Measurement Quantization Unites Classical and Quantum Physics

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Unifying quantum and classical physics has proved difficult as their postulates are conflicting. Using the notion of counts of the fundamental measures—length, time, and mass—a unifying description is resolved. A theoretical framework is presented in a set of postulates by which a conversion between expressions from quantum and classical physics can be made. Conversions of well-known expressions from different areas of physics (quantum physics, gravitation, and optics) exemplify the approach and mathematical procedures. The postulated integer counts of fundamental measures changes our understanding of length, suggesting that our current understanding of reality is distorted.

POPULAR SUMMARY

In our effort to understand and describe the physical world around us, many researchers have attempted to connect quantum and classical physics, our best understanding of the very small and its relation to the very large. Ideally, success would be distinguished with a single expression or model from which everything may be understood and described. In this paper, a model is presented that allows the development of expressions that describe the very small and the very large.

To date the main focus of this effort has been gravity, the only force that has no apparent relation to the present model of the other forces and no fundamental understanding as to why it exists. This paper begins with a geometric expression that describes gravity. The model may also be used to derive new expressions and values for the Planck units of length, time, and mass using only macroscopic measures. The two sets of relations are found to be equivalent providing a parallel understanding of the postulates of this model.

Importantly, the model exposes a fundamental constant of nature that correlates angular measure and momentum. Results from experiments in quantum entanglement are used to substantiate this claim, which with the above model is expanded to demonstrate the central distorting effect in length measurement. The effect is incorporated with Heisenberg's uncertainty principle as well as Planck's and Einstein's expressions for energy.

I. INTRODUCTION

On the nature of electromagnetic radiation (ER) and its unique properties in relation to blackbody spectral emissions, Planck's work introduced the notion of quantized energy packets that led to a better understanding of light. He postulated that ER adhered to a strict quantal rule in the absorption and emission rules with photon energies given by $E=nhv$ where n is the number of packets, h is Planck's constant, and v is frequency [1]. The constant h was later understood as the smallest action that could exist in nature and with it Planck developed expressions for the fundamental units of length, time, and mass:

$$l_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} m \text{ [2,3]}, \tag{1}$$

$$t_p = \left(\frac{\hbar G}{c^5}\right)^{1/2} s \text{ [2,3]}, \tag{2}$$

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} kg \text{ [2,3]}. \tag{3}$$

Planck's work on the idea of fundamental measures provides insight into understanding the divide that separates quantum and classical physics. While successful models separately exist in each of these disciplines, there has been no mechanism to interrelate them and there exists some speculation as to whether a correlation exists [4].

The more promising developments include M-theory [5], Loop Quantum Gravity (LQG) [6] and Noncommutative Geometry (NCG) [7]. Each have provided insight in contributing to a Theory of Everything (ToE) [8], but have not produced verifiable physics results. This appears to stem from an inability to resolve discrete physical constants when using statistical, iterative, integrative, recursive and similar higher-order operators.

Another inhibitor to a successful ToE is the chosen framework. Frameworks that arise from nondimensionalized [9] expressions can be beneficial in gaining access to fundamental terms, but often fail in their implementation. They incorporate structures, both statistical and integrative approaches for example, that inhibit or restrict access to underlying discrete values. Without a framework defined relatively in terms of the smallest measures and implemented with the lowest-order operators, realizing significant expressions can be impeded or entirely excluded [10].

Models, such as M-theory and the many approaches based on this model, manifolds, compactification and heterotic M-theory, seem unlikely to succeed. The foremost cited understanding of M-theory is based on matrix theory [11], a quantum mechanical model on which the formulations of Edward Witten's idea [12] are described. In contrast, LQG finds success with a more Planck-like approach built on a background independent framework called a spin foam [13]. However, like M-theory, LQG's implementation invests heavily in the use of higher-order operators. Constructs, such as expressions arising around quantum constraint (i.e. resolution of kinematic Hilbert space or spatial diffeomorphisms and more specifically the Hamiltonian), create a layer of abstraction that complicates the resolution of the quantized terms in LQG.

Planck's formulation, in contrast, while built on a nondimensionalized framework of fundamental units of measure and succeeds in revealing important fundamental relationships, does not establish a grounded understanding of the origin of these units. The unstated bias in each of the leading models is that the underlying space-time structure adheres to rules that are more quantum than discrete – indivisible units of length, time and mass. This opposing perspective presented here is what separates this approach. Referred to as Informativity, this model is an approach based on the idea that phenomena are described as discrete units, that is, integer values of a fixed amount of length, time, and mass. Informativity is built entirely on a background independent framework. Observations of light are considered in geometric terms that may be used to describe gravity as a whole-unit interpretation of a physical phenomenon. In this manner, Informativity allows for the development of expressions of a predictive nature in several areas including both quantum and classical. For one, using a Bell state model presented by Shwartz and Harris [12], expressions are presented that describe the presence of a distorting effect at work in the measure of G and $\hbar$. Understanding this effect leads to the resolution of contradictions in the evaluation of certain expressions and provides a foundation with which to develop expressions that describe phenomena both very large and very small.

