# Model of the Universe based on the Repulsive Dark Matter

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### Abstract

This article describes the new model of the Universe that is an alternative for the wellknown Big Bang (BB) model. The recently published paper <sup>[1]</sup>, where the authors have presented data on the oldest star in the Milky Way galaxy halo, challenges the validity of the BB model claim about the age of the Universe. It is thus apparent that there is a need to develop an alternative model for the Universe that would not have this problem and provide a better agreement with observations. The model presented in this paper offers such a new alternative by assuming that the Universe is not expanding and is filled with a static gravitating "dark matter" (DM) that is transparent and therefore does not absorb light. This matter provides a framework in which the visible matter moves similarly as defects or vacancies move in a crystal floating from the bulk to the surface. It is further assumed that the visible matter may have been created from this dark transparent matter by an unspecified process sometime in the past, or is being constantly created with a smaller rate. After aggregation to stars and galaxies the visible matter is driven out to the edge of the Universe where it disintegrates and generates the immense Gamma Ray Bursts (GRB). This radiation then may contribute to the generation of new matter throughout the Universe similarly as the assumption that the matter is being constantly created in the Fred Hoyle's model of the Universe<sup>[2]</sup>. The DM model provides equations for the observed recession velocities of distant galaxies, and for many remaining parameters that follow directly from the Hubble constant such as: the size and the mass of the Universe, the maximum observable luminosity modulus, the maximum observable Z shift, the maximum galaxy recession velocity, the size of the average galaxy, etc.. An important relation, also derived from the model, is the relation between the Hubble constant and the temperature of the cosmic microwave background radiation (CMBR). This relation allows a precise calculation of Hubble constant from this temperature. The developed theory is compared with the available data of the GRBs, the Supernova Cosmology project, and the BATSE catalog, and a very good agreement is obtained.

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**Key words:** Repulsive DM Model of the Universe, Hubble Constant, Hubble mass, Galaxy maximum recession velocity, Galaxy maximum age, Gamma Ray Bursts, CMBR temperature, Relation of Hubble constant to CMBR temperature.

### 1. Introduction

"Astrophysicists are always wrong, but never in doubt", R. P. Kirshner.

The model of the Universe that has attained the large popularity is the so called Big Bang (BB) model. This model was developed by a simplistic extrapolation from the famous Hubble discovery and from many later measurements that are clearly showing most of the observed galaxies receding from Earth with an increasing velocity in a linear proportion to their distance from Earth. However, during the process of model refinement and further development several problems have surfaced and are discussed in many publications<sup>[3]</sup>. An increasingly complex modifications and improvements of the model, usually by adding new tunable parameters, are thus necessary to explain these problems away. There are also well known critics of the BB theory that are bringing forward strong arguments against its validity<sup>[2]</sup>, which is in agreement with the recent Hubble telescope observations of vast regions of young star formations. However, a new difficulty for the BB model has appeared following the data published in a paper in 2007<sup>[1]</sup> where the authors describe the discovery of a star in our Milky Way galaxy halo that is 13.2 Gyr old. The authors commented that this age is within the current BB model age limit of 13.7 Gyr, so this should not be a problem. However, it now seems that the model does not give enough time for the hydrogen to condense into stars, burn through the two star generations, and then form the galaxies with halos. There should be many galaxies identical to our own with the same age as a consequence of the *ad hoc* assumption of the initial rapid superluminal inflation postulated in the BB theory. These galaxies are located far away and must now have the same halos and old stars in them as well. Since we see these galaxies almost fully developed and the light from them has traveled to us perhaps 13.1 Gyr, this results in an unreasonably short galaxy formation times of 0.5~0.6 Gyr since the BB. Finally, as will become clear later, the galaxies explode near the edge of the Universe generating GRBs. Therefore, in order to travel there yet another time needs to be added to the average galaxy age. It is thus more reasonable to assume that the average galaxy, which includes our Milky Way, is at least 20~40 Gyr old. This certainly does not fit the BB model and a new model of the Universe needs to be developed.

The work described in this paper is an extension of the previously published work where the uniform dark matter density was assumed. This assumption is now removed and a much better agreement with the available data of the GRBs and the Supernova Cosmology project is obtained <sup>[4]</sup>. The theory provides equations for the duration of the GRBs and the frequency of the GRB occurrences that correlates well with the temperature of the cosmic microwave background radiation CMBR. The new model also allows to calculate the Hubble constant from the CMBR temperature, supplies values for the size and the mass of the Universe, the dark matter pressure at the origin of the Universe, the limit for the maximum observable Z shift, the maximum value for the luminosity modulus that can possibly be observed, the maximum galaxy recession velocity, the size of the average galaxy, and the time to the Milky Way galaxy destruction. None of these interesting parameters that uniquely follow from the Hubble constant and the current recession velocity of the Milky Way galaxy are available from the standard Big Bang model of the Universe.

#### 2. Model assumptions

The key assumption of the new Universe model is that the visible matter represents only a small portion of the total matter of the Universe and can be essentially neglected in its long range gravitational effects. The most of the matter of the Universe is therefore dark. This dark (transparent) matter (DM) will be considered attractive to itself everywhere but causing a repulsive force to visible matter. In the previously published paper the author has assumed that the DM has a uniform density <sup>[5]</sup>. This assumption is now relaxed and the DM is considered compressible having the density  $m_0$  and the pressure  $P_0$  at the origin. More details about the physical nature of the DM mass density, about the origin of its repulsive force, and about its relation to the CMBR temperature are given in the last section of this paper. The propagation speed of the pressure disturbances in the DM abstract and flat physical space-time will be considered constant, equal to the speed of light, and satisfying the well-known formula for the speed of sound:  $c = \sqrt{P/m}$ . This is consistent with the relation  $P = mc^2$ , when P is identified with the energy density. It will also be considered that the radiation that is moving at the speed of light relative to the DM reference frame does not have any gravitational mass <sup>[6]</sup>. The visible matter natural space-time description will be based on the Riemann metric hypothesis of curved space-time and the DM space will be considered having a finite size.

The constants such as the speed of light, the Hubble constant, the gravitational constant, the Milky Way recession velocity relative to the CMBR, etc., are the results of measurements and are not the part of assumptions.

