a Bell Inequality [Ref. 4]**

<p>A Venn Diagram is constructed out of three overlapping circles A, B and C. Seven distinct non-overlapping areas can be labelled:</p> <p style="margin-left: 40px;">abc</p> <p style="margin-left: 40px;">ab (which excludes abc above)</p> <p style="margin-left: 40px;">ac (which excludes abc above)</p> <p style="margin-left: 40px;">bc (which excludes abc above)</p> <p style="margin-left: 40px;">a (which excludes all the above)</p> <p style="margin-left: 40px;">b (which excludes all the above)</p> <p style="margin-left: 40px;">c (which excludes all the above)</p> <p style="margin-left: 40px;">where:</p> <p style="margin-left: 40px;">circle A contains $a + ab + ac + abc$</p> <p style="margin-left: 40px;">circle B contains $b + ab + bc + abc$</p> <p style="margin-left: 40px;">circle C contains $c + ac + bc + abc$</p>
--

AB' corresponds to the area on the Venn Diagram given by $(a + ab + ac + abc) - (ab + abc)$

$= a + ac$. This is the area within circle A which is not also contained in circle B.

BC' corresponds to the area on the Venn Diagram given by $(b + ab + bc + abc) - (bc + abc)$

$= b + ab$. This is the area within circle B which is not also contained in circle C.

AC' corresponds to the area on the Venn Diagram given by $(a + ab + ac + abc) - (ac + abc)$

$= a + ab$. This is the area within circle A which is not also contained in circle C.

So we can make an inequality: $a + ac + b + ab \geq a + ab$ as the RHS quantity is subsumed within the LHS.

This gives $AB' + BC' \geq AC'$, and as AB' is equivalent to BC' then the inequality to break becomes: $2 AB' \geq AC'$.

In the computer simulation shown in Appendix B this inequality **is broken** as

$0.0366 + 0.0366$ is less than 0.1248 .

Susskind used quantum mechanical calculations, using Projection Operators, to show that these three areas were:

$0.073 + 0.073$ is less than 0.25 and the inequality **is broken**.

The close correspondence of my simulation results and Susskind's calculations shows my computer modelling of the simulation to be on the correct lines but first, an explanation of why my results differ from those of Susskind by a scale factor of 2. Take one set of measurements on Alice's detector for a bunch of randomly generated particles, where half of the particles register 1 and the other half register -1. In other words the proportion of +1 measurements is 0.5. For integer unit measurements on a hemisphere, the aggregate of all the +1 vectors is a simple count in the ah hemisphere, giving the proportion 0.5. But when the measurement made is a sum of projections of unit vectors onto an exact vector axis \mathbf{av} , then the average loading on the \mathbf{av} axis will be only half of the count made for the integer case as the average particle vector in the ah hemisphere only projects half way along the \mathbf{av} vector, for the particles with $A = 1$. That accounts for the scale factor of 2. Simulation 2 proportions for breaking the inequality need to be scaled up by a factor of 2 to attain a unitary value for the total number of particles as a proportion.

In the computer Simulation 2 summing raw integer measurements, the result is also 0.25. For the computer Simulation 2 aggregating non-integer projections, this output value is halved (to 0.125) and should be scaled by a factor of x2 to preserve a unitary total proportion.

Summary and conclusions

Two simulations have been carried out in this paper. Simulation 1 gives the Quantum Mechanical correlation of 0.707 between Alice's and Bob's measurements when their detectors are aligned at an angle of 45° to one another. To do this, the exact unit vectors of particle underlying the unit A and B measurements were generated. That allowed fractional projections of the unit vectors onto the exact detector vectors to be calculated. This result corresponded with the Quantum Mechanical value of the correlation, and also corresponds with the normal statistical definition of the correlation between two exact vectors to be $\cos \theta$. $\cos 45^\circ$ is approximately 0.707.

Simulation 2 used a similar method to Simulation 1 in that exact unit particle vectors were generated and fractional projections onto exact detector vectors were calculated. In this case, correlations were not calculated but instead fractional projections were accumulated to correspond to areas in a Venn Diagram which could be mapped into a Bell's Inequality calculation. The accumulated areas broke the Bell's Inequality giving the exact values which were generated by Quantum Mechanical calculations [Refs. 3 and 4]. This shows that QM calculations, not surprisingly as they use Projection Operators, are deriving aggregates of fractional projections, but doing so without knowing the individual underlying exact unit vectors of particles.