II. METHODS

The approach is based on the idea that three fundamental measures may be relatively defined: length, time, and mass. The measures are identified by symbols l_f, t_f, and m_f, but at this stage are not assigned values. The subscript f is used to distinguish these fundamental measures of Informativity from Planck's units, Eqs. (1–3).

Although the values of l_f and t_f are unspecified, their ratio may be understood and constrained by the elapsed time on an atomic clock relative to a distance traveled by a pulsed laser beam in vacuo, where $c=l_f/t_f$ with respect to any inertial frame. It is recognized that this ratio is fixed given the experimental support for the invariance of the value of c. Where $n_{Lf}l_f=n_{Tf}t_fc$, a count of l_f , i.e., n_{Lf}, will equal a count of t_f, i.e., n_{Tf}. Note that l_f or t_f may take any value and as such an arbitrary value may be chosen such that it is the largest value for which no smaller value can be observed. Other physical quantities, in turn, are obtained as counts of the fundamental measures.

The model does not provide a description of mass as a discrete unit of measure. Hence, the phenomenon of mass will be treated as being either a whole or fractional count of m_f. Apart from this modified definition for mass, each of the measures is taken as relative to a fundamental unit, which is only meant to say that they are countable.

Note that time has been subtly tied to distance and for that reason our definition is not only inclusive of the three spatial dimensions but extends to the temporal dimension. Without time, there exists no means to define space.

We summarize these two statements, which with two others formalize our model:

O_1: Quantities exist which are whole-unit counts of the fundamental measures l_f and t_f.
O_2: Mass may be a whole or fractional count of m_f.
O_3: Any remainder of a whole-unit calculation of l_f or t_f describes an action.
O_4: Distances for which the Pythagorean Theorem applies, the shortest side a is fixed with a count of one l_f, against which counts of l_f along sides b and c are made.

An underlying premise of the model is that in O_1 and O_2 all measures are defined relatively. With a unit system applied, there exists an agreed upon framework by which phenomena may be described by counting fundamental measures. O_3 recognizes the possibility that some expressions may solve for fractional counts of fundamental measures. Fractional counts violate the premise by which l_f and t_f are established. Therefore, any expression that describes a change in distance equal to the remainder of a measure must also describe an action.

O_4 presents a tool, the Pythagorean Theorem, for estimating length. Where the measure is relatively defined, the theorem incorporates a reference which must be equal to a unit count of one; that is $a=1$ unit is the reference. The other short side b is any given unit count of distance measures whereas the long side c (hypotenuse) is the distance measure of unknown unit count. Each side is a count of the reference measure defined by a. This establishes a foundation for a background independent framework that acknowledges the need for a reference within the definition. The expression $1^2+b^2=c^2$ describes an unknown distance c relative to b in terms of a count of a. As desired, solutions to c result in values that are fractional leaving us to test the hypothesis that gravity may be described as the lost excess over and above the whole-unit count of distance measures.

While we recognize that the measurement of quantities is limited, this does not mean they are not significant. Validating their significance against existing data is one goal of the model. Furthermore, although we use the Pythagorean Theorem to understand distance, there is no specific argument to suggest that another geometric expression would not serve the same purpose. The theorem provides insight only so long as it allows the presentation of expressions that cannot be reduced by another means. Finally, the term *fundamental* was chosen as a general term with the connotation that a measure has the characteristic of being countable and that measurement may be characterized as a count of fundamental measures. The term also serves to distinguish the units of measure adopted by Informativity from those given in Planck's base system.

Note that Planck's formulations are referenced for context, but are considered only as a guide in the development of our model. The model adopts values for the fundamental measures that are resolved entirely within Informativity.

