Correlation of – cos θ between measurements in a Bell's Inequality experiment simulation calculated using local hidden variables

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Abstract

This paper shows that the theoretical correlation between Alice's and Bob's measurements in a simulated Bell's Inequality experiment with local hidden variables is – cos θ. That is without the need for detection loopholes nor missing data. Instead of putting all the onus on the particles to embody randomness on detection, this paper transfers at least some of this onus onto the magnets. The randomness caused by the magnets implies that 'counterfactual definiteness' is inappropriate for simulations of Bell's inequality experiments.

Introduction

The unsurprising result that correlation (A, B) = – cos θ is derived from first principles. It is a very reasonable result because cos θ is a standard form for the correlation between two vectors differing in direction by angle θ, but nevertheless it is important to derive it in the current case.

A model of the electron including the local hidden variable for electron spin is required. Also, a structure for a detector magnet is required, including a discussion of what is meant by setting a magnet at a given angle.

Spin in entangled pairs, local hidden variables and magnets

The obvious candidate for the local hidden variable of an electron is its chiral handedness. The left-handed (LH) chiral electron has spin - 0.5 while the right-handed (RH) chiral electron has spin + 0.5. A LH electron can emit a photon with spin -1 and change form to become a RH electron. Spin is conserved by the interaction as the total spin before and after the interaction in -0.5. Likewise, a RH electron can emit a photon with spin +1 and change form to become a LH electron. And similarly for LH and RH positrons. A chiral left-handedness indicates a difference in structure from the right-handed form. This is different from a left-handed helicity for observer 1, which may simultaneously take on a right-handed appearance for observer 2. Chiral 'left' is permanently chiral 'left', although observers observe a particle's helicity with the chirality being hidden or inferred.

The spin of an electron is nominally about an axis. The orientation of the axis is always unknown in practice although it can be assumed to be known in a local hidden variable simulation of an experiment. The most information in practice that is available about spin is that for paired particle creation, say for an electron and positron, arising from an interaction where the incoming total spin was zero, then the outgoing particles have equal and opposite spins, summing to zero angular momentum. The directions of the axes are also the same for both particles: that is, treating the axes as vectors with the arrowheads removed, or ignoring the signs of the axis vectors.

If a small, free-standing macro magnet is brought close to a powerful magnet, then it may precess and radiate away energy until it lines up with the direction of the stronger magnet. Electrons do not behave like this. Instead, if they do undergo an interaction, they emit one photon, as described above, and completely reverse chiral handedness. This can be repeated, for the same magnet orientation, but the electron is now stable and will not flip again. The electron may flip if brought near a differently oriented magnet, but the outcome is that the electron never changes the direction of its axis during the course of the experiment (and far beyond the experiment). The vector of the electron spin axis changes from **s** to **– s** and back to **s** in repeated interactions but instead of calling that a change in vector spin direction the electron is given an N or S pole label like the macroscopic magnet, and the label flips from N to S to N in successive interactions while the axis stays constant (ignoring the vector sign).

It needs to be illustrated more clearly what it means for an individual electron to receive a label N or S and to do this requires some discussion of randomness. There is no reason to assume that any one incoming electron has its spin axis pointing in any particular direction, although it is true [making the assumption that local hidden variables exist] that an incoming pair of particles will have the same axis and one will be an N and the other an S during their times of flight. In quantum mechanics, on the other hand, the pair of particles share a common entangled state of 0.5 |S> + 0.5 |N> until one of them interacts, which is an approach which eschews individual particles' local hidden variables.

If two electrons are prepared pointing say north in the 2D space of the laboratory floor, this does not mean that their spin axes are parallel and it also does not mean that their axes are pointing exactly north. Their spin axes are unknown. Say for electron 1 an N label is put at one end of its spin axis and an S label at the other end. Ditto for electron 2. To prepare the electrons, the electrons are interrogated each to see if it is the N label or the S label which is nearer the north wall of the laboratory. If the N label is to be nearer the North wall, it may require a switch of the N and S labels to achieve this. If the labels do need to be switched, then an 'interaction' is required to effect the switch. This corresponds to Alice or Bob making a measurement of +1. If no switch of labels is required, this is equivalent to Alice or Bob making no measurement. However, in practice, Stern-Gerlach detectors can circumvent this lack of a measurement and instead turn that into an actual interaction, recorded as -1. So there is an interaction recorded for every particle with no inefficiency by wastage of information.

