The physical nature of the basic concepts of physics

1. Linear Momentum (i)

Guido F. Nelissen

guido.nelissen@gmail.com

Abstract

The principles of the conservation of ‘linear momentum’ and the conservation of “energy” are the corner stones of the present theory of physics. The true nature of these concepts and the underlying physical mechanisms of their conservation have, however, never been properly cleared out. Even the great European physicists, Descartes, Leibniz and D’Alembert had lengthy discussions [1] on whether ‘kinetic energy’ or ‘linear momentum’ were the true property considered by the conservation laws.
In the present physics the linear momentum of a body is mathematically defined as the product of its mass and its velocity and its conservation is explained as a consequence of Newton’s first law of motion.
In this paper the author reveals the physical nature of the linear momentum of a moving particle system and the physical reason for its conservation in the absence of external interactions.

1. The present law of the conservation of linear momentum

1.1 The conservation of the linear momentum of a single particle

The notion of “momentum” finds its origin in Newton’s second law of motion, which reads “The rate of change of momentum of a body is equal to the resultant force impressed to it and acts along the direction of that force”, or:

$$\mathbf{F} = d\mathbf{p}/dt = d(m.\mathbf{v})/dt = m.d\mathbf{v}/dt = m.\mathbf{a}$$

It follows from this second law, that in the absence of external forces ($\mathbf{F} = 0$), the linear momentum remains constant:

$$\mathbf{F} = d\mathbf{p}/dt = d(m\mathbf{v})/dt = 0$$

so that: $\mathbf{p} = m\mathbf{v} = \text{constant}$
which is in fact a quantitative expression of Newton’s ‘first law of motion’.

1.2 The conservation of the linear momentum of a system of particles

Newton’s third law of motion gives the quantitative relationship between the mutual forces on

(i) Updated version of my paper http://viXra.org/abs/1610.0268.

colliding particles: "*Whenever a body exerts a force on another body, the latter exerts a force of equal magnitude and opposite direction on the former*".

Since 'force' equals the rate of change of momentum ($\mathbf{F} = d\mathbf{p}/dt$), Newton's third law can also be expressed in terms of momentum as: "*Whenever a body exert a force on another body, the rate of change of momentum of both bodies are of equal magnitudes and opposite directions".* Which means that if:

$d\mathbf{p}_1/dt = \mathbf{F}$

then:

$d\mathbf{p}_2/dt = -\mathbf{F}$

so that:

$d\mathbf{p}_1/dt + d\mathbf{p}_2/dt = d(\mathbf{p}_1 + \mathbf{p}_2)/dt = 0$

Which means that the total linear momentum of two colliding particles remains constant:

$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$

This demonstrates that Newton's third law of motion means that for a multi-particle system (on which there are no external forces), the internal collisions between particles cannot change the total linear momentum of the particle system, because at each internal collision the linear momentum remains constant.

2. The axiomatic character of Newton's laws of motion

Newton's second law of motion has a certain ambiguity in it, as it defines in one equation three different concepts in function of one another:
- Force, as the acceleration of a given mass ($\mathbf{F} = m\mathbf{a}$),
- Acceleration, as the force per unit mass: ($\mathbf{a} = \mathbf{F}/m$),
- Mass, as the force per unit acceleration: ($m = \mathbf{F}/\mathbf{a}$).

This means that, even if you can define acceleration independently from Newton's second law, namely as the rate of change of the velocity per unit time ($\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2$), you still have 'force' (ii) and 'mass' (iii) that are ambiguously defined in function of each other.

Newton's laws of motion are empirical laws, based on the scrupulous observation of macroscopic bodies. They are in that way comparable to the laws of classical thermodynamics, which are based on macroscopic observations.
According to the late A. B. Pippard (former J. H. Plummer Professor of Physics in the University of Cambridge) [2] "*In classical thermodynamics, the method of approach takes no account of the atomic constitution of matter, but seeks rather to derive from certain basic postulates, the laws of thermodynamics, relations between the observed properties of substances".* And further "*Classical thermodynamics makes no attempt to explain why the laws have their particular form, that is, to exhibit the laws as a necessary consequence of other laws of physics which may be regarded as even more fundamental. This is one of the*

(ii) This will be analyzed in my paper "The physical nature of force".
(iii) This will be analyzed in my paper "The physical nature of mass".

problems which is treated by statistical thermodynamics".
Just like the classical laws of thermodynamics, Newton's laws of motion for macroscopic bodies are not exhibited as a logical consequence of their microscopic constitution. Rather, their sole function is to explain their observable, macroscopic behavior as the consequence of a few empirical 'laws'.
A. B. Pippard observes thereby that "*Not all practitioners of the physical sciences (in which term we may include without prejudice chemists and engineers) have this particular ambition to probe the ultimate mysteries of their craft, and many who have are forced by circumstances to forgo their desire. .. For often enough in the pure sciences, and still more in the applied sciences, it is more important to know the relations between the properties of substances than to have a clear understanding of the origin of these properties in terms of their molecular constitution*."
In this paper I will reveal the true physical nature of the linear momentum of a moving particle system, as a natural consequence of the motion of its basic components.