# 3. Mathematical background of the model

This section describes the logical consequences that follow directly from the assumptions stated above. The validity of assumptions can be, of course, justified only by comparing the derived consequences with observations and measurements.

Since the long range gravitational effects of visible radiating mater and all of the radiation can be neglected, the space-time metric can be considered static, spherically symmetric, and described by the following differential metric line element <sup>[7]</sup>:

$$ds^{2} = g_{tt} (cdt)^{2} - g_{rr} dr^{2} - \rho^{2} g_{tt} d\Omega^{2}$$
(1)

where:  $d\Omega^2 = d\vartheta^2 + \sin^2 \vartheta \cdot d\varphi^2$ ,  $g_{tt} = \exp(2\varphi_v)$ ,  $g_{tt}g_{rr} = 1$ , and *c* is the local intergalactic speed of light. The cosmological Newton gravitational potential for the visible matter,  $\varphi_v$ , normalized to  $c^2$  is calculated using the well-known equation:

$$\varphi_{\nu}(r) = -\frac{4\pi\kappa}{c^2\rho(r)} \int_{0}^{\rho(r)} m(\rho)\rho(r)^2 d\rho$$
<sup>(2)</sup>

where  $\kappa$  is the Newton gravitational constant. Due to the deformation of the observed natural radius *r* by the DM gravity, the physical radius  $\rho(r)$  must be used in the formula and this parameter is found from the differential equation that follows from the metric:

$$d\rho = \sqrt{g_{rr}} dr = e^{-\varphi_{r}} dr \tag{3}$$

Because any particular galaxy now represents only a small test body in this Universe, the well-known and many times verified Lagrange formalism will be used to describe the motion of such galaxies. The Lagrangian is therefore as follows <sup>[7]</sup>:

$$L = e^{2\varphi_{v}} \left(\frac{cdt}{d\tau}\right)^{2} - e^{-2\varphi_{v}} \left(\frac{dr}{d\tau}\right)^{2} - \rho^{2} e^{2\varphi_{v}} \left(\frac{d\Omega}{d\tau}\right)^{2}$$
(4)

For a purely radial motion the Lagrangian can be simplified and the first integrals of the

corresponding Euler-Lagrange equations easily found using the initial condition at the origin where the recession velocity is zero and where:  $d\tau = dt$ . The results are:

$$\frac{dt}{d\tau} = e^{-2\varphi_{v}} \tag{5}$$

$$\left(\frac{dr}{d\tau}\right)^2 = c^2 - c^2 e^{2\varphi_v} \tag{6}$$

Eliminating  $d\tau$  from these equations, because  $d\tau$  is not an observable parameter, leads to the formula for the recession velocity:

$$\frac{dr}{dt} = c \ e^{2\varphi_v} \sqrt{1 - e^{2\varphi_v}} \tag{7}$$

For the relatively near objects, where the cosmological gravitational potential  $\varphi_{\nu}$  is still small, it holds that:  $\rho(r) \cong r$ , and  $m(\rho) = m_0$ . This simplifies Eq.7 as follows:

$$\frac{dr}{dt} \cong c\sqrt{-2\varphi_v} = \sqrt{\frac{8}{3}\pi \kappa m_0} r = H_0 r$$
(8)

From this result it is then clear that the recession velocity is linearly proportional to the natural coordinate distance r of such objects from the origin and that the Hubble constant  $H_0$  is related to the DM density  $m_0$  at the origin according to the following equation:

$$H_0 = \sqrt{\frac{8}{3}\pi \kappa m_0} \tag{9}$$

The recession velocity and the Hubble constant are referenced to the DM coordinate system, so the value of the Hubble constant should be corrected and referenced to the Earth's centered coordinate system where it is actually measured. However, the correction is very small and it will be neglected. Earth and its Milky Way galaxy are located relatively near the center of the Universe in comparison to its immense size.

In order to proceed further in the model development it is necessary to find the relation for the DM density  $m(\rho)$  as a function of the physical radius. This is obtained by adapting the well-known approach described, for example, by Zel'dovich <sup>[8]</sup> where the DM pressure gradient can be expressed as a function of the physical radial distance:

$$\frac{dP}{d\rho} = -\frac{4\pi \kappa m(\rho)}{\rho^2} \int_0^\rho m(\rho) \rho^2 d\rho$$
(10)

After substituting for the DM pressure from the relation:  $c = \sqrt{P/m}$ , and defining the

normalized mass density function:  $m_n(\rho) = m(\rho) / m_0$ , Eq.10 can be rearranged with the help of the Green's function as:

$$m_n(\rho) = \exp\left(-A_0 \int_0^\rho m_n(\xi) \left(\xi - \xi^2 / \rho\right) d\xi\right)$$
(11)

where  $A_0$  is a constant equal to:  $A_0 = 4\pi \kappa m_0 / c^2$ . There is no known analytic closed form solution for this equation, so it is necessary to use the numerical iterative approach or find an approximating function. The approximating function approach was selected for the next steps to avoid very long computing times during iterations. The selected function, however, underestimates the true value of the DM mass density at large  $\rho$ , but the error has only a small overall effect. The first two iterations and the approximating function:

$$m_{a}(\rho) = \exp\left(-\frac{\rho^{2}}{\rho_{h}^{2}} + \frac{3}{10}\frac{\rho^{4}}{\rho_{h}^{4}} - \frac{4}{35}\frac{\rho^{6}}{\rho_{h}^{6}} + \frac{61}{1260}\frac{\rho^{8}}{\rho_{h}^{8}} - \frac{4507}{231000}\frac{\rho^{10}}{\rho_{h}^{10}} + \dots\right)$$
(12)

are shown in a graph in Fig.1. More details of the approximating function derivation are given in the Appendix. The introduced parameter defined as:  $\rho_h = 2c/H_0$ , is called the Hubble distance or the Hubble physical radius:  $\rho_h = 2.720 \cdot 10^{26} m = 28.76 \cdot 10^9 Ly$ .



Fig.1: The first two iterations (dashed and dot-dashed traces) and the approximating function describing the dark matter mass density as a function of the physical radius where:  $x = \rho / \rho_h$ .

