Comment on „On the Equality of Relative Velocities Between Two Objects for Observers in Different Rest Frames “ by Buenker

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Abstract

In the cited paper author (Robert J. Buenker) “proves” invalidity of the special relativity length contraction prediction, relaying on muon experiments and using incorrect calculation of the relative velocity. This phenomenon is common in anti-relativistic texts: arguments against the special theory of relativity authors base on misunderstandings (misconceptions) of the theory.

Keywords: special relativity, relative velocity, misconception, time dilatation, coordinate transformation, velocity addition, length contraction, Sagnac effect, anti-relativists

Relative velocity

Author [1] starts with the strange statements about the relative velocity:

“Applications of the theory of special relativity (SRT) are discussed in which the relative velocity of two objects needs to be determined from the vantage point of two different rest frames. It has generally been assumed without proof that such relative velocities are the same for all observers.”

This is strange because the concept of relative velocity is not generally used in SRT, but the invariance of the relative velocity is definitely not “generally assumed”, there is no such an invariance. What is the relative velocity? In Newton’s physics we can define the relative velocity between two objects as difference of their velocity vectors in some (inertial) reference frame. Such a definition becomes problematic in general relativity due to obvious mathematical problems. Relative velocity concept is frequently “hidden” in the problems of calculations of the collision time for two particles. It is important to understand that relative velocity concept follows from the definition of velocity and is completely unrelated to the concept of relativity – notice that we need just one single reference frame to define it. It doesn’t depend on the speeds (magnitude of the velocity vector) of objects (photons are included). To illustrate this we can imagine a simple experiment in the lab frame. Two balls, initially at distance \( L \), start to move along the x-axis with velocities \( u_1 \) and \(-u_2\). The collision time is

\[
\frac{L}{u_1 - (-u_2)} = \frac{L}{u_1 + u_2},
\]

and we see that difference of velocity vectors appears

\[
\frac{\Delta x_1}{\Delta t} - \frac{\Delta x_2}{\Delta t} = \frac{\Delta x_1 - \Delta x_2}{\Delta t} = \frac{L}{\Delta t}.
\]
Notice that we have the same denominator $\Delta t$ for both velocities—we define those quantities in the same (lab) frame of reference. Notice also that speeds could be relativistic, including the photon speed ($c$). Thus, the relative velocity magnitude is between zero and $2c$. And this fact results in numerous attacks on SRT, for example, there are claims that Einstein was wrong in his paper (1905) because of appearance of quantity $c \pm v$ in his formulas for the moving rod length. But notice that Einstein considers the moving rod and the photon sent from one end of the rod (B) to the other (B), point A moves with the velocity $v$ (in lab frame), photon moves with the velocity $-c$, both velocities are measured in the lab frame, so, we have the collision time $L/(c + v)$. Notice that $c + v$ is not a velocity of photon in lab frame, it is the relative velocity between the photon and the point A (end of rod). If someone don’t like the concept of the relative velocity he will refer to a term collision time, but that doesn’t change the physical content. There is a famous favorite example of anti-relativists—Sagnac effect, where the quantity $c \pm v$ appears, and critics (distracted by misconceptions or just by lack of knowledge) claim that light is “anisotropic”. But again, $c \pm v$ are not the speeds of photon, measuring equipment rotates and $c \pm v$ appears as a consequence of the collision time measurements.

The frequent statements are that $c \pm v$ appears as a consequence of the Galilean addition velocity rule, but this is incorrect. Such a rule appears as a consequence of transformation of coordinates (in Galilean, special, or any other possible relativity). So far we had just one single reference frame and there is no any “transformations” or “velocity rules” here. All quantities (differences of space coordinates and time intervals) are measured in the same frame. But there are situations where we have to transform quantities. If trains moves with the velocity $v$ (in the reference frame A, ground) and we have a passenger in the train with velocity $w$ (in the rest frame of the train, B) then we can say that the relative velocity between the train and passenger is $v_1 = w$ (in B). What is the relative velocity $v_2'$ in A? The answer depends on the relativity theory we use. In Galilean relativity we have in $A$ $w_2' = v + w$ (velocity addition rule), so $v_2' = w' - v = w'$. Notice that both velocities, $w'$ and $v$, are measured in A, so, again, relation $v_2' = w' - v$ is not a “velocity addition rule”, it is just the definition of the relative velocity in the frame A. In the special relativity we have the same definition (we are in single frame A) $v_2' = w' - v$, but now we must use different velocity addition rule to find $w'$

\[
    w' = \frac{v + w'}{1 + \frac{vw}{c^2}},
\]

so

\[
    v_2' = w' - v = \frac{v + w}{1 + \frac{vw}{c^2}} - v \neq w. \quad (1)
\]

Comment

The author (Buenker) in his paper “calculates” the relative velocity in the frame A:

\[
    v_2' = \frac{w' - v}{1 - \frac{vw}{c^2}} = w,
\]
which is obviously an invalid procedure. Here author (Buenker) uses the velocity addition rule instead of the definition of relative velocity. In fact, he is missed to properly define the relative velocity in his article. Then follows the “proof” for the general case. To apply his (wrong) result, author (Buenker) comments the muon experiments. To simplify, if the relative velocity between muon (frame 1) and target (Earth’s surface, frame 2) is \( u \) in the muon rest frame then it is \( u \) in the target rest frame, too, which (according to the author [1]) leads to (\( L \) is for distance, \( T \) is the muon’s lifetime)

\[
u = \frac{L_1}{T_1} = \frac{L_2}{\gamma T_1} \Rightarrow L_2 = \gamma L_1,
\]

meaning that there is the “length expansion” instead of the length contraction.


Conclusion

The special relativity theory is highly confirmed by numerous experiments. There is an explosion of anti-relativistic texts in articles, books, Internet, etc. My personal experience is that every such a text (till now) is based on some misunderstanding of the theory. Interesting, authors of such a texts are usually really persistent in their misconceptions. Here we have a typical misunderstanding about the concept of relative velocity and even the concept of the velocity addition rule in special relativity, which is easily to detect and correct. The author [1] is really active in publishing anti-relativity texts, he even wrote a book.

Literature