On Complex interval linear system

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Abstract

Linear system of equations with crisp values are crucial filed of research. Several different technique are available to solve this type of equations. The parameter values are actually uncertain in nature because data are collected from experiment. Another aspect is error in calculation. To avoid errors and uncertain nature of the parameter values, we use interval analysis. In this work, we are addressed solution methods for complex interval linear system. We propose a new method for finding solution of complex linear system of equations (CLSE). Moreover we study the numerical experiments using the proposed different methods.

Keywords: Interval, Complex Interval Arithmetic, Linear System

AMS Subject Classification $65E05, 65G40$

1 Introduction

Engineering and scientific problem described by the linear system of equations involving uncertain model parameters. Original and significant research in this field directed towards the use of intervals to represent the uncertain quantities in such systems.A famous and very old example of an interval enclosure is given by the method due to Archimedes.He considered inscribed polygons and circumscribing polygons of a circle with radius 1 and obtained an increasing sequence of lower bounds and at the same time a decreasing sequence of upper bounds for the area of the corresponding disc. Thus stopping this process with a circumscribing and an inscribed polygon, each of m sides, he obtained an interval containing the numberπ. By choosing m large enough, an interval of arbitrary small width can be found in this way containing π.One of the first research article about interval arithmetic as a tool of numerical computation was published in Russian 1951 explicitly[8,10].

System of linear equations can be solved by different numerical scheme like Gauss-Elimination method, Gauss-Jacobi iteration method, Gauss-seidel iteration method, matrix inversion method, matrix factorization method when the coefficient matrix and the right vector are in crisp. When the coefficient matrix and the right vector are in the interval, we may find the solution using real interval arithmetic operation. Kolev [1] presented a method for outer interval solution of linear parametric systems. Skalna [2] introduced methods for solving systems of linear equations of structure mechanics with interval parameters. Popova [3] also proposed a method on the solution of parametrised linear systems. Walter Kraemer [4] investigated computing and visualizing solution sets of interval linear systems. Majumdar and Chakraverty [6] addressed a new solution technique for the system of linear equation in interval form.In this present paper, we introduced two iterative scheme , for the solution of complex interval systems which gives a interval solution.In the next couple of sections, we discuss interval complex arithmetic operation, complex interval linear system and proposed two iterative techniques to solve CLSE. Finally, we present a couple of numerical examples to solve a CLSE by our proposed iteration methods.

2 Interval complex arithmetic

The universally accepted interval real arithmatic read as [5,7,9]

$$[a, b] = \{x | x \in \Re, a \leq x \leq b\} \tag{1}$$

where a is the left value and b is the right value of the interval respectively. We define $\frac{a+b}{2}$ the centre and $w = b - a$ is the width of the interval $[a, b]$.
Let $[a, b]$ and $[c, d]$ be two elements then the following arithmetic operation are well known

$$[a, b] \circ [c, d] = [min\{a \circ c, a \circ d, b \circ c, b \circ d\}, max\{a \circ c, a \circ d, b \circ c, b \circ d\}] \tag{2}$$

where $\circ$ is an arithmetic operation (i.e. addition, subtraction, multiplication, division).
Complex interval can be defined as rectangles[8]

$$\alpha + \beta i \equiv [\alpha_1 + \beta_1 i, \alpha_2 + \beta_2 i] = \{\alpha + i\beta \in \mathbb{C} | \alpha_1 \leq \alpha \leq \alpha_2, \beta_1 \leq \beta \leq \beta_2\} \tag{3}$$

or as circles

$$< p, r >= \{z \in \mathbb{C} || p - z| \leq r\} \tag{4}$$

3 Complex interval linear system

Let us consider the complex linear system of equation as

$$\mathbf{CX} = \mathbf{Q} \tag{5}$$

where $[\mathbf{C}] = [C^-, C^+]$ complex interval matrix of order n and $[\mathbf{Q}] = [Q^-, Q^+]$ is the interval vector of n components. This linear system is called complex interval linear system. we can also write a linear system of interval equations explicitly as given below

$$\begin{pmatrix} [a_{11}, b_{11}][a_{12}, b_{12}] & \cdots & [a_{1n}, b_{1n}] \\ [a_{21}, b_{21}][a_{22}, b_{22}] & \cdots & [a_{2n}, b_{2n}] \\ \vdots & \vdots & \vdots \\ [a_{n1}, b_{n1}][a_{n2}, b_{n2}] & \cdots & [a_{nn}, b_{nn}] \end{pmatrix} \begin{bmatrix} [x_1, y_1] \\ [x_2, y_2] \\ \vdots \\ [x_n, y_n] \end{bmatrix} = \begin{bmatrix} [e_1, f_1] \\ [e_2, f_2] \\ \vdots \\ [e_n, f_n] \end{bmatrix} \tag{6}$$