Another advantage of using the approximating function is that the DM concentration tail extending past the maximum radial distance can be easily cut off by suitably truncating the power series expansion in the exponent. This feature is advantageous if it is considered that the visible matter debris from explosions of galaxies are accumulating at the edge of the Universe and are forming a loosely bound shell there. Of course, it is possible to add more terms than shown in Eq.12; however, this will not be pursued any further in this paper, because the accuracy of the approximation was found reasonable as will be discussed later by comparison with observations.

Once the mass density function is known it is easy to find the normalized gravitational potential for the visible matter using the formula in Eq.2, and for the dark matter using the Green's function formula derived also from the Gauss law as follows:

$$\varphi_d(\rho) = A_0 \int_0^{\rho} \frac{d\xi}{\xi^2} \int_0^{\xi} m_a(\zeta) \zeta^2 d\zeta - 3.303 = A_0 \int_0^{\rho} m_a(\xi) (\xi - \xi^2 / \rho) d\xi - 3.303$$
(13)

Both potentials are plotted in the graphs shown in Fig.2.



Fig.2: The dependencies of normalized gravitational potentials for the visible matter (solid trace) and for the dark matter as functions of the physical radius. The integration constants were adjusted such that the potentials at infinity are zero.

In the next step of the model development it is necessary to find the formula for the Z shift, since this is the parameter that is directly measured by astronomers. The Z shift typically consists of three components: the star gravity induced red shift, the cosmological potential induced red shift, and the Doppler red shift resulting from the recession velocity. The star gravity red shift does not have to be considered here, because after the star or the galaxy explosion has occurred most of the principal source of the gravitational field has been converted to radiation and radiated away and only the remnants or the afterglow produce the light that is observed. The cosmological potential induced red shift does not have to be considered the galaxies

are in a radial free fall and this compensates for the shift <sup>[5]</sup>. The only remaining Z shift component is thus the Doppler red shift resulting from the radial recession velocity  $v_r$ . The Doppler red shift observed on Earth is:

$$Z(\rho) = \frac{\sqrt{1 - v_r^2 / c_r^2}}{1 - v_r / c_r} - 1 = \frac{\sqrt{g_t}}{1 - \sqrt{1 - g_t}} - 1 = \frac{\exp(\varphi_v)}{1 - \sqrt{1 - \exp(2\varphi_v)}} - 1$$
(14)

where  $c_r$  indicates the light speed at the galaxy location in reference to Earth. The graph of the Z dependency on the natural coordinate radius r is shown in Fig.3.



Fig.3: The dependency of Z shifts on the natural coordinate radial distance. The maximum Z shift that can be observed is  $Z_{mx} = 10.35$ . The visible matter does not exist at larger distances than:  $r_{mx} = 22.11 \cdot 10^9 Ly$ , since it disintegrates a the Universe's edge.

The radial distance r, also called in this paper the natural radial distance, which is the observable parameter, is calculated according to Eq.3 as follows:

$$r(\rho) = \int_{0}^{\rho} \exp(\varphi_{\nu}) d\rho$$
 (15)

It is also convenient to introduce the average Universe radius  $\rho_a$ , which is the radius corresponding to a sphere with the constant mass density  $m_0$  that has the same total dark mass as the Universe, and the maximum Universe radius  $\rho_{mx}$  that corresponds to the radial distance where the visible matter potential has its minimum.

The numerically computed values for these parameters, including also the Hubble distance for a comparison, all expressed in light years (Ly), are:

$$\rho_h = 28.76 \cdot 10^9 Ly$$
,  $r_h = 18.84 \cdot 10^9 Ly$  (16)

$$\rho_a = 34.73 \cdot 10^9 Ly , \qquad r_a = 20.34 \cdot 10^9 Ly \qquad (17)$$

$$\rho_{mx} = 44.03 \cdot 10^9 Ly$$
,  $r_{mx} = 22.11 \cdot 10^9 Ly$  (18)

The radius of the observable Universe, where the DM ends and only within which the visible matter can exists is therefore:  $r_{mx} = 22.11 \cdot 10^9 Ly$ . The interesting parameter is the DM pressure at the center of the Universe, therefore, near our Earth. The pressure is equal to:  $P_0 = 7.80 \cdot 10^{-10} Pa$ . This is an extremely low value but nevertheless the gradient of this pressure is causing galaxies and all the visible matter to float to the edge of the Universe where they disintegrate. The recession velocity following Eq.7 is shown in a graph in Fig.4 as a function of the natural radial distance. The graphs also include the current velocity of the Milky Way galaxy and the velocity at its formation 40 Gyr ago.



Fig.4: Numerically computed galaxy recession velocity in km/sec as a function of the natural coordinate radius in light years measured from the center of the Universe (purple trace), the limiting vacuum speed of light  $c_r$  (green trace), the current Milky Way recession velocity: 552 km/sec (dashed trace), and the Milky Way galaxy recession velocity: 34.18 km/sec during its formation 40 Gyr ago (dotted trace). The galaxies disintegrate at the distance of: 22.11 bLy.

For completeness and before making comparisons with observations, it is interesting to find the values of the remaining parameters that directly follow from the single Hubble galaxy recession velocity constant. This is in contrast to the Big Bang model that needs at least three to six adjustable parameters and a postulate of a sudden superluminal inflation that has no known physical cause to obtain an agreement with observations. The total dark mass of the Universe calculated from the field is:

$$M_{du} = \frac{c^2}{2\kappa} \int_0^\infty \left(\frac{\partial \varphi_d(\rho)}{\partial \rho}\right)^2 \rho^2 d\rho = -2\pi \, m_0 \int_0^\infty m_a(\rho) \varphi_d(\rho) \rho^2 d\rho = 1.290 \cdot 10^{54} \, kg \qquad (19)$$

This value is perhaps more accurate than the direct integration of the mass approximating function:  $M_{dua} = 1.052 \cdot 10^{54} kg$ , since this satisfies the energy-mass equivalence rule. Both results are larger than the Hubble mass defined as:  $M_h = 4c^3 / \kappa H_0 = 7.327 \cdot 10^{53} kg$ . An agreement between  $M_{du}$  and  $M_{dua}$  is obtained when it is considered that about 19% of dark matter is shielding the visible matter, predominantly at the edge of the Universe. The definitions of Hubble parameters and an example of their use in calculations are given in section **5** and in the Appendix.