III. RESULTS

A. Length measurement and gravitational acceleration

For long side c and short sides $a=1$ and b of any chosen integer count of a right-angle triangle (Fig. 1), we may resolve a count for the length measure representing the uncertain distance,

$$c=\left(1+b_{Lf}^2\right)^{1/2}. \tag{4}$$

Any non-whole-unit count relates to a change in distance and may be described by rounding up *(repulsion)* or down *(attraction)*. The remainder lost to rounding will be denoted by

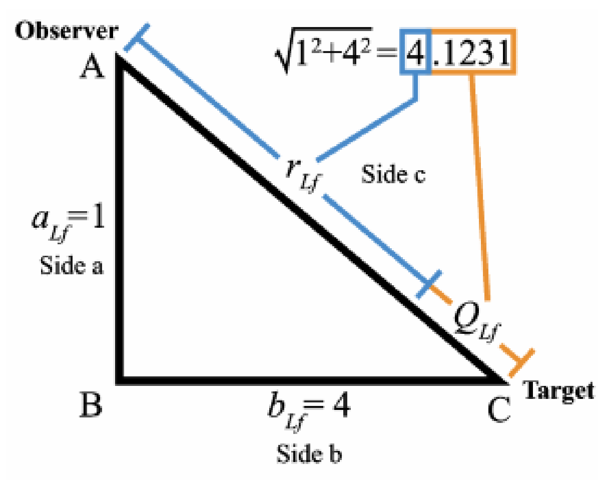

FIG. 1.Count of distance measures between an observer and target where $b_{Lf}=4$.

Q_{Lf}. For all solutions, Q_{Lf} is less than half and thus attractive. There are no explored examples of repulsion. The model provides a count of distance measures that is closer by

$$Q_{Lf} = \left(1 + b_{Lf}^2\right)^{1/2} - b_{Lf}\,, \tag{5}$$

at every instant in time. For example, if $b_{Lf}=4$, then $Q_{Lf}/r_{Lf}=((\surd 17)-4)/4=0.1231/4$. Because side c always rounds down, we find that r_{Lf} always equals b_{Lf}. In the following, we shall always refer to the 'observed measure count' as r_{Lf}. Moreover, note that the reference measure against which all counts are measured is defined by $a_{Lf}=1$. With this we have composed an expression for gravity such that the loss of the remainder relative to the whole-unit count is Q_{Lf}/r_{Lf}.

Together Q_{Lf} and r_{Lf} are conjectured to represent an important dimensionless ratio that describes gravity. We proceed with that hypothesis by presenting the ratio in meters per second squared (m/s^2), where we multiply by l_f for meters and divide by t_f^2 together describing the distance loss at the maximum sampling rate of one sampling every t_f seconds per second,

$$\frac{Q_{Lf}\, l_f}{r_{Lf}\, t_f^2}\,. \tag{6}$$

We now note that this quantity is scaled and hence requires a scaling constant; we multiply by the speed of light c and divide by a scaling constant S. Setting $r=r_{Lf}l_f$ and $c=l_f/t_f$, Eq. (6) reduces to

$$\frac{Q_{Lf}\, l_f}{r_{Lf}\, t_f^2}\frac{c}{S} = \frac{Q_{Lf}\, c^2}{r_{Lf}\, t_f S} = \frac{Q_{Lf}\, l_f c^2}{r_{Lf}\, l_f t_f S} = \frac{Q_{Lf}\, c^3}{rS}\,. \tag{7}$$

As the ratio c/S may be understood as $1/kg$ or a maximum count of m_f per kilogram, it may also be thought of as the corresponding mass frequency associated with gravity. Where $S=3.26239$, this expression is now equivalent to G/r^2 to five significant digits for all distances greater than $10^3\, l_f$. Where quantum differences are not a consideration, we may set Eq. (7) equal to G/r^2 and therefore

$$\frac{Q_{Lf}\, c^3}{rS} = \frac{G}{r^2}\,, \tag{8}$$

$$Q_{Lf}\, r c^3 = GS\,. \tag{9}$$

We may interpret S as momentum; hence the units for these expressions will match accordingly. Nevertheless, recognizing that S is a dimensionless scalar is an important and critical detour that shall be central to the discussion below. Two applicable interpretations will be shown. We first investigate S as a momentum, and then perform a similar analysis as an angular measure.

Consider Eq. (9); after rearranging and reducing the term on the right with $r=r_{Lf}l_f$, we use $\lim_{b\to\infty} f(Q_{Lf}r_{Lf}) = 1/2$ as noted in Appendix A. In passing, the term $Q_{Lf}r_{Lf}$, referred to as the *Informativity differential*, plays a key role in describing how fractional values less than the theoretical limit describe a distorting effect in measurement. Consideration of the *Informativity* *differential* at a limit is a matter of convenience, but to maintain a precise expression, values for Q_{Lf} and r_{Lf} should always be entered specific to the phenomenon being observed. The values determined can cover the entire physical regime from one l_f to infinity. From Eq. (9), we have

$$\frac{c^3}{G}=\frac{S}{Q_{Lf}r}=\frac{S}{Q_{Lf}r_{Lf}l_f}=\frac{2S}{l_f}. \tag{10}$$