The conclusion is that (1) QM correlations use fractional projections to disattenuate the Bell limits on correlation and (2) Bell's Inequalities are also broken by use of fractional projections of underlying or hidden exact unit vectors of particles onto exact vectors of detectors.

These two points come together to show that QM effects are achieved by estimating what the disattenuated correlations are between exact vectors.

The attenuated correlation of 0.5 in Simulation 1 was achieved by a simple correlation of raw measures of A and B without any fractionalisation. Measurements A and B are the magnitudes of unit vectors of particles pointing in a hemisphere centred on a detector vector, so throwing away most of the information about direction by compactifying that data into a + or - sign and leaving the magnitude at unity. So A and B are basically the quantised measurements of a compactified system corresponding to a fuzzy vector about the detector angle. With the fuzziness extending to cover a hemisphere. It is no surprise, therefore, that the correlation for the fuzzy vectors is attenuated from 0.707 to 0.5.

In Simulation 2, when the raw unit magnitudes of A and B were aggregated, Bell's Inequality was not broken. So there is a clear relationship between the simple adding of raw measurements of A and B and the failure to achieve the QM correlation and likewise the failure to break a Bell's Inequality.

Another conclusion is that the quantum mechanical calculations which break Bell's Inequality are amazingly good, yet are clearly not spooky. What is weird is why anyone should think that a quantum correlation can be achieved by a simple correlation of raw A and B measurements. Why was that search ever initiated?

The results in this paper are consistent with the non-existence of a correlation of 0.707, for $\theta = 45^\circ$, between fuzzy vectors on a hemisphere and, likewise, the non-existence of a correlation of 0.707 between measurements A and B, using only simple arithmetic as in the CHSH statistic.

Next steps

The latest experiment to measure the CHSH statistic, intended to be loophole free [Ref. 6, 2015], used 245 trials, or pairs of particles, to obtain a statistic which has a p value of 0.039. The CHSH statistic is $S = 2.42 \pm 0.20$. To give an approximate equivalence with the simulations in this paper simply divide S by 4, so $S=2$ corresponds to a correlation of 0.5 while an $S=2.828$ corresponds to a correlation of 0.707.

The significance level of the real CHSH experiment may not necessarily be meaningful based on a small number of particles. However, the experiment's report notes the great pains that

were taken to avoid bias: *“We employ an event-ready scheme that enables the generation of high-fidelity entanglement between distant electron spins. Efficient spin readout avoids the fair sampling assumption (detection loophole) ...”*.

Some possibilities are that ...

- (a) Maybe unconscious human bias has existed in experiments to implement the CHSH experimental design in the long drive towards finding what may be non-existent quantum correlations on **fuzzy** vectors that are, however, widely assumed to exist. Note that Simulation 1 has shown a genuine quantum correlation between **exact** vectors using local and real hidden variables, but contrariwise not between fuzzy vectors on a hemisphere. Note that Simulation 2 has accurately replicated, for **exact** vector loadings and local hidden variables, the quantum mechanical calculations required to break a Bell’s Inequality; and contrariwise not for fuzzy vector aggregates.
- b) Despite Simulations 1 and 2 in this paper, and the Bell Inequality, indicating that the CHSH statistic, appropriate to fuzzy vectors, should not exceed 2 unless weird effects are occurring, maybe in a real experiment truly weird factors are at work.
 - (i) McEachern (Ref. 8) argues that controlling artificially generated noise, in a computer simulation, can enable the disattenuated quantum correlation to be obtained for measurements on single memory bits.
 - (ii) Christian (Ref. 7) controversially argues, that the disattenuated quantum correlation may be obtained with local and real conditions under the assumption that space has a constant spatial curvature (S^3).
- (c) Maybe electrons do have non-local effects which, somehow, weirdly lead to disattenuated correlations between fuzzy vectors. There is no guarantee that an electron is physically located at a point in between interactions especially if electrons may speculatively be suggested to be composed of sub-particles. (Ref. 9)
- (d) Space may have compactified extra dimensions [for example, in string theory] which may allow apparently non-local effects when observed in three dimensions (Ref. 9).