The minimum normalized potential of the visible matter at the Universe's edge is:  $\varphi_{v\min} = -1.7436$ , resulting in the maximum for the observable Z shift:  $Z_{mx} = 10.35$ . This value surprisingly agrees with the re-ionization Z shift:  $Z_{ri} = 10.4 \pm 1.2$  of the BB model. The maximum time to the Milky Way galaxy explosion observed by the observers on Earth that may be travelling with it is:  $\tau_{max} = 100.0 \ Gyr$ . This is, of course, much longer than the Hubble time:  $\tau_h = 1/H_0 = 14.38 \ Gyr$ . The time to destruction is calculated using the integral following from Eq.3 and Eq.6:

$$\tau_{\max} = \frac{1}{c} \cdot \int_{\rho_g}^{\rho_{\max}} \frac{d\rho}{\sqrt{\exp(-2\varphi_v(\rho)) - 1}}$$
(20)

where  $\rho_g$  is the current physical distance of our galaxy from the center of the Universe calculated based on the measured Milky Way recession velocity relative to the cosmic microwave background radiation reference frame:  $v_g = 552 km/sec$ .

There is no equation in this model to find the age of the DM Universe, in particular the age is not related to the Hubble time. The age has to be deduced from some other observations and considerations as is, for example, discussed in the already referenced paper <sup>[1]</sup> and as commented on in the introduction. Nevertheless, it is possible to estimate

that the maximum galaxy lifetime is approximately equal to:  $\tau_{glife} = 140 \ Gyr$ . This is the maximum time limit for any intelligence to develop an intergalactic travel capability. The lifetime of any particular galaxy is, of course, dependent on the place of its creation, which would substantially reduce this limit.

### 4. Comparison with observations

The astronomers typically evaluate their observations in terms of the apparent and intrinsic stellar magnitudes by introducing the luminosity modulus:  $\mu_s = m_{sa} - M_{si}$  and plotting it as a function of the Z shift, which can be precisely measured. The modulus can be expressed in terms of the luminosity distance, which is defined as:  $d_L = \rho(r)(Z+1)$ , where  $\rho(r)$  is the physical distance. The resulting equation is then as follows:

$$\mu_{s}(r) = 5\log_{10}\left(\frac{\rho(r)(Z+1)}{10\,pc}\right)$$
(21)

where pc is the distance of one parsec. It is then not too difficult to find the theoretical prediction of luminosity modulus as a function of the Z shift. It is only necessary to invert the formula in Eq.14 and find the physical radius as a function of the Z shift, which is then substituted into the formula in Eq.21. The theoretical dependency of the luminosity modulus on Z shift is shown in Fig. 5 together with the measured values published by Kowalski<sup>[4]</sup> and Schaefer<sup>[9]</sup>. It is also possible to plot the luminosity modulus directly as a function of the natural radial distance as shown in Fig.6. For completeness the plots also include the dependency of the galaxy recession velocity and the speed of light on the natural radial distance. The agreement of theoretical predictions with the measurement is, considering the simplicity of the theory, stunning. The agreement is noticeably better in comparison with the uniform DM density model introduced in the previous publication. This result thus unquestionably confirms the correctness of the developed model, validates the model assumptions, and at the same time raises new doubts about the validity of the BB model. It is also worth mentioning that the upward bending of the luminosity modulus curve in the range of  $5 \cdot 10^9 \sim 2 \cdot 10^{10} Ly$ , or similarly a slight upward bending of the luminosity modulus curve in Fig.5 in the range of: 0.5 < Z < 10 is not caused by the accelerated Universe expansion, which the main stream BB astrophysicists claim that exists.



Fig.5: Measured 304 Supernova (squares) and 69 GRB (circles) data points of luminance

modulus  $\mu_s$  plotted together with the corresponding theoretical values of modulus (black dots) as functions of the Z shift. This diagram is sometimes also called the Hubble diagram and it is the direct comparison of observations with the theory.



NATURAL COORDINATE DISTANCE [light years]

Fig.6: Measured 304 Supernova (squares) and 69 GRB (circles) data points of modulus  $\mu_s$  plotted together with the corresponding theoretical values of modulus (black dots) as functions of the natural radial distance. The recession velocity is also shown on the same graph (plus signs) with the speed of light (triangles). The DM density at the origin used in calculations was:  $m_o = 0.8686 \cdot 10^{-26} kgm^{-3}$ , as derived from the Hubble constant:  $H_0 = 68.0 km \ s^{-1} Mpc^{-1}$ . It is interesting, however, to observe in the plots in Fig.6 that most of the GRB explosions occur in the region where the galaxies decelerate, just before the maximum Universe natural radial distance of:  $r_{mx} = 22.11 \cdot 10^9 Ly$  is reached. If the GRBs were caused by the galaxy collisions, as is typically claimed, the probability of GRB occurrences throughout the Universe's volume would be approximately uniform or more likely skewed towards the nearby older galaxies and some GRBs would occur relatively near Earth. It is possible to estimate, based on the current observed rate of explosions, that during the entire Earth's existence at least 43 GRBs within the radius of 10 million light years would have occurred. The GRB explosions in such a near proximity to Earth would certainly destroy all the life every time and sterilize Earth forever.

Again, the fit of the modulus data to the theory seems very good with a perfect match in many cases. This result, therefore, provides the experimental support for the theory and for the correctness of the metric used in Eq.1. The maximum recession velocity of:  $v_{mx} = 0.385c$  occurs at the distance of:  $r_{vmx} = 9.095 \cdot 10^9 Ly$ . The maximum luminosity modulus at the Universe's edge that can ever be observed is:  $\mu_s(\max) = 50.926$ . It is fascinating to see that this value has already almost been reached. This particular GRB signal must have, therefore, come to us from the region that is very close to the edge of the Universe.

The compressibility of the DM allows for the propagation of DM disturbances in the physical space-time with the speed of light c. The corresponding radial speed of light in the natural space time, as observed from Earth that is currently positioned relatively close to the center of the Universe, is, of course, equal to:  $c_r = c g_u = c \exp(2\varphi_v)$ .