Multiply both sides by t_f and reduce to obtain a mass,

$$m_f=t_f\frac{c^3}{G}=t_f\frac{2S}{l_f}=\frac{2S}{c}\ \ kg\,. \tag{11}$$

Hence the momentum of a fundamental measure of mass *(light)* moving at c may be expressed as

$$\rho=m_f c=\left(\frac{2S}{c}\right)c=2S\ kg\ m\ s^{-1}\,. \tag{12}$$

We understand S as being half the momentum of a fundamental measure of mass. This may also be written as

$$2S=m_f c=\frac{m_f l_f}{t_f}\ \ kg\ m\ s^{-1}\,. \tag{13}$$

Any count of l_f must equal the count of t_f, hence requiring that S must correspond to m_f being fractional. There exists no prerequisite that Informativity expressions be composed of whole-unit counts of m_f; see O_2 of Sec. II. With this resolved, we now consider S as an angular measure. With the Pythagorean Theorem supporting an understanding of S as momentum, then a circle supports an understanding of S as an angle.

Consider Eq. (1) organized such that $c^3/G=\hbar/l_f^2$. Take Eq. (10), replace c^3/G with $\hbar/l_f^2$ and replace $\hbar=h/2\pi$. Hence

$$S=\frac{l_f}{2}\left(\frac{c^3}{G}\right)=\frac{l_f}{2}\left(\frac{\hbar}{l_f^2}\right)=\frac{\hbar}{2l_f}=\frac{h}{4\pi l_f}, \tag{14}$$

where $S=\hbar/2l_f$. Then the arc length of a circle of radius l_f and angle S is

$$L=r\theta=l_f\left(\frac{\hbar}{2l_f}\right)=\frac{\hbar}{2}. \tag{15}$$

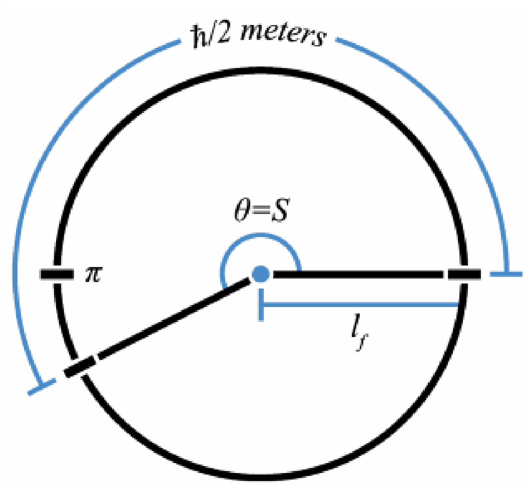

FIG. 2.Arc length of a circle of radius l_f and subtending angle $\theta=S$ radians.

In Fig. 2, we find that an arc-length with

$\theta=2S$ radians is precisely the value of $\hbar$ as meters. Each of the terms has a suitable geometric description:

- l_f radius of a fundamental circle in meters
- $2S$ angle in radians that subtends a segment with an arc length of $\hbar$ meters
- $\hbar$ arc-length of a segment corresponding to the momentum of a fundamental measure of mass

Applicability to an existing geometric expression is just the first of several tests. Next, we consider support for the equivalence of these two interpretations. We begin by resolving S in terms of our initial description of gravity from Eq. (10),

$$S = \frac{l_f c^3}{2G} \ kg \ m \ s^{-1} . \tag{16}$$

Next consider $S=\hbar/2l_f$ as resolved in Eq. (14). With $l_f c^3/2G$ a momentum and $\hbar/2l_f$ an angular measure, we set them equal giving

$$\frac{l_f c^3}{2G} = \frac{\hbar}{2l_f} . \tag{17}$$

Isolating l_f to the left-hand side and

$$l_f = \left(\frac{\hbar G}{c^3}\right)^{1/2} m \ \text{[2,3]}. \tag{18}$$

The expression clarifies our conjecture of the equivalence between the two interpretations: momentum and angular measure. The expression also demonstrates that the comparison is in fact a modified form of Planck's universally recognized formulation for length.

Finally, to verify this interpretation, we now seek a quantity where S is:

- an invariant characteristic of light at a threshold
- described as $l_f c^3/G$ with respect to momentum
- described as $\hbar/2l_f$ with respect to angular measure

Indeed, a quantity was measured by Shwartz and Harris in 2011 regarding the quantum entanglement of light at the degenerate state [12]. Using polarization entangled photons in pure Bell states at X-ray wavelengths, they were able to take advantage of the intersections of the component curve (as a function of the square of the current density) to resolve pump angles θ_p where the magnitudes of the components of each Bell state are equal. With this, solutions to the phase matching and current density equations were resolved to determine the sign of the components at the intersection. Then solving the phase matching equations for the signal θ_s and idler θ_i with respect to the atomic planes, substituting the related electric fields, the current density is a function of just the pump angle. With these conditions in place, the momentum of a fundamental measure of mass is then equal in value to the angle of the signal and idler $l_f c^3/2G$ with respect to the atomic planes where the pump is at its maximum angle.