3. Congruent and thermal translational motion

Newton's basic laws of motion don't specify the internal structure of the considered 'bodies', which means that he regarded them as monolithic objects. All physical 'objects' are however in reality composite, dynamic particle systems. To analyze the conservation of linear momentum for such dynamic particle structures, we have to decompose the velocities of their individual particles into their fundamental velocity components.
In the present textbooks, the fundamental velocity components of particle systems are defined in relation to their center of mass (CM):

- the velocity of the CM in the chosen reference frame (which is called the CM-velocity '$\mathbf{v}_{CM}$') and,
- the individual velocities of the particles relative to the CM (which are called the 'internal' or 'thermal' velocity components '$\mathbf{q_j}$').

It is thereby important to realize that the so-called 'center of mass' is not a physical concept, but that it is a virtual point in space-time that indicates the average position of the components of a mass particle system [3], which allows us to calculate the motion of that body, as if all its mass were concentrated in that virtual point.

To understand the physical meanings of the CM- and the internal velocities of a particle system, it is therefore advisable to consider the behavior of its components in function of the degree of coherence of their motions.

4. The conservation of linear momentum

4.1 Congruent translational velocity

The CM-velocity ($\mathbf{v_{cm}}$) is classically obtained by differentiating the position vector of the CM ($\mathbf{r}_{cm}$) with regard to time:

$$\mathbf{r}_{cm} = \sum m_j \mathbf{r_j}/\sum m_j = \sum m_j \mathbf{r_j}/m$$

$$\mathbf{v_{cm}} = d\mathbf{r}_{cm}/dt = \sum m_j (d\mathbf{r_j}/dt)/\sum m_j$$

So that: $\mathbf{v_{cm}} = \sum m_j \mathbf{v_j}/m$

In this equation, in which 'm' is the total mass of the particle system, the individual mass (m_j) of each particle is used as a weighting factor, which means in fact that each particle with a mass 'm_j' and a velocity '$\mathbf{v_j}$' is implicitly considered as a sub-particle system consisting of 'm_j' (basic) particles with unit mass (iv) that all move in bulk, i.e. in a 'congruent' way with exactly the same 'congruent' velocity '$\mathbf{v_j}$'.

This means that the factor 'm**v**', which is in the present textbooks generally denoted as the 'linear momentum', is actually a physical indication of the vector sum of the velocities of all the (basic) particles with unit mass, so that the CM-velocity is in fact the resultant velocity of all the particles with unit mass divided by the total mass (m).

In nowadays physics, the 'unity of mass' is the arbitrarily chosen SI-unit of 'kilogram'. If we want to unveil the true nature of linear momentum, we have to put these arbitrarily chosen mass units aside and have to define the mass unit in a natural way, which is the mass of the smallest possible component, which we can define as the basic quantity/quantum of mass, in analogy with the quantum of energy (hν). In that way, the mass of a material body is expressed as the total amount of these mass units. This viewpoint is as a matter of fact also in line with Newton's view, who also defined mass as "the quantity of matter" [4].

From this viewpoint, the CM-velocity ($\mathbf{v_{CM}}$) of a particle system is in fact the 'common', 'coherent' or 'congruent' velocity component with which all the unit particles of that particle system move in a coherent way, that is with exactly the same speed in exactly the same direction ($\mathbf{v_c} = \mathbf{v_{CM}}$). This 'congruent', 'common' or 'coherent' velocity $\mathbf{v_c}$ is sometimes called the 'external' or 'bulk' velocity of the particles of a particle system. The fundamental characteristic of this congruent velocity is that it is exactly the same for all the particles of a particle system (Fig. 1.1) [5].

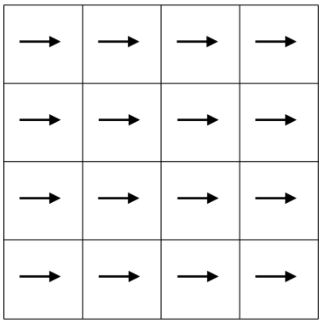

Fig. 1.1

One important conclusion from this definition of the CM-velocity as the 'common', 'congruent' or 'coherent' velocity component ($\mathbf{v_c}$) of all the particles of a particle system, is that it cannot be affected by the internal motions or the internal collisions of the particles within the particle system, so that, without external intervention, the congruent velocity of a particle system is bound to remain constant.