The reason for the GRB explosions has not been established with the certainty yet, however, one of the contributing factors is the reduction of gravitating mass of stars in the galaxy when they approach the edge of the Universe. The gravitating mass depends on the cosmological potential for the visible matter as was explained in more detail in the author's previous paper <sup>[5]</sup>. The most likely possibility, however, is that the visible matter aggregates at the boundary of the Universe forming there a shell from debris, or a shell from clouds of hydrogen plasma, or even a shell from clouds of elementary particles such as neutrons, since the visible matter gravitational potential has its minimum there. The

neutrons could then be the predominant source of the cosmic neutrinos detected here on Earth. This structure has a temperature of the CMBR and the new galaxy arrivals collide with it converting most of their mass into radiation. Finally, it is possible to consider that the reduction of the DM pressure at the edge of the Universe cannot keep the visible matter compacted together any longer and causes it to disintegrate. However, the extremely low value of the DM pressure does not seem to make this possibility very likely. Nevertheless, it might be interesting to investigate if the DM pressure, or the deep negative DM potential, could somehow be dynamically created here on Earth and induce the controlled nuclear fission of any visible matter as a source of energy. The correct understanding of the physics of gravity and the Universe is thus very important for the advancement of technology.

The frequency of the GRB explosions per day can roughly be estimated assuming that they are caused by the annihilation of the Milky Way size galaxy central masses equal approximately to:  $4.0 \cdot 10^6$  Suns. From the CMBR temperature of:  $T_b = 2.725K$ , using the Stephan-Boltzmann law, it is simple to calculate the total heat energy radiated from the interface back into the Universe considering that there is approximately  $2 \cdot 10^{11}$  Suns in the average galaxy. The mass of the galaxy is therefore:  $M_G = 2 \cdot 10^{11} M_s$ . The efficiency of the mass conversion to heat is assumed:  $\eta = 1.0\%$ , since most of the energy from the explosions is converted to gamma rays. The efficiency of the GRB detection is assumed:  $\xi = 50\%$  due to the Earth shielding effect. The observation time of one day expressed in seconds is:  $t_d = 8.64 \cdot 10^4$  sec. The equation for the count of the GRB explosions per day is then as follows:

$$N_{GRB/day} = \frac{\xi}{\eta} \frac{8}{15} \pi^6 \left( \frac{k_B T_b}{\sqrt{g_{tt}}} \right)^4 \frac{\rho_{mx}^2 g_{tt} t_d \sqrt{g_{tt}}}{c^4 g_{tt}^2 h^3 (M_G / \sqrt{g_{tt}})}$$
(22)

where  $k_B$  is the Boltzmann constant. There is no cosmological potential effect on  $k_B$ , similarly as there is none on the Planck constant <sup>[5]</sup>. The effect on the remaining parameters is included in the formula as indicated. The surface area of the interface was calculated using the metric given in Eq.1. After simplification and substitution of values for the parameters, the GRB count per day is:

$$N_{GRB/day} = \frac{\xi}{\eta} \frac{8}{15} \pi^6 \left( \frac{k_B T_b}{hc \sqrt{g_{tt}}} \right)^4 \frac{\rho_{mx}^2 h t_d}{M_G} = 0.88$$
(23)

The obtained result is in a reasonable agreement with the observed frequency of the long duration GRB pulse occurrences. The detected CMBR is therefore the image of the boundary region of the Universe and not the remnant of the Big Bang. The supporting observational evidence for this conclusion is obtained from the angular dependence of the CMBR power spectrum ripples <sup>[10]</sup>. The arriving galaxy explosion disturbances propagate along the surface of the Universe's boundary shell forming the well-known circles on the CMBR background, which are detected as shown in Fig.7.



Fig.7: NASA WMAP data of angular dependence of the CMBR power spectrum ripples caused by the galaxy explosions at the edge of the Universe<sup>[10]</sup>.

The calculation of the gamma ray flux measured on Earth from the conversion of the galaxy central masses to energy is less reliable, since the amount of absorption of the gamma rays on their way to Earth is not known. It is assumed that the absorption is considerable since it may be contributing to the generation of new visible matter.

The length of the long duration GRB pulses, on the other hand, can be readily found in the current DM model. The calculation is best performed in the physical space-time where it is simple to find the minimum physical radius to which any large mass can be compacted to and divide the result by the speed of light. After the physical time length of the explosion is found it is then only necessary to add the cosmological time dilation factor to it to obtain the Earth's observed duration. The minimum radius of the galaxy central mass is:  $\rho_{\min} = R_s / 4$ , as was derived previously elsewhere <sup>[7]</sup>, with  $R_s$  being the Schwarzschild radius of the galaxy central mass. The resulting equation for the GRB pulse duration observed on Earth is thus as follows:

$$\tau_{grb} = \frac{\rho_{\min}}{c\sqrt{g_{tt}}} = \frac{\kappa M_s 4.0 \cdot 10^6}{2c^3 \sqrt{g_{tt}}} = 56.34 \cdot \text{sec}$$
(24)

The value of  $g_{tt}$  substituted into Eq.24 is the value at the Universe's edge:  $g_{tt} = e^{-3.4872}$ . This result agrees well with the statistic obtained from the BATSE catalog <sup>[11]</sup> as is shown in Fig.8, which the standard BB model does not predict. This also experimentally disproves the existence of Black Holes, which are now replaced by a very compact masses without event horizons <sup>[5,7]</sup>, and can, therefore, explode. The peak of the short GRB pulse durations corresponds to the explosions of Quasars that did not have enough time yet to develop into the full size galaxies, or corresponds to the first generation of massive stars. The short pulse duration agrees again well with the prediction obtained from Eq.24 when the average mass of Quasars is substituted into the formula. This provides once more a good experimental support for the presented model of the Universe.



Fig.8: Statistical distribution of the long and short GRB pulse durations as published in the BATSE 4B catalog<sup>[11]</sup>.