There are five pump angles representing two of the Bell states that can generate entangled photons and $l_f c^3/2G$ is uniquely distinguished where θ_p is at its maximum. Shwartz and Harris recognize these Bell states, where $|H>$ is the polarization of the electric field of the X-ray in the scattering plane and $|V>$ is the polarization orthogonal to the scattering plane which contains the incident **k** vector and the lattice **k** vector ***G***. Subscripts *p*, *s*, and *i*, respectively, denote the pump, signal and idler. Note also that the values in Table I are solely from the paper of Shwartz and Harris.

TABLE I. Angle setting in radians of the **k** vectors of the pump, signal and idler for maximally entangled states at the degenerate frequency with corresponding Shwartz and Harris values (Ref. [12]).

Bell's State	θ_p	θ_s	θ_i
$(\lvert H_s, V_i> + \lvert V_s, H_i>)/\surd 2$	$(l_f c^3/2G)-\pi$ (0.1208)	π - $(l_f c^3/2G)$ (-0.1208)	π - $(l_f c^3/2G)$ (-0.1208)
	2π - $(l_f c^3/2G)$ (3.02079)	$(l_f c^3/2G)$ (3.26239)	$(l_f c^3/2G)$ (3.26239)

The expressions arise from Eq. (16) each describing an equidistant angle either side of *0, π* or *2π* and are precisely identical in value to the Shwartz and Harris measurements. Using the most recent CODATA [2] as a guide for the value of l_f, we find that

$$S = \frac{l_f c^3}{2G} = \frac{1.616199\ 10^{-35}\ (299792458)^3}{2 * 6.67408\ 10^{-11}} = 3.26239\ \ radians\ . \tag{19}$$

But, where we have made use of Planck's relation in Eq. (14), $S=\hbar/2l_f=3.26250$. We conclude that our understanding of l_f as expressed by Informativity precisely matches the values presented in the Shwartz and Harris model, but our understanding of $\hbar$ when applying Planck's expression is incomplete. The issue that affects Planck's reduced constant will be resolved in Section *D*.

The correlation between *S* and the angular measures of the Shwartz and Harris Bell state is not unexpected. Where the signal and idler are resolved specifically to obtain the polarization angles necessary for entanglement, seeking the pump angle follows naturally, thereby resolving each of the conditions where entanglement may occur. Informativity is not a coincidental alignment of one of these values, but a means of resolving the maximum angular measure corresponding to light in terms of the fundamental measure m_f. With that, resolving the angular measures for each of the limits described by the Bell state follows in a straight-forward manner.

Where the expression *2S* describes the momentum of a fundamental measure of mass, the term *S* describes an angle. Both interpretations are valid. The juxtaposition of units describes a relationship that is conflicting, similar to Einstein's relation $E=mc^2$, which expresses energy in joules as a form of mass in kilograms, and vice versa. This presentation demonstrates that momentum and angular measure are one and the same.

With this understanding we consider replacing the scalar term *S* with θ_{si}. The term alludes to recognizing the angular measure of the signal and idler under some conditions and momentum under others. Although both interpretations are applicable, θ_{si} is retained emphasizing that we are not working with a theoretical value, but an invariant macroscopic measure. Additional research regarding the measure of θ_{si} has been reported [14], where the error in angular measurement is estimated to be less than 2 micro-radians.

In reflecting on Planck's work, one might argue that this approach is a basic presentation of a derivation of Planck units. One must take note that the role of fundamental measures at this point is a mathematical construct, a proposed interpretation of the existing argument. The measures exist only in their expression until formally resolved in the next section. Whereas CODATA estimates may be used to guide our understanding of S, up to this point no theoretical values are assumed. Our confidence in correlating S to θ_{si} rests in the correctness of the two interpretations of S and their correlation accounts for Planck's expression for length.

One might also view this approach as an innovative alternative to Newtonian vector calculus thus side-stepping what might be an otherwise traditional understanding of gravitation. However, this argument would also work against an underlying premise of this paper, that higher-order operators (other than the four basic arithmetic operators) disguise the fundamental constructs. Where a treatment using vector calculus would resolve the traditional presentation, the quantum relationship to θ_{si} would be lost or at least well-disguised.