$\mathbf{v_c} = \sum m_j \mathbf{v_j}/m = \mathbf{v_{CM}} = \text{constant}$

This means that the congruent velocity of all the individual elements of a particle system can be represented by the only one vector with one given length and one given direction.

(iv) This will be analyzed in my paper "The physical nature of mass".

4.2 The total amount of congruent translational velocity

Since in a closed system, the total mass ‘m’ (which may be regarded as the total number ‘m’ of basic particles with unit mass) is constant, this means that the total sum of the congruent velocities of all the basic particles with unit mass is also constant:
$m.\mathbf{v_c} = \sum m_j\mathbf{v_j} = \mathbf{p_c} = \mathbf{p} = \text{constant}$

In this equation, the expression ‘$m.\mathbf{v_c}$’ is generally known as the ‘linear momentum’ of a particle system with total mass ‘m’ and a congruent or CM-velocity ‘$\mathbf{v_c}$’ and this equation is therefore generally known as ‘the conservation of linear momentum’ (**p**).

In the former section we have demonstrated that the linear momentum of a moving particle system, is equal to the sum of the (identical) congruent velocities of all its particles with unit mass ($\sum m_j\mathbf{v_j} = m.\mathbf{v_c}$). This means that the linear momentum is a mathematical expression of the total amount of congruent translational velocity/motion and that the conservation of linear momentum is a mathematical expression for the conservation of the total amount of congruent translational velocity/motion of the particle system.
This insight into the true nature of the conservation of linear momentum corresponds exactly to Newton’s point of view who, in his “Principia Mathematica” called it the conservation of the total ‘quantity of motion’ [6]. In French the linear momentum is therefore still called ‘*quantité de mouvement*’.

Since a ‘quantity’ or an ‘amount’ is generally expressed as a number, we will in this paper use the concept of the conservation of the total ‘amount of motion’ of a particle system to indicate the conservation of the sum of the magnitudes of the velocities of all its basic components. We will on the other hand use the general concept of the conservation of ‘motion’ if all the individual velocities of these basic particles, their magnitudes as well as their directions, are conserved.

In this way the conservation of linear momentum: $\sum m_j\mathbf{v_j} = m.\mathbf{v_c} = \mathbf{p_c} = \text{constant}$, means in fact that in an isolated mass particle system, the total amount of congruent translational velocity/motion ‘$\mathbf{p_c}$’ is conserved in both magnitude and direction and that as well for each unit mass particle in particular as for the particle system as a whole. Newton’s first law of motion as well as the conservation of linear momentum can therefore be expressed in a general way as “the conservation of the total amount of congruent translational velocity/motion”.

4.3 Internal translational motion

The velocity components ($\mathbf{q_j}$) with which the individual particles of an ideal particle system move relative to the center of mass of the particle system to which they belong, are defined as the ‘internal’ or ‘thermal’ velocities of the particles. They are by definition the remaining translational velocity components of the particles, when we subtract their common (or congruent) velocity component ($\mathbf{v_c}$) from their individual velocities ($\mathbf{v_j}$) in the chosen reference frame:

$$\mathbf{q_j} = (\mathbf{v_j} - \mathbf{v_c}) = (\mathbf{v_j} - \sum m_j\mathbf{v_j}/m)$$

It follows from this definition, that the thermal velocities are isotropicly distributed over all possible directions (Fig 1.2) [5].

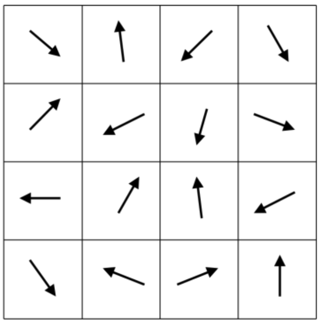

Fig. 1.2

The isotropic character of these velocities means that they cannot in any way produce a resultant velocity:

$$\mathbf{q} = \sum m_j \mathbf{q_j}/m = 0$$

This means that the total thermal linear momentum of a particle system is bound to be zero:

$$\mathbf{p_q} = \sum m_j \mathbf{q_j} = 0$$

The total amount of internal translational motion, which we have defined as the sum of the magnitudes of the thermal velocities of the basic particles with unit mass, is however not zero:

$$p_q = \sum m_j q_j \neq 0.$$

And neither their average internal speed of the particles is zero:

$$q = \sum m_j q_j/m \neq 0$$

In the present textbooks of physics, the notion of "the total amount of internal translational motion" of an ideal gas is not commonly used, instead the notion of internal or thermal kinetic 'energy' is used (v):

$$K_T = mv^2/2 = (3/2)NkT$$

and the average thermal speed of the molecules is defined on the basis of this thermal energy, as the root mean square speed of the molecules:

$$v_{rms} = \sqrt{3kT/(m/N)}$$

5. The flow characteristics of congruent translational velocity

5.1 Mass flow

In engineering sciences "the amount that flows across a given section per unit time" is defined as "the flow rate" or shortly "the flow" (Q). In electro-magnetism; the word 'flux' (Φ) is used (electric flux, magnetic flux) to indicate the same characteristic.
In principle the amount of any physical quantity can be used to define the flow or the flux: mass flow/flux, momentum flow/flux, heat flow/flux, charge flow/flux, energy flow/flux, etc.