So now there are two electrons prepared with their labels appropriately adjusted with their N labels nearer to the North wall than their S labels. Say it is now required to simulate preparing the two electrons to point towards the West wall of the laboratory. Without changing their spin axes each particle is tested to flip, or not as necessary, their N and S labels so the two N labels are nearer the West wall than are the S labels. In practice this requires also putting a magnet against the West wall aligned in an East-West direction. As these two electrons are not entangled there is no expectation that both electrons will interact in the same way. So even though both electrons were originally prepared pointing northwards, there is no knowing which one, if any, was already also pointing west. For two entangled electrons, if one electron needed a switch of labels to point West [or any chosen new direction], the other electron would not need a switch to point West. This is because entangled spin axes are parallel and oppositely labelled.

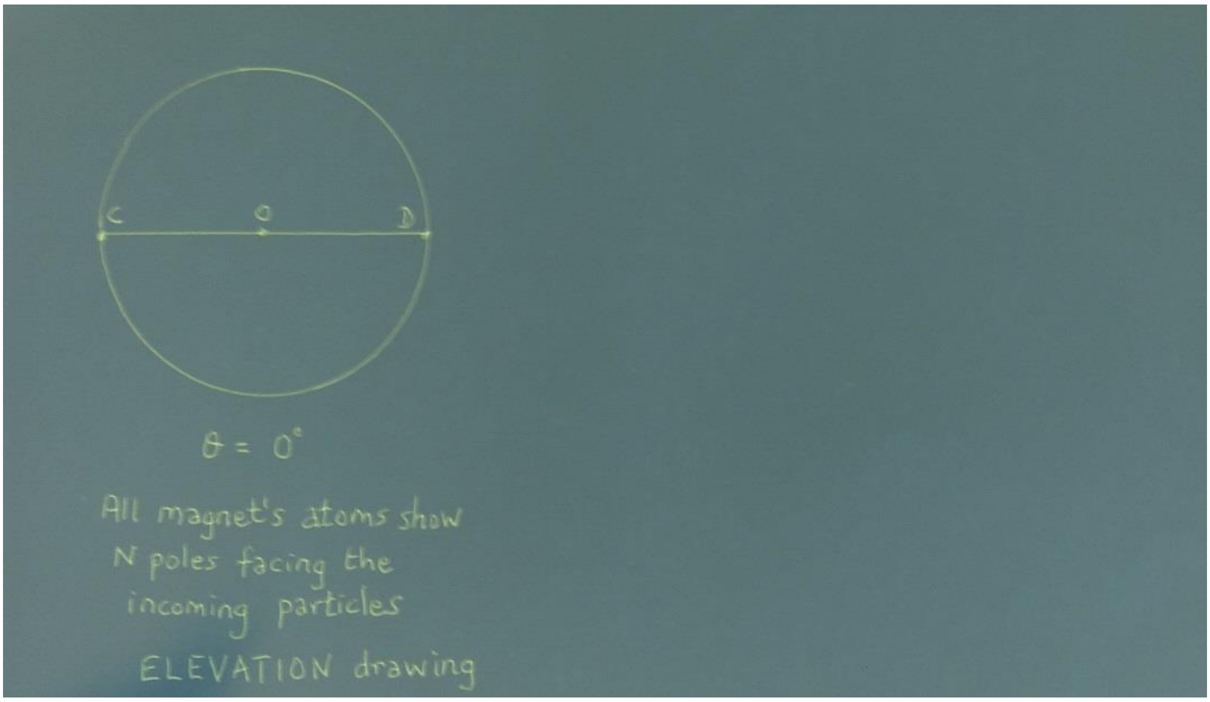
Structure and angle settings for the detector magnets and derivation of r (A, B) = – cos θ from first principles