B. Fundamental measures

With the θ_{si} correlation, we may now resolve the fundamental measures l_f, t_f, and m_f, not as a theoretical construct, but with physical expressions constrained by the characteristics of light and gravity, consisting entirely of macroscopic measures. We start with Eq. (10) by solving for l_f,

$$l_f = \frac{2G\theta_{si}}{c^3} = \frac{2*6.67408\ 10^{-11}*3.26239}{(299792458)^3} = 1.61620\ 10^{-35}\,m\,, \tag{20}$$

where time follows from the definition $t_f=l_f/c$. Replacing l_f with Eq. (20) gives

$$t_f = \frac{l_f}{c} = \frac{2G\theta_{si}}{c^4} = \frac{2*6.67408\ 10^{-11}*3.26239}{(299792458)^4} = 5.39106\ 10^{-44}\,s\,. \tag{21}$$

Finally, reordering time to resolve mass.

$$m_f = t_f\frac{c^3}{G} = \frac{2\theta_{si}}{c} = \frac{2*3.26239}{299792458} = 2.17643\ 10^{-8}\,kg\,, \tag{22}$$

where interpreting $S=\theta_{si}$ as a momentum yields the appropriate units. Most importantly, whereas the value for θ_{si} is obtained from a macroscopic measurement, Planck's approach is achieved as a theoretical construct. Establishing that these values are the same provides a new foundation with which to build a model based entirely on physical measurements. Also note, whereas l_f and t_f are proposed to be the smallest significant measures, m_f is not; m_f does play a central role in many expressions because it is a product of l_f and t_f, i.e. $m_f=2\theta_{si}t_f/l_f$, and for that reason the term is retained.

The Informativity formulation parallels Planck's although the expressions are presented in quantized terms and are entirely formulated under the background independent framework of Informativity. An additional characteristic of the fundamental measures is that they are not a product of Planck's formulations, Planck's constant or any quantum term. Rather, values are derived using only the geometric expression $Q_{Lf=}G\theta_{si}/rc^3$. The expression depends on c, r, G and

θ_{si} and all are resolved macroscopically. Where $r=r_{Lf}l_f$ and $lim_{r=\infty}f(Q_{Lf}r_{Lf}) =1/2$ from Appendix A, the relation is rearranged to give $\theta_{si}=l_f(c^3/2G)$. If the expression were not derived by observations regarding light and gravity consisting entirely of macroscopic measures, the fundamental measures would be unconstrained.

Planck's approach provides a valid way to constrain the fundamental measures, but the solution provides no mechanism to confirm the approach through measurement indirectly confirming the significance of the measures. The Informativity approach recognizes the significance of θ_{si} and uses that to resolve l_f, t_f, and m_f. Whereas both approaches arrive with the same conclusion, the ability to resolve the fundamental measures as a product of macroscopic phenomenon is an important and decisive difference.

It should finally be noted that the fundamental measures l_f and t_f can never be directly resolved with identical or greater precision. This would defy their definition. The fundamental measures are the references against which everything is defined. It would be neither possible nor meaningful to measure a reference where the most appropriate reference is the reference itself. Fundamental measures can only be inferred as a characteristic of nature that indicates their significance.

C. Understanding Newton's constant

Using Eq. (8) at the quantum scale to calculate G will show a difference with Newton's presentation. Is G variable [15]? No. The difference is a reflection of the precision between the geometric model of Informativity in comparison to Newton's presentation. Newton's expressions do not include the geometric distortion effect inherent in the *Informativity differential* $Q_{Lf}r_{Lf}$ as numerically assessed in Table II.

TABLE II. Informativity difference in G/r^2.

	$50\ l_f$	$100\ l_f$	$200\ l_f$	$300\ l_f$	$500\ l_f$	$1000\ l_f$
Difference	0.00100%	0.00250%	0.00062%	0.00028%	0.00010%	0.00003%

For clarity, we work through a calculation in detail. A distance of 1 meter is intentionally selected such that $G/r^2=G/1=G$. Using Eq. (20) for l_f, the inverse gives us a count in 1 meter such that $b=6.18735\ 10^{34}$; that is,

$$Q_{Lf} = c - b = \sqrt{1+b^2} - b = \sqrt{1+(6.18735\ 10^{34})^2} - 6.18735\ 10^{34} = 8.08100\ 10^{-36}, \quad (23)$$

$$\frac{Q_{Lf}c^3}{r\theta_{si}} = \frac{8.08100\ 10^{-36} * (299792458)^3}{1*3.26239} = 6.67407\ 10^{-11} m / kg\ s^2. \quad (24)$$

With only our understanding as prescribed by the Pythagorean Theorem and expressed in Eq. (7), this is G/r^2 at a distance of 1 meter, and therefore numerically equal to G. At its base, the formulation depends on an understanding of l_f, which may be resolved entirely from the expression presented in Eq. (20). The value is also identical to the most recent CODATA estimate of Planck's formulation of length. The CODATA [2,16] estimates of Planck units have changed over recent years, but those estimates continue to give support to the expressions of Informativity.