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APPENDIX A Excel Visual Basic program to calculate the correlation between Alice's and Bob's measurements; and an example of the output

Sub AJFModel1()

```

'-----
' Excel visual basic program to try to produce results for Bell curves: for theta = 45 deg
'-----
' Produces correlation 0.707 which exceeds sawtooth value of 0.5.
'-----
'Integer values of A and B taken as absolute magnitudes of fuzzy vectors on a hemisphere
'centred on vectors a and b ... gives sawtooth Bell curve
'-----
'Integer values of A and B fractionally projected onto exact vectors a and b ... gives disattenuated quantum correlation
'-----
' Remove the following two lines from the body of the program to give the quantum value
' a = Sgn(a) 'REMOVE THIS LINE TO GET qm CORRELATION 1ST OF 2
' b = Sgn(b) 'REMOVE THIS LINE TO GET qm CORRELATION 2ND OF 2
'-----

'PRELIMINARY WORK:

' set the dimensions of variables, their sizes and types
Dim ithPair As Long ' ithPair is a long integer used as the index for the ith pair of particles
Dim TotalNoOfPairs As Long ' a long integer used to store the total number of pairs to be generated in the run of the program
Dim x, y, z, xa, ya, za, xb, yb, zb, xe, ye, ze, length As Double ' extra precise real numbers
Dim a, b, sa, sb, ssa, ssb, sab, sn, meanA, meanB, sdA, sdB, corrAB, total As Double ' extra precise real numbers

' "Randomize" initializes the random-number generator, to avoid using the same set of random numbers in every run
Randomize

'define constants to be used in the program
TotalNoOfPairs = 1000000 ' this sets the total number of pairs of particles which are to be generated
DELTA0 = 0.000000000000001 ' used to help to avoid dividing by zero
DELTA1 = 0.999999999999999 ' used to help to avoid dividing by zero
Pi = 3.1415926535 ' pi radians is equivalent to an 180 degree angle

' set accumulator counters for means, sds, N and correlation to zero before use
sa = 0
sb = 0
ssa = 0
ssb = 0
sab = 0
sn = 0

'END OF PRELIMINARY WORK
'-----

'MAIN PART OF PROGRAM

' generate particle pairs in a For/Next loop
For ithPair = 1 To TotalNoOfPairs

' PICK SPECIFIC VECTORS FOR DETECTORS A AND B AND PARTICLES, E

' PICK A POLARIZATION VECTOR for detector a (Alice)
' Alice's unit vector has components xa, ya and za

xa = 0
ya = 0

```

```
za = 1
```

```
' PICK A POLARIZATION VECTOR for detector b (Bob)
' Bob's unit vector has components xb, yb and zb
```

```
xb = 0
yb = Sqr(2) / 2
zb = Sqr(2) / 2
```

```
' PICK A POLARIZATION VECTOR for particle pair e.
' Note that the two particles have the same vector, e, so that we are aiming for result +cos(theta) not -cos(theta) *****
```

```
' The particles' vector has components xe, ye and ze.
```

```
'pick uniform random numbers x between -1 and +1
3
```

```
x = Rnd * 2 - 1
y = Rnd * 2 - 1
z = Rnd * 2 - 1
```

```
length = x * x + y * y + z * z
```

```
If length < DELTA0 Then GoTo 3
If length > DELTA1 Then GoTo 3
```

```
xe = x
ye = y
ze = z
```

```
length = Sqr(length) 'the particle vector is not unity, but lies within a unit sphere
```

```
'CALCULATIONS BASED ON ONE PARTICLE PAIR
```

```
'calculation of angle between stations a and b
```

```
x = xa * xb + ya * yb + za * zb
If Abs(x) > DELTA1 Then GoTo 3
theta = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1) ' excel's formula to give the arccos function
'need now to turn theta (decimal in radians) into IntegerTheta (an integer angle in degrees)
```

```
'calculation of A
```

```
' a dot e = xa * xe + ya * ye + za * ze
' A = a dot e where a and e need to be unit vectors
'here, a is a unit vector, but e has size = length, so
a = (xa * xe + ya * ye + za * ze) / length ' length is magnitude of particle vector
a = Sgn(a) 'REMOVE THIS LINE TO GET qm CORRELATION 1ST OF 2
```

```
'calculation of B
```

```
b = (xb * xe + yb * ye + zb * ze) / length
b = Sgn(b) 'REMOVE THIS LINE TO GET qm CORRELATION 2ND OF 2
```

```
'INCREMENT the accumulators
```

```
sa = sa + a
sb = sb + b
ssa = ssa + a * a
ssb = ssb + b * b
sab = sab + a * b
sn = sn + 1
```

```
10 Next ithPair ' jump to top of FOR/NEXT loop for generation of the next pair of particles
```

```
theta = theta * 180 / Pi
```

'END OF MAIN PART OF PROGRAM

'-----

' WRITE RESULTS TO SPREADSHEET : means, SDs, correlation and number of pairs of particles