The described Repulsive Dark Matter theory has only one adjustable parameter to fit the data, as already mentioned, the Hubble constant:  $H_0 = 68.0 km \ s^{-1} Mpc^{-1}$ . This is in stark contrast to the existing Big Bang model, which requires at least three or more parameters

that need to be adjusted and an *ad hoc* postulate of a sudden superluminal inflation that is difficult to justify by the well-established laws of physics. The more adjustable parameters there are, the less reasonable the model becomes, since with a large number of adjustable parameters it is always possible to fit any data.

A significant confirmation of validity of the presented DM model, however, would come from the prediction of the galaxy rotation curves. In order to accomplish this task it is necessary to consider that the DM is partially depleted with its concentration reduced in the vicinity of the galaxy central mass. The depletion of the dark matter is due to the introduction of the DM compressibility and results from the mutual repulsive force between these two types of matter. This phenomenon was difficult to justify in the previous model of the uniform DM density, but arises naturally in the deformable DM model concept. The gravitational force that is generated from the depleted region, which is depleted relative to the locally approximately uniform background, can thus be viewed as additional attractive force acting on the visible matter. A significant depletion, however, occurs only in the very near vicinity of the compact masses, but a small fraction of depletion extends to larger distances. Equation for the DM concentration in the neighborhood of the mass  $M_s$  can be derived following the same concept as in the derivation of Eq.10.

$$\frac{\rho^2}{m(\rho)}\frac{dm(\rho)}{d\rho} = -\frac{4\pi\kappa}{c^2}\int_0^\rho m(\rho)\rho^2 d\rho + \frac{\kappa M_s}{c^2}$$
(25)

In the case of a galaxy, the non-uniform distribution of visible matter in the galaxy's arms, the mutual gravitational star interaction, and the compensating centrifugal forces due to the arms' rotation including the local rotation or counter rotation of DM itself that may be dragged along by the arms, unfortunately complicate the calculations of the DM distribution further and as a result the complete and accurate model of the galaxy rotation is not expected to be easy to develop. This work is deferred to future publications.

Despite of these complications, however, the DM model of the Universe offers an interesting calculation that is related to the sphere of the gravitational influence of each star or the entire galaxy. At a certain radius the DM mass density will again return to the local background level and at that point the derivative of the density will be equal to zero. From Eq.25 then follows that at that radius it will hold:

$$M_s = 4\pi \int_0^{\rho_i} m(\rho) \rho^2 d\rho$$
(26)

Since the change of the DM density over the local background in most of the volume of the sphere of gravitational influence is very small, it is reasonable to consider that  $m(\rho) \approx m_0$ . This simplification then yields the equation for the radius. Also, for the visible matter potential at the edge of the sphere we have from the Gauss law:

$$\rho^2 \frac{d\varphi_v}{d\rho} = \kappa M_s - 4\pi \kappa \int_0^{\rho_i} m(\rho) \rho^2 d\rho = 0$$
(27)

It is therefore clear that at the edge of the sphere of gravitational influence the gravitational field is completely compensated and shielded by the DM. For the star of the mass equal to the mass of our Sun the diameter of the sphere of gravitational influence is thus simply determined as follows:

$$d_{is} = 2 \sqrt[3]{\rho_h^2} \frac{\kappa M_s}{2c^2} = 802.3 \cdot Ly.$$
 (28)

If we now consider that the arms of the Milky Way galaxy on average consist of stars similar to our Sun it is clear that the Milky Way disc thickness should be approximately equal to or slightly larger than the diameter of the sphere of the gravitational influence of individual stars considering only a moderate sphere overlaps. It is estimated that the Milky Way galaxy thickness is approximately equal to:  $d_{ig} = 1000 \cdot Ly$ . This value is reasonably close to the value calculated from the formula in Eq.28, considering that any vertical star distribution in the galaxy's arms was neglected and that the average mass of the stars in the galaxy may be greater than the mass of our Sun. Similarly, if it is considered that the Milky Way galaxy center harbors a compact star with the mass equal to:  $3 \cdot 10^6 \cdot M_s$ , the diameter of the galaxy's sphere of the gravitational influence is found from the same formula as in Eq.28 and is equal to:

$$d_{iG} = 2 \sqrt[3]{\rho_h^2 \frac{\kappa M_s 3 \cdot 10^6}{2c^2}} = 1.157 \cdot 10^5 \cdot Ly.$$
(29)

The astronomers estimate that the Milky Way galaxy diameter is: 100,000 Ly. Again, this value is in a reasonable agreement with observations. However, the interaction of the DM mass with the visible matter is not linear and the mutual interaction of the moving stars in

the galaxy's arms is also complicated. It is, therefore, interesting that despite the large number of stars in the galaxy, estimated to be approximately:  $2 \cdot 10^{11}$ , no stable structure is formed past the radius of the gravitational influence of the central body. If all the stars were concentrated in one central mass then the diameter of the gravitational influence would be 40.5 times larger. Therefore, it appears that the motion of stars in the galaxy's arms and their large aggregate mass is disruptive enough that when the stabilizing effect of the central body is shielded by the DM mass no stable structure except the galaxy halo can develop at larger distances. It is thus also clear that most of the galaxies on a large scale are not bound among themselves by the gravitational forces and will not evolve into a single giant Universe galaxy during their long lifetime as it might be otherwise expected. The galaxies can form clusters or strings only if their mutual distances do not exceed the 40 times their diameter. The most galaxies thus move independently in a free fall to the edge of the Universe, which is consistent with the original assumption of the repulsive DM model. The DM shielding effect thus seems to be the necessary requirement for the existence of individual galaxies in the first place. The gravitational field of the visible matter therefore does not extend to infinity. It is also interesting that the mass of the galaxy central star, the Hubble distance, and the galaxy diameter are all related by the simple formula introduced in Eq.29. It would be interesting to confirm the validity of this formula by investigating if the size of the galaxies depends on their distance from Earth, since the DM density is reduced at larger distances.

As a last topic of this section it is also necessary to mention the gravitational waves. It is perhaps without any doubt that the gravitational waves in the tensor form or in a simplest case in the scalar form must exist and are produced. However, the repulsive interaction between the visible matter and the dark matter may significantly affect their long range propagation and detection due to the various screening and matter depletion effects. So, until a more sophisticated dynamic model of this interaction is developed it is difficult to claim with certainty that the gravitational waves can be detected by the currently proposed methods <sup>[12]</sup>, in particular from the extragalactic sources. To this date only two gravitational wave events were detected: <u>http://vixra.org/abs/1606.0203</u>.