D. Understanding Planck's constant

To build on our understanding of G presented in Section C, we investigate how the use of macroscopic and quantum terms affects the calculation of physical measures. We begin by formulating a known Informativity expression that may serve as a reference. Dividing Eq. (9) by G, substituting $r=r_{Lf}l_f$, and factoring c^3/G, we obtain

$$\theta_{si} = \frac{Q_{Lf} r c^3}{G} = \left(\frac{c^3}{G}\right) Q_{Lf} r_{Lf} l_f = \frac{c^3 l_f}{2G} = 3.26239 \ \ radians \,. \tag{25}$$

The measure for θ_{si} matches the angular measurements made by Shwartz and Harris. We derived l_f because of the correlation of S and θ_{si}. Comparing Planck's formulation in Eq. (1) where $c^3/G=\hbar/l_f^2$ and substituting it into Eq. (25) yields

$$\theta_{si} = \left(\frac{c^3}{G}\right) Q_{Lf} r_{Lf} l_f = \left(\frac{\hbar}{l_f^2}\right) Q_{Lf} r_{Lf} l_f = \frac{\hbar}{2 l_f} = 3.26250 \ \ radians \,, \tag{26}$$

where G and r^2 are macroscopic factors whereas $\hbar$ has a quantum origin; all values of fundamental units, such as l_f, are neither macroscopic nor quantum, but treated as constants. For convenience, the macroscopic limit of the *Informativity differential* is taken, and not its quantum limit. This is acceptable when working with macroscopic terms, but when working with quantum terms, including the Planck constant, that limit produces inaccurate results. The *Informativity differential* needs to be retained and expressions properly calculated regarding the actual distance of the measured interaction. With respect to the conditions that lead to the measure of $\hbar$, distance as a count of l_f may be resolved by solving for r_{Lf} using its expression Eq. (C7) derived in Appendix C,

$$r_{Lf} = \theta_{si} l_f \sqrt{\frac{1}{\hbar(\hbar - 2\theta_{si} l_f)}} = 84.85536 \,. \tag{27}$$

Note that r_{Lf} must be a whole-unit count, $85l_f$. Resolving a quantum distance provides an understanding as to why Planck's constant as it is currently defined is appropriate in expressions such as the fine-structure constant [17]. Hence, use of Planck's reduced constant at an *Informativity differential* distance of $85l_f$ produces the correct result. However, the expression is not appropriate in expressions consisting entirely of macroscopic measures. With G, both factors vary depending on the *Informativity differential*.

We understand this effect better by solving for $\hbar$ using Eq. (26) and then comparing that to the currently recognized value of $\hbar=1.05457\ 10^{-34}$ [2]. For this purpose, we see in Table III that the variation in $\hbar$ changes quickest within the first few l_f.

TABLE III. Informativity difference in Planck's reduced constant $\hbar$.

	$85\ l_f$	$100\ l_f$	$200\ l_f$	$300\ l_f$	$500\ l_f$	$14\ bly$
Difference	0.00001%	0.00097%	0.00285%	0.00319%	0.00337%	0.00347%

If we wish to use $\hbar$ with macroscopic terms, then we need to resolve the value of $\hbar$ at a macroscopic distance. A good example would be Planck's theoretical relations for length, time, and mass, which involve macroscopic terms G and c. Solving Eq. (26) for $\hbar$ at a macroscopic distance $lim_{b\to\infty}f(Q_{Lf}r_{Lf})=1/2$, then

$$\hbar = \frac{l_f^2\theta_{si}}{Q_{Lf}r_{Lf}l_f} = \frac{l_f\theta_{si}}{Q_{Lf}r_{Lf}} = 2\theta_{si}l_f = 1.05454\ 10^{-34}\ Js\ . \tag{28}$$