Magnets obtain and keep their magnetism mainly via angular momentum of their atoms, rather than from the intrinsic spins of their electrons. For this paper it is simplest to describe a magnet as a 'bar' magnet in the shape of a sphere. A useful analogy for some of this discussion is sunlight reflected off the moon. A full moon can represent a magnetic N pole so that light is associated with N pole atoms and dark is associated with S pole atoms within the moon-shaped magnet. The moon is seen as if it were a flat plane, despite it being a sphere. If the moon rotated by say 30 degrees then a gibbous moon with more light than dark would be seen. In this section the exact illuminated area on the moon's projected flat plane is calculated for various angles of rotation. It gives away no secrets that this relationship is obviously given by cos θ, where θ is the angle that the moon has rotated.

There would be no randomness of outcome of an interaction if one knew which atom in the magnet was initiating photon emission and what was the label of that atom was, N or S. One does not even need to know any axis direction to be certain of the outcome for an individual interaction if one knows the N of S state of the incoming electron and detecting atom. Under these conditions the use of 'counterfactual definiteness' would be permissible as there is no chance in the outcome. This information really derives from the case where two detector atoms in the magnets have an identical axis, that is, they are aligned in parallel. The experiments, however, use magnets for which the N and S labels are not known for individual atoms in the magnet pole and the facing area of the magnet can contain a mixture of N and S facing atoms. This uncertainty prevents the usage of 'counterfactual definiteness'.

Alice's magnet is chosen to point with vector zero degrees. The magnet therefore shows its face as completely covered with all atoms aligned as N poles facing the incoming electrons (Figure A). Bob's magnet will receive all the entangled partner particles (Figure B).

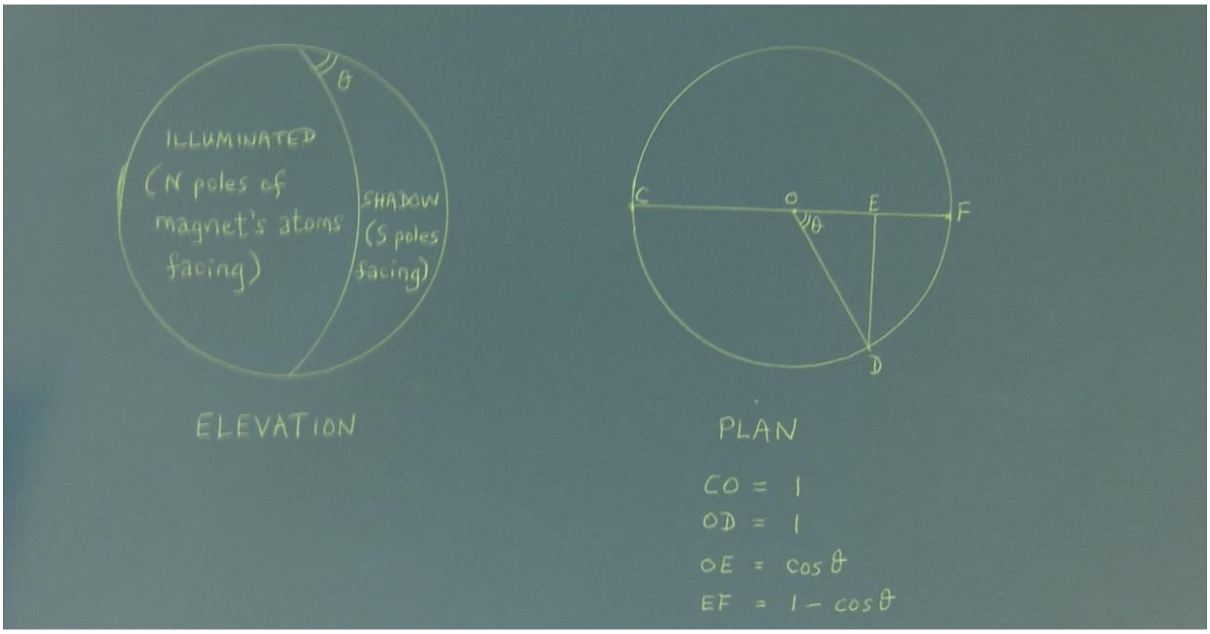
Figure A: Alice's detector magnet in elevation: analogous to a full moon