In conclusion of this section it is once more necessary to emphasize that the DM frequently described in the literature, where it is used to explain the gravitational lensing,

where it is also claimed that it causes additional forces needed to keep the galaxies together, and where it is needed to obtain an agreement with the measurements of the galaxy rotating curves, is different than the DM described in this paper. In this paper the DM is causing the shielding effect for the standard gravitational forces and therefore the DM measurements published in the literature are most likely describing the missing or depleted repulsive DM and other forces resulting from the galaxy arms motion.

# 5. The origin of the DM repulsive force

This section provides explanation for the possible origin of the DM repulsive force, derives the specific value for the DM mass quantum, and derives an interesting relation between the Hubble constant and the CMBR temperature.

The Hubble constant introduced in Eq.9 is related to the Universe's DM mass density at the origin as follows:

$$H_0 = \sqrt{\frac{8}{3}\pi \kappa m_0} \tag{30}$$

From the simple dimensional analysis and the distance comparisons in Eqs.16,17,18, also follows that the Hubble radius or the Hubble distance is correctly defined as:

$$\rho_h = 2c / H_0 \tag{31}$$

and not as is sometimes claimed in the mainstream literature equal to:  $\rho_h = c/H_0$ . Similarly, the Hubble DM mass of the Universe can be calculated as:

$$M_{h} = \frac{4\pi}{3} \rho_{h}^{3} m_{0} = \frac{4c^{3}}{\kappa H_{0}}$$
(32)

and finally, for the Hubble time it is customary to write:

$$\tau_h = 1/H_0 \tag{33}$$

The Hubble constant can, therefore, be considered also as a nature's lowest frequency and used for the frequency normalization.

The interesting parameter is the Planck-Hubble mass quantum:

$$m_{qh} = H_0 h / c^2 = 1.625 \cdot 10^{-68} kg = 9.114 \cdot 10^{-33} eV$$
(34)

and its dimensionless ratio to the Hubble DM mass of the Universe:

$$N_{ph} = \frac{M_h}{m_{qh}} = \frac{4c^5}{\kappa H_0^2 h} = 4.520 \cdot 10^{121}$$
(35)

This very large number can be established as the fundamental constant of the Universe.

Another and much more interesting fundamental DM mass quantum can be obtained by assuming that the DM is almost a mass-less crystal-like structure consisting of cells with the standing wave vibrations. An example of such vibrations is the trapped photons that, as is well known, exert a repulsive force on the cell walls. This is consistent with the idea of repulsive DM force acting on the visible matter. The cell mass is then derived from these vibrations and any other mass that could be the constituent of the cells will be neglected. The cells may have random shapes and sizes, but in their simplest representation may be considered on the average as cubes. The formula for the cell mass can then be written as:

$$m_{q\lambda} = \sqrt[4]{\frac{3}{8} \frac{m_0 h^3}{c^3}} = 1.369 \cdot 10^{-38} kg$$
(36)

This formula is derived from the consideration that each cell has an average volume:  $V_{q\lambda} = \xi (\lambda/2)^3$  and contains the energy  $E_{q\lambda} = m_{q\lambda}c^2$  equal to:

$$E_{q\lambda} = m_0 c^2 \xi \left(\frac{\lambda}{2}\right)^3 = \frac{hc}{\lambda}$$
(37)

The parameter  $\xi = 3$  represents the threefold spatial degeneracy corresponding to the three orthogonal cell vibrations with the same frequency. When this mass is compared with the mass of neutrinos, the particular neutrino would have to have its mass equal to:

$$m_{ni} = 7.682 \ meV$$
 (38)

Such a low neutrino mass has already been predicted elsewhere <sup>[13]</sup>.

However, when the mass  $m_{q\lambda}$  is converted to energy and the energy to temperature at the edge of the Universe, where the visible and the dark matter are in a thermal equilibrium due to their strong mutual gravitational interaction there, the result is surprisingly close to the temperature of the CMBR observed on Earth:  $T_b = 2.7255 \pm 0.0006^{\circ} K$ <sup>[14]</sup>. The Earth observed temperature  $T_{b\lambda}$  corresponding to  $m_{q\lambda}$  is calculated according to the formula:

$$T_{b\lambda} = \frac{m_{q\lambda}c^2 g_{tt}}{k_B} = 2.727^{\circ}K$$
(39)

where for  $g_{tt}$  it was again substituted:  $g_{tt} = e^{-3.4872}$ . This result, therefore, clearly shows

that the Hubble constant and the CMBR temperature are not the mutually independent parameters. The close agreement of this result with the measurement provides once more a clear experimental support for the correctness of the repulsive DM model of the Universe. The Hubble constant can thus be directly and precisely calculated from the measured CMBR temperature according to the following formula:

$$H_{0b} = \frac{8}{3} \sqrt{\frac{\pi \kappa c^3}{h^3}} \left(\frac{k_B T_b}{c^2 g_t}\right)^2 = 67.922 km \ s^{-1} Mpc^{-1}$$
(40)

and the mass quantum from the formula:

$$m_{q\lambda} = k_B T_b / c^2 g_{tt} = 7.677 meV$$
(41)

with the corresponding resonant DM frequency:

$$v_b = \frac{k_B T_b}{g_u h} = 1.856 T Hz \tag{42}$$

which the standard BB model of the Universe cannot offer. All the Hubble parameters are, therefore, determined by the CMBR temperature. These relations are very interesting and may be important for the evaluation of various models of the Universe where the precise value of Hubble constant may help to distinguish between the correct and incorrect theories that are attempting to describe the reality.