The system of equations in equation (6) can be solved by direct elimination method which we generally do for a crisp data. Now complex interval values are operated through the interval arithmetic rules. Computation involved in this procedure are hard to control. It is a difficult task to perform and the margin of uncertainty increases drastically. To overcome the above difficulties (to a certain extent) we now propose a new method.

4 Proposed Methods

Equation (6) can be solved by iterative scheme like Jacobi numerical method. The new form of representation of equation (6) is as follows

$$\begin{aligned} [x_1, y_1]^{k+1} &= \frac{[e_1, f_1] - [a_{12}, b_{12}][x_2, y_2]^k - [a_{13}, b_{13}][x_3, y_3]^k - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^k}{[a_{11}, b_{11}]} \\ [x_2, y_2]^{k+1} &= \frac{[e_2, f_2] - [a_{21}, b_{21}][x_1, y_1]^k - [a_{23}, b_{23}][x_3, y_3]^k - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^k}{[a_{22}, b_{22}]} \\ &\vdots \\ [x_n, y_n]^{k+1} &= \frac{[e_n, f_n] - [a_{n1}, b_{n1}][x_1, y_1]^k - [a_{n2}, b_{n2}][x_2, y_2]^k - \cdots - [a_{n(n-1)}, b_{n(n-1)}][x_{n-1}, y_{n-1}]^k}{[a_{nn}, b_{nn}]} \end{aligned} \tag{7}$$

The above procedure will be more efficient if we replace the vector $[x, y]^k$ in the right side of Equation (7) element by element as in Gauss-Seidel type method. So Equation (7) may then be written as

$$\begin{aligned} [x_1, y_1]^{k+1} &= \frac{[e_1, f_1] - [a_{12}, b_{12}][x_2, y_2]^k - [a_{13}, b_{13}][x_3, y_3]^k - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^k}{[a_{11}, b_{11}]} \\ [x_2, y_2]^{k+1} &= \frac{[e_2, f_2] - [a_{21}, b_{21}][x_1, y_1]^{k+1} - [a_{23}, b_{23}][x_3, y_3]^k - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^k}{[a_{22}, b_{22}]} \\ &\vdots \\ [x_n, y_n]^{k+1} &= \frac{[e_n, f_n] - [a_{n1}, b_{n1}][x_1, y_1]^{k+1} - [a_{n2}, b_{n2}][x_2, y_2]^{k+1} - \cdots - [a_{n(n-1)}, b_{n(n-1)}][x_{n-1}, y_{n-1}]^k}{[a_{nn}, b_{nn}]} \end{aligned} \tag{8}$$

5 Numerical experiment

Example 1: Let us consider the following interval complex linear system:

$$\begin{pmatrix} [2+i,3+i][0+i,1+i] \\ [1+i,2+i][2+i,3+i] \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} [0+i,120+i] \\ [60+i,240+i] \end{bmatrix} \tag{9}$$

Applying the proposed method, solution of the given interval system of linear equation is
$x = [-6.96 - 9.28i, 8.3793 - 20.4483i]$ and
$y = [26.5200 - 4.6400i, 64.1847 - 10.2241i]$.
The graphical solution set for both the interval is look like as Figure.1 and Figure.2.