'calculate means and correls etc

```
meanA = sa / sn
meanB = sb / sn
sdA = ssa / sn - meanA * meanA
sdA = Sqr(sdA)
sdB = ssb / sn - meanB * meanB
sdB = Sqr(sdB)
corrAB = sab / sn - meanA * meanB
corrAB = corrAB / (sdA * sdB)
total = sn
```

' set which row and column will be used as starting point to write results to spreadsheet

Range("a1").Select

```
ActiveCell.Offset(0, 1).Formula = "theta = "
ActiveCell.Offset(11, 2).Formula = "mean of A"
ActiveCell.Offset(12, 2).Formula = "mean of B"
ActiveCell.Offset(13, 2).Formula = "SD of A"
ActiveCell.Offset(14, 2).Formula = "SD of B"
ActiveCell.Offset(15, 2).Formula = "correl of A and B"
ActiveCell.Offset(16, 2).Formula = "Total N of pairs"
```

ActiveCell.Offset(0, 2).Formula = theta

' ActiveCell.Offset(9, 5).Formula = total 'total number of pairs of particles

```
ActiveCell.Offset(11, 3).Formula = meanA
ActiveCell.Offset(12, 3).Formula = meanB
ActiveCell.Offset(13, 3).Formula = sdA
ActiveCell.Offset(14, 3).Formula = sdB
ActiveCell.Offset(15, 3).Formula = corrAB
ActiveCell.Offset(16, 3).Formula = total
```

' NOW USER CAN GO TO RESULTS SPREADSHEET TO INSPECT 2X2 TABLE AND CORRELATION

'-----

End

End Sub

Example of output from the Excel Visual Basic program in Appendix A

theta = 45

Integer values of A and B taken as absolute magnitudes of **fuzzy** vectors on a hemisphere centred on vectors **a** and **b**

i.e. correlation between fuzzy vectors on a hemisphere **ah** and **bh**

mean of A	0.00024
mean of B	-0.00043
SD of A	0.999999971
SD of B	0.999999908
correl of A and B	0.499454164
Total N of pairs	1000000

Integer values of A and B fractionally projected onto **exact** vectors **a** and **b**

i.e. correlation between exact vectors **av** and **bv**

mean of A	0.000911218
mean of B	0.001414591
SD of A	0.577522836
SD of B	0.57779221
correl of fractional projections of A and B	0.707258632
Total N of pairs	1000000

APPENDIX B Excel Visual Basic program to reproduce Susskind's version of a Bell Inequality; and an example of the output

```
Sub AJFModel2()
```

```
'-----
' Excel visual basic program to break Bells Inequality
'-----
' a = Sgn(a)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 1 OF 3
' b = Sgn(b)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 2 OF 3
' c = Sgn(b)          ' REMOVE THIS LINE TO OBTAIN BREAKAGE OF BELL'S INEQUALITY .. LINE 3 OF 3
'-----

'PRELIMINARY WORK:

' set the dimensions of variables, their sizes and types
Dim ithPair As Long ' ithPair is a long integer used as the index for the ith pair of particles
Dim TotalNoOfPairs As Long ' a long integer used to store the total number of pairs to be generated in the run of the program
Dim x, y, z, xa, ya, za, xb, yb, zb, xe, ye, ze, length As Double ' extra precise real numbers
Dim xc, yc, zc As Double ' extra precise real numbers
Dim a, b, c, sa, sb, sc, sn, total As Double ' extra precise real numbers

'"Randomize" initializes the random-number generator, to avoid using the same set of random numbers in every run
Randomize

'define constants to be used in the program
TotalNoOfPairs = 1000000 ' this sets the total number of pairs of particles which are to be generated
DELTA0 = 0.0000000000000001 ' used to help to avoid dividing by zero
DELTA1 = 0.9999999999999999 ' used to help to avoid dividing by zero
Pi = 3.1415926535 ' pi radians is equivalent to an 180 degree angle

' set accumulator counters for means, sds, N and correlation to zero before use
sa = 0
sb = 0
sc = 0
sn = 0

'END OF PRELIMINARY WORK

'-----

'MAIN PART OF PROGRAM

' generate particle pairs in a For/Next loop
For ithPair = 1 To TotalNoOfPairs

' PICK SPECIFIC VECTORS FOR DETECTORS A AND B AND PARTICLES, E

' PICK A POLARIZATION VECTOR for detector a (Alice)
' Alice's unit vector has components xa, ya and za

    xa = 0
    ya = 0
    za = 1

' PICK A POLARIZATION VECTOR for detector b (Bob)
' Bob's unit vector has components xb, yb and zb

    xb = 0
    yb = -Sqr(2) / 2
    zb = -Sqr(2) / 2
```

' PICK A POLARIZATION VECTOR for detector c (Charlie)
 ' Charlie's unit vector has components xc, yc and zc

xc = 0
 yc = 1
 zc = 0

' PICK A POLARIZATION VECTOR for particle pair e.
 ' Note that the two particles have the same vector, e, so that we are aiming for result +cos(theta) not -cos(theta) *****

' The particles' vector has components xe, ye and ze.