#### 6. Conclusions

In this paper the previously developed uniform DM density model of the Universe was generalized to a new model where the DM is deformable thus permitting the propagation of disturbances with the speed of light. The model provides equations for the recession velocity of galaxies and a number of other parameters such as the total DM mass of the Universe, the size of the Universe, the maximum galaxy recession velocity, the maximum observable luminosity modulus, the maximum observable red shift, and the relation between the CMBR temperature and the Hubble constant. All of these parameters are determined by only three constants: the Hubble constant, the gravitational constant, and the speed of light. The recession of galaxies in this model resembles the motion of defects or vacancies in a solid matter that seem to float from the bulk to the surface, the Universe's edge, where they disintegrate. It is therefore reasonable to conclude that the disintegration of the galaxy centers is the cause for the long duration GRB pulses. The

short duration GRB pulses seem to result from the disintegration of Quasars. The pulse duration was calculated from the mass and the size of the typical galaxy central bodies and a good agreement with observations was obtained. It was also concluded that the galaxy central masses cannot be the Black Holes but are very compact massive objects without the event horizons <sup>[5,7]</sup>. The GRB radiation back to the Universe's bulk may be the cause for the creation of new matter through the Universe, which then condenses to new stars and new galaxies, repeating endlessly the cycle of destruction and creation. The CMBR temperature seems to also correlate well with the number of the destroyed galaxies per day. The detected CMBR radiation pattern is thus the image of the Universe's edge region and not the remnant of the BB. The developed alternative model provides values for the luminosity modulus as function of the radial distance from the center of the Universe. This function was compared with the extensive data available from the GRBs, and the Supernova Cosmology project, and an excellent agreement between the theory and observations was obtained. Finally, the model also provided a rough estimate for the size of the average galaxy and determined its approximate relation to the Hubble distance and the mass of the galaxy's central body. The agreement of theory with observations thus suggests that the model is correct. It is, therefore, clear that the new model presents a good alternative for and a considerable challenge to the main stream BB theory. The new model also avoids the number of implausible and very strange assumptions, which the BB model must have; the sudden creation of all the Universe's visible matter from a single point singularity, a sudden space inflation but only between the galaxies not within the individual atoms of matter, the endless Universe expansion with galaxies accelerating without a force acting on them again only in places where it seems to fit the narrative, and finally the Universe's mass disappearance to nothingness after its final conversion to radiation. All of these strange assumptions including few others that are also well known are discussed elsewhere <sup>[3]</sup> in the published literature.

### Appendix

This section illustrates the use of the Hubble radius  $\rho_h$  and the average Hubble radius  $\rho_a$  defined previously that naturally followed from the Hubble constant to simplify the derived formulas by transforming them into the dimension-less forms. For example, by

introducing the dimension-less radius:  $x = \rho / \rho_h$ , and then substituting for the mass:  $m(\rho) = m_0 \exp(\psi(\rho))$  into the formula in Eq.11 the result, after differentiation, becomes:

$$x\psi(x)'+2\psi(x)'=-6x\exp(\psi(x))$$
(A1)

where the prime denotes the derivative with respect to x. Similarly, using the substitution:  $m(\rho) = m_0 \exp(\psi(\rho) - \kappa M_s/c^2\rho)$ , Eq.25 is transformed into:

$$x\psi(x)''+2\psi(x)'=-6x\exp(\psi(x)-\alpha/x)$$
(A2)

where the constant  $\alpha$  is equal to:  $\alpha = \kappa M_s / c^2 \rho_h$ . Eq.A1 is well known as the Emden's equation <sup>[8,15]</sup>, and its solution was approximated here by expanding the function  $\psi(x)$  into a power series and comparing the coefficients with the same powers of x on both sides of the equation. The result that was used in Eq.12 is:

$$\psi(x) = -x^2 + \frac{3}{10}x^4 - \frac{4}{35}x^6 + \frac{61}{1260}x^8 - \frac{4507}{231000}x^{10}...$$
 (A3)

The calculation of the total DM mass of the Universe is also simplified, as is shown below, but shown only for the case of the uniform DM mass density for the sake of simplicity. Assuming that the uniform DM density extends to the distance  $\rho_a$ , we have for the total mass calculated from the field the following relation:

$$M_{du} = \frac{c^2}{2\kappa} \int_0^\infty \left(\frac{\partial \varphi_d(\rho)}{\partial \rho}\right)^2 \rho^2 d\rho \tag{A4}$$

where the potential has been normalized to  $c^2$ . The potential inside of the DM region is calculated from the formula derived from the Gauss law:

$$\varphi_{di} = \frac{2}{3} \frac{\pi \kappa m_0}{c^2} \rho^2 + const = x^2 + const$$
(A5)

and outside of the DM region according to the formula that follows from the Newton gravitational potential:

$$\varphi_{do} = -\frac{4}{3}\pi\kappa m_0 \frac{\rho_a^3}{\rho} = -\frac{2}{x}\frac{\rho_a^3}{\rho_h^3}$$
(A6)

which satisfies the zero potential condition at infinity. Similarly for the visible matter the potential inside of the DM region it is:

$$\varphi_{vi} = -2x^2 \tag{A7}$$

Because at the edge of the DM region the potential must be a continuous function the constant can be determined and the DM potential inside of the DM region written as:

$$\varphi_{di} = x^2 - \frac{3}{4} H_0^2 \frac{\rho_a^2}{c^2}$$
(A8)

From Eq.A4 then follows for the DM mass of the Universe:

$$M_{du} = \frac{H_0^4}{8\kappa c^2} \int_0^{\rho_0} \rho^4 d\rho + \frac{H_0^4}{8\kappa c^2} \int_{\rho_0}^{\infty} \frac{\rho_a^6}{\rho^2} d\rho = M_h \left(\frac{1}{5} + 1\right)$$
(A9)

A significant portion of the DM mass is, therefore, derived from the field energy inside of the DM region, which was in the previous paper neglected <sup>[5]</sup>. Perhaps this should not be neglected when a more accuracy is needed as it was done in the formula derived in Eq.19. The field energy is equal to the DM mass in this model following the relation:  $E = Mc^2$ . Once all the DM mass of the Universe is known the Universe's average radius  $\rho_a$  can be found:

$$\rho_a = \rho_h (1 + 1/5)^{1/3} = \rho_h \cdot 1.062658...$$
(A10)

Finally, we can also determine the minimum DM potential at the origin where x = 0:

$$\varphi_d(0) = -3(1+1/5)^{2/3} = -3.387729...$$
 (A11)

Perhaps this is the absolute minimum of the DM potential that can ever exist in the Universe. The value found for the potential in Eq.13 did not exceed this limit.

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