Alice's magnet detector is represented in Figure A as a full moon in elevation, completely illuminated, where 'illumination' is analogous to atoms with N poles facing the incoming electrons. The totality of electrons detected as +1 by Alice are now post-selected, in this procedure, as it is known that they all were originally presenting their N faces to the detector. For convenience of calculation, only acute angles of detectors are treated here and, further, rather than showing that the correlation between Alice's and Bob's measurements are – cos θ it is to be shown that the correlation would be +cos θ if the partner pairs were identical rather than exactly opposite in nature. So it is assumed that the exactly same electrons are sent to Bob, rather than the exactly opposite positrons.

Next the proportion of N versus S atoms in Bob's 'gibbous moon' detector need to be calculated (Figure B and Tables 1 and 2).

Figure B: **Bob's detector magnet in elevation (left) and plan (right): analogous to a gibbous moon**



The equator line OD for Alice's detector (Figure A, ELEVATION diagram) corresponds to the line OD in Bob's detector PLAN diagram (Figure B). In Bob's detector, OD has swept through an angle θ from its starting position at OF. {Electron spin axes cannot be rotated through any angle other than 180^o. The atoms in magnets can, however, be rotated through angles. One can hold a bar magnet and point it's North pole in any direction.}

For the horizontal line COEF in Bob's ELEVATION diagram, CO and OE are illuminated (N) whereas EF is in shadow (S). CO is arbitrarily assigned unit length. In Bob's PLAN diagram, OD also has unit length as it is the radius of the same circle of which CO is also a radius. By simple trigonometry, OE = cos θ and EF = 1 - cos θ. So in Bob's ELEVATION diagram, the illuminated length of line COEF is 1 + cos θ and the shadow part of the line is of length 1- cos θ.

The total area illuminated/shaded can be built up by summing over many infinitesimal-width horizontal lines parallel to COEF. Assume one of these parallel lines is C'O'E'F'. Assume also that C'O' has length α. By the same arguments as above the illuminated length of line C'O'E'F' is α + α cos θ and the shaded length is α - α cos θ. For all horizontal lines, therefore, the illuminated and shadow lengths are in the ratio: (1 + cos θ) to (1 - cos θ). Thus the illuminated area of the gibbous moon relative to the area in shadow also has the same ratio: (1 + cos θ) to (1 - cos θ).

Next in this procedure, a set of particles identical to those recorded as +1 (N for these particles) by Alice are sent to Bob's detector. All those interacting with the illuminated area (N in the magnet) are recorded by Bob as +1; all those interacting with the area in shadow (S in the magnet) are recorded by Bob as -1. It is not actually known in practice in which region any particular particle interacts, so these simulated measurements also are made to depend likewise on chance outcomes.

Likewise, a set of particles identical to those recorded as -1 (S for these particles) by Alice are sent to Bob's detector. All those interacting with the illuminated area (N in the magnet) are recorded by Bob as -1; all those interacting with the area in shadow (S in the magnet) are recorded by Bob as +1. Again, it is not known in which region any particular particle interacts.

From these measurements a 2x2 table of Alice's (A) and Bob's (B) outcomes can be constructed (Table 1).

Table 1: 2x2 table of results for Alice's and Bob's measurements

	B = 1	B = -1	Total
A = 1	1 + cos θ	1 - cos θ	2
A = - 1	1 - cos θ	1 + cos θ	2
Total	2	2	4

The correlation between A and B outcomes is cos θ is calculated in Table 2.