(v) This will be analyzed in my paper "The physical nature of pressure, temperature, thermal energy and thermodynamic entropy".

In fluid dynamics one of the basic applications of the concept of 'flow' is the equation of the mass flow (Q_m) which gives the amount of mass (m) that flows across a given section per unit time, which is equal to the number (N) of unit mass particles (m_1) per unit time:
$Q_m = Nm_1/t = m/t = \rho V/t = \rho AL/t = \rho Av$
It is similar to the electric current (I) which is in fact nothing but "the charge flow" (Q_q) which is the amount of unit-charge particles (e) that flow across a given section per unit time, which in metallic conductors is the number of electrons (N_e) per unit time:
$Q_q = N_e/t = q/t = I$

5.2 Total amount of mass flow

In a general way, "mass per second" indicates a growth or a decrease of mass in time, rather than a motion with a given magnitude and a given direction.
But the mathematical expression of 'mass flow' is also a typical engineering characteristic expressing the mass per unit time that flows through a specific arrangement of pipes or ducts.
In the specific case of a fluid with N unit particles, each with mass 'm_1', that moves with a velocity '**v**' in a piping system with a total length '**L**', the mass flow of the fluid can be expressed in function of the length of the piping system:
$Q_m = Nm_1/t = Nm_1(\mathbf{v_c}/\mathbf{L}) = m.(\mathbf{v_c}/\mathbf{L}) = (m/\mathbf{L}).\mathbf{v_c}$

In this expression, the factor 'm/**L**' represents the (average) mass per unit length in its direction of motion, whereas '**v**' represents the congruent velocity of the particles of the fluid. In that way, the mass flow, or 'the mass per unit time' is equal to 'the mass per unit length' in its direction of motion times its congruent velocity.
This viewpoint allows us to reveal the true nature of the 'linear momentum' of a moving mass particle system: if the mass of the particles in a unit length times their congruent velocity is equal to the mass flow 'Q_m', then the total mass of the particles in the whole length of the particle system, times their congruent velocity must be equal to the 'total amount (or capacity) of (congruent) mass flow' ($\mathbf{Q_{mL}}$) of that particle system:
$\mathbf{Q_{mL}} = Q_m.\mathbf{L} = (m\mathbf{v_c}/\mathbf{L}).\mathbf{L} = m.\mathbf{v_c} = \mathbf{p}$ (expressed in M = kg.m/s = N/s)

This demonstrates that the 'linear momentum' of a mass particle system is in fact nothing else than a mathematical expression of its total amount of congruent mass flow (or its total amount of congruent translational motion).

6. Conclusion: the physical nature of linear momentum

In the present physics the linear momentum of a body is arbitrarily defined as the product of its mass times its vector velocity in a given reference frame, and its conservation is explained as a consequence of Newton's first law of motion.

In this paper I have explained the physical meaning of the velocity of a particle system, as the 'common' translational velocity ($\mathbf{v_c}$) with which all the particles of the particle system move in a coherent way, that is with the same speed in the same direction, and which cannot in any way be affected by the internal/thermal motions of the particle system, so that without external interaction, the congruent velocity of a particle system must remain constant.

This allowed me to demonstrate that the 'linear momentum' of a moving particle system is a mathematical expression of its total amount of coherent mass flow, which is an engineering term that expresses the total amount of congruent translational motion.

This leads us to the conclusion that the conservation of linear momentum of a mass particle system is a mathematical expression of physical fact that a particle system cannot change its total amount of congruent translational velocity/motion at the given velocity level by its own.

REFERENCES

[1] Resnick & Halliday “Physics Part 1” p 222 – 223, John Wiley & Sons Inc. (1966).

[2] Pippard A. B. “The Elements of Classical Thermodynamics” p 1 – 4, Cambridge University Press (1966).

[3] Orear Jay “Physics” p 200, Macmillan Publishing Co., Inc. (1979).

[4] Goldberg Stanley “Understanding Relativity” p 53, Birkhäuser Boston Inc. (1984).

[5] P.W. Atkins “Physical Chemistry” p 99 Oxford University Press (1999).

[6] Resnick & Halliday “Physics Part 1” p 191, John Wiley & Sons Inc. (1966).