'pick uniform random numbers x between -1 and +1
 3

x = Rnd * 2 - 1
 y = Rnd * 2 - 1
 z = Rnd * 2 - 1

length = x * x + y * y + z * z

If length < DELTA0 Then GoTo 3
 If length > DELTA1 Then GoTo 3

xe = x
 ye = y
 ze = z

length = Sqr(length) 'the particle vector is not unity, but lies within a unit sphere

'CALCULATIONS BASED ON ONE PARTICLE PAIR

'calculation of angle between stations a and b

x = xa * xb + ya * yb + za * zb
 If Abs(x) > DELTA1 Then GoTo 3
 theta = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1) ' excel's formula to give the arccos function
 'need now to turn theta (decimal in radians) into IntegerTheta (an integer angle in degrees)

'calculation of A

' a dot e = xa * xe + ya * ye + za * ze
 ' A = a dot e where a and e need to be unit vectors
 'here, a is a unit vector, but e has size = length, so
 a = (xa * xe + ya * ye + za * ze) / length ' length is magnitude of particle vector
 'a = Sgn(a) 'REMOVE THIS LINE TO GET QM CORRELATONS 1ST OF 3

If a < 0 Then GoTo 10

'calculation of B

b = (xb * xe + yb * ye + zb * ze) / length
 'b = Sgn(b) 'REMOVE THIS LINE TO GET QM CORRELATONS 2ND OF 3

'INCREMENT the accumulators

sa = sa + a
 If b < 0 Then GoTo 110
 sb = sb + b

110 'calculation of C

c = (xc * xe + yc * ye + zc * ze) / length
 'c = Sgn(c) 'REMOVE THIS LINE TO GET QM CORRELATONS 3RD OF 3

If c < 0 Then GoTo 10
 sc = sc + c

10 Next ithPair ' jump to top of FOR/NEXT loop for generation of the next pair of particles

theta = theta * 180 / Pi

'END OF MAIN PART OF PROGRAM

'-----

' WRITE RESULTS TO SPREADSHEET :

' set which row and column will be used as starting point to write results to spreadsheet
Range("a1").Select

ActiveCell.Offset(11, 2).Formula = "sum of A+"
ActiveCell.Offset(12, 2).Formula = "sum of B+ where A is also +"
ActiveCell.Offset(13, 2).Formula = "sum of C+ where A is also +"

ActiveCell.Offset(16, 2).Formula = "Total N of pairs"

ActiveCell.Offset(11, 3).Formula = sa
ActiveCell.Offset(12, 3).Formula = sb
ActiveCell.Offset(13, 3).Formula = sc

ActiveCell.Offset(16, 3).Formula = TotalNoOfPairs
ActiveCell.Offset(11, 5).Formula = sa / TotalNoOfPairs
ActiveCell.Offset(12, 5).Formula = sb / TotalNoOfPairs
ActiveCell.Offset(13, 5).Formula = sc / TotalNoOfPairs
ActiveCell.Offset(9, 3).Formula = "sum"
ActiveCell.Offset(9, 5).Formula = "Proportion"

' NOW USER CAN GO TO RESULTS SPREADSHEET TO INSPECT 2X2 TABLE AND CORRELATION

'-----

End
End Sub

Example of output from the Excel Visual Basic program in Appendix B

Integer values of A, B and C taken as integer magnitudes of fuzzy vectors on a hemisphere centred on vectors **a**, **b** and **c**

i.e. sums of fuzzy vectors on a hemisphere

sum of A+	500143	0.500143
sum of B+ where A is also +	125318	0.125318
sum of C+ where A is also +	249611	0.249611

Integer values of A, B and C fractionally projected onto exact vectors **a**, **b** and **c**

i.e. sums of exact vectors with fractional magnitudes

sum of A+	250942.5605	0.25094256
sum of B+ where A is also +	36604.27625	0.036604276
sum of C+ where A is also +	124758.1586	0.124758159