Table 2: Workings for calculation of correlation between Alice's and Bob's measurements

A	B	AB	A^2 (=B^2)	f (=freq.)	fA	fA^2 (=fB^2)	fAB
1	1	1	1	1 + cos θ	1 + cos θ	1 + cos θ	1 + cos θ
1	-1	-1	1	1 - cos θ	1 - cos θ	1 - cos θ	-1+cos θ
-1	1	-1	1	1 - cos θ	-1+cos θ	1 - cos θ	-1+cos θ
-1	-1	1	1	1 + cos θ	-1- cos θ	1 + cos θ	1 + cos θ
				Σ= 4	Σ= 0	Σ= 4	Σ=4 cos θ

Correlation *	=	ΣfAB / ΣfA^2	=	cos θ			

* *Corrrelation formula for the special case where the means are zero and the standard deviations are equal.*

To restore the actual correlation of – cos θ, merely flip the labels of + 1 and – 1 for Bob in Table 1 and re-calculate the correlation. A further tidying is to make the total number of pairs one instead of four by dividing the terms by four. So the cells in Table 1, which is based on pairs of exactly identical particles, can be rearranged to be filled with terms 0.25 (1 –cos θ) and 0.25 (1 + cos θ) as appropriate. Calculation of the new correlation between A and B gives – cos θ for particles with exactly opposite properties.

Discussion

The construction of outcomes of A and B in Table 1 used local hidden variables (Ref. 1) but did not use 'counterfactual definiteness'. The term 'counterfactual definiteness' can be taken to mean that "we may think of outcomes of measurements that were not actually performed as being just as much part of reality as those that were made" (Ref. 1). In simulations of Bell's Experiment, the term is taken to mean that if Alice's magnet was aligned at angle θ and Bob's magnet was also aligned at angle θ, then if Alice measured +1 for one entangled particle, Bob would with certainty measure -1 for the partner entangled particle.

Each particle presented an N or S face to the detector magnets, and the detector magnet was divided into two regions where the magnet atoms were assumed to be known as N or S. The measurements A and B were assumed using the logic that like poles repel and would require a photon emission which indicates a +1 measurement. Unlike poles attract and so they are in a relatively stable positions, which indicates no photon emitted which is convertible by an efficient detector into a -1 measurement. In these calculations it was assumed that Bob's angled magnet introduced a random element into measurements. Bob's measurements were calculated with some uncertainty for each individual particle. The measurements could have been calculated without randomness, but there was no purpose in taking that extra step because there is no way in practice to choose the particular atom in a magnet with which to interact. In practice, it is impossible to know which atom, and its spin axis vector, an incoming particle will interact with. Also, it is the randomness of measurement inherent in the superposition of N and S atoms in the magnet which allows the –cos θ correlation to arise; using counterfactual definiteness would prevent the - cos θ result.

It is normal in quantum mechanics to assume a superposition of opposite spin states for an entangled pair of microscopic particles. In this paper, however, it is the macroscopic magnet which is behaving as if its facing pole was a superposition of N and S states. When the angle is set at θ, Bob's magnet, nominally acting as a north pole, acts with a superposition of N and S states as follows:

Bob's magnet's supposition of magnetic states = | 0.5 (1 + cos θ) N > + | 0.5 (1 - cos θ) S >

It is this superposition which allows the correlation between A and B of - cos θ to occur. The result of this paper cannot be verified experimentally as being caused by Bob's physical magnet's superimposed states as Bell's experiments have already been taken supposedly to confirm the particles' entangled states.

The model of the magnet used here is maybe a throwback to a pre-Rutherford toy atom: a spherical blob, with N and S magnetic poles. The macroscopic magnet used here reflects the shape of the individual toy atoms in the magnet. It is easiest to visualise the toy model for rotated magnets with $\theta = 0^o$, 90^o and 180^o. For these three angles, Bob's magnet presents the states: |N>, | 0.5 N > + | 0.5 S > and |S> respectively, to the incoming particles. And using these three angles as a firm basis it is maybe not so difficult to visualise interpolating between these three positions using a generalised superposition for angle θ:

$| 0.5 (1 + \cos\theta)\ N > + \ | 0.5 (1 - \cos\theta)\ S >$.

Instead of putting all the onus on the particles to embody randomness on interaction with the detector magnet, this paper transfers at least some of this onus onto the magnets themselves.

Reference

1. Bell's theorem. https://en.wikipedia.org/wiki/Bell%27s_theorem

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