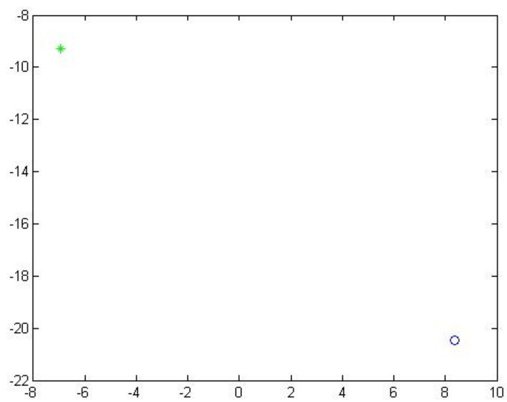

Figure 1: Solution sets for x

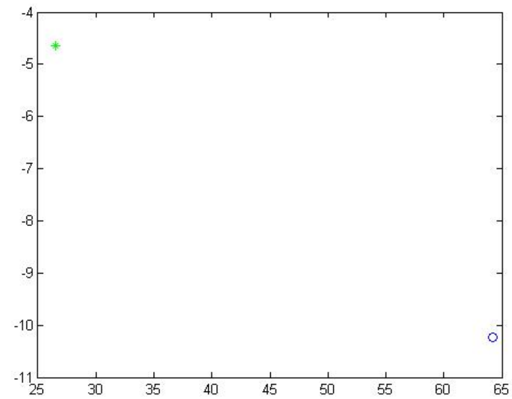

Figure 2: Solution sets for y

Example 2: We take another interval 3×3 complex linear system:

$$\begin{pmatrix} [6+i,8+i][1+i,3+i][0+i,2+i] \\ [-1+i,1+i][2+i,4+i][0+i,2+i] \\ [2+i,4+i][3+i,5+i][1+i,3+i] \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} [2+i,6+i] \\ [0+i,5+i] \\ [3+i,8+i] \end{bmatrix} \tag{10}$$

Applying the proposed method in a similar way, we obtained the solution as
$x = [0.3237 - 0.1040i, 0.1529 - 0.1882i]$
$y = [0.2659 - 0.7283i, 0.0706 - 1.3176i]$
$z = [1.7630 + 1.0405i, 2.4706 + 1.8824i]$. The graphical solution set for x, y, z are as follows

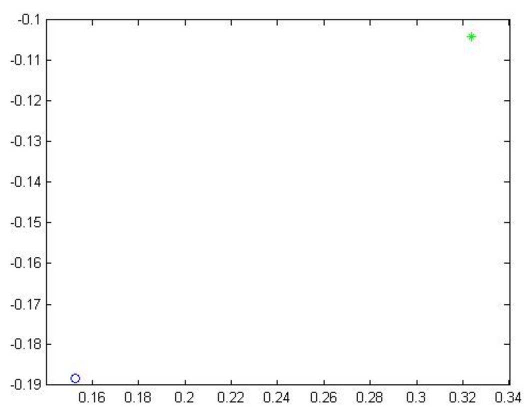

Figure 3: Solution sets for x

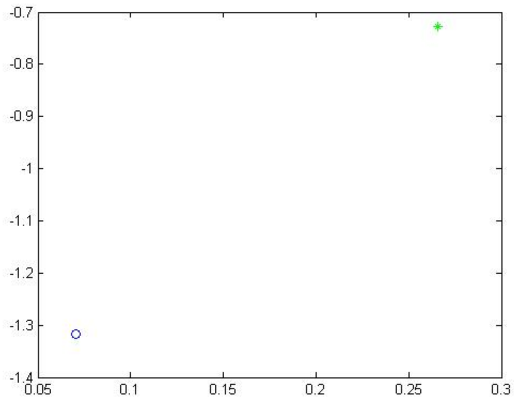

Figure 4: Solution sets for y

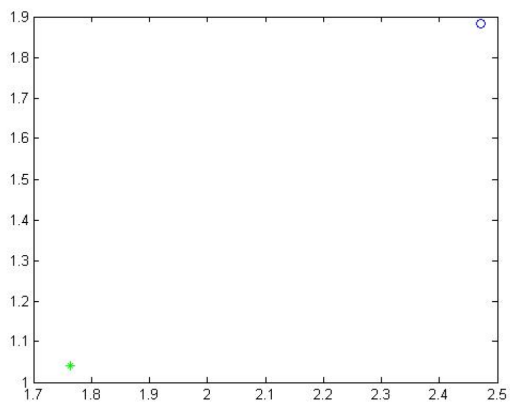

Figure 5: Solution sets for z

6 Conclusions

In this presentation, we proposed two iterative scheme for the solution of complex linear interval system of equations which is relatively new approach compare to the other methods. These proposed iterative scheme give a interval solution.

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