With this value, the distance-adjusted Informativity and Planck formulations are now mathematically equivalent and the variation in G and $\hbar$ cancel out:

$$\frac{2G\theta_{si}}{c^3} = \left(\frac{\hbar G}{c^3}\right)^{1/2} m\ , \tag{29}$$

$$\frac{2G\theta_{si}}{c^4} = \left(\frac{\hbar G}{c^5}\right)^{1/2} s\ , \tag{30}$$

$$\frac{2\theta_{si}}{c} = \left(\frac{\hbar c}{G}\right)^{1/2} kg\ . \tag{31}$$

Each expression may be reduced to

$$4G\theta_{si}^2 = \hbar c^3\ . \tag{32}$$

Where uncertainty exists in the derivation of l_f, t_f, and m_f, we may express the fundamental measures in terms of θ_{si} instead of $\hbar$. This expression confirms that any geometric distortion in $\hbar$ is proportionally compensated with the same in G. We are also more aware of the important role played by the *Informativity differential* and have confirmed that these two very different approaches arrive at precisely the same result. Note further that this is a well-grounded physical expression that may be used to resolve each of the fundamental measures, thus providing significance to each. Finally, with a distance adjusted value for $\hbar$, we can return to the Shwartz and Harris results as presented in Table I and cast them in terms of the ratio of arc length and diameter of a circle.

TABLE IV. Angles in radians for the k vectors of the pump, signal, and idler for the maximally entangled states at the degenerate frequency with corresponding Shwartz and Harris values (Ref. [12]).

Bell's State	θ_p	θ_s	θ_i
$(\|H_s, V_i> + \|V_s, H_i>)/\sqrt{2}$	$(\hbar/2l_f)-\pi$ *(0.1208)*	π - $(\hbar/2l_f)$ *(-0.1208)*	π - $(\hbar/2l_f)$ *(-0.1208)*
	2π - $(\hbar/2l_f)$ *(3.02079)*	$(\hbar/2l_f)$ *(3.26239)*	$(\hbar/2l_f)$ *(3.26239)*

With $\hbar = 1.0454\ 10^{-34}$ Js from Eq. (28) and $l_f = 1.61620\ 10^{-35}$ m from Eq. (20) (where their ratio corresponds to units in radians as resolved in Eq. (15)), these expressions precisely match the Shwartz and Harris values [12]. Whether presented in macroscopic or quantum terms, we find the angular measures presented in Table IV may be resolved.

E. Fundamental measures correlated

In Eqs. (20–22), solutions to the fundamental measures are resolved and while they are appropriate for use in macroscopic terms, it is not representative of a distance-sensitive formulation. Here, we resolve distance-sensitive expressions and demonstrate their use in several well-known expressions. We begin with Eq. (11) and expand the right-hand term restoring the *Informativity differential* to mass that was factored out in Eq. (10) with the limiting process (Appendix A),

$$m_f = \frac{2\theta_{si}}{c} = \frac{\theta_{si}}{Q_{Lf} r_{Lf} c}\,. \tag{33}$$

We may now translate mass to a length and time by applying the fundamental transforms (Appendix B) $\Delta(l_f \rightarrow m_f)$ where $l_f c^2/G = m_f$ (B2) and $\Delta(t_f \rightarrow m_f)$ where $t_f c^3/G = m_f$ (B3) to obtain

$$l_f = \frac{\theta_{si} G}{Q_{Lf} r_{Lf} c^3}\,, \tag{34}$$

$$t_f = \frac{\theta_{si} G}{Q_{Lf} r_{Lf} c^4}\,. \tag{35}$$

We may further reduce the length with $G/c^3 = t_f/m_f$ (B3), time with $G/c^2 = l_f/m_f$ (B2), and mass with $c = l_f/t_f$ (B1).

$$l_f = \frac{\theta_{si} t_f}{Q_{Lf} r_{Lf} m_f} = \frac{t_f}{m_f}\left(\frac{\theta_{si}}{Q_{Lf} r_{Lf}}\right), \tag{36}$$

$$t_f = \frac{\theta_{si} l_f}{Q_{Lf} r_{Lf} m_f c^2} = \frac{\theta_{si} l_f t_f^2}{Q_{Lf} r_{Lf} m_f l_f^2} = \frac{Q_{Lf} r_{Lf} m_f l_f}{\theta_{si}} = m_f l_f \left(\frac{Q_{Lf} r_{Lf}}{\theta_{si}}\right), \tag{37}$$

$$m_f = \frac{\theta_{si} t_f}{Q_{Lf} r_{Lf} l_f} = \frac{t_f}{l_f}\left(\frac{\theta_{si}}{Q_{Lf} r_{Lf}}\right). \tag{38}$$

Thus, we have each of the fundamental measures in their most robust form. Where $\lim_{b\rightarrow\infty} f(Q_{Lf} r_{Lf}) = 1/2$ then each expression may be reduced to

$$l_f m_f = 2\theta_{si} t_f\,. \tag{39}$$

There are several possible arrangements of the terms. Take for instance a description of light $l_f/t_f = 2\theta_{si}/m_f$. As noted early, any count of l_f must equal a count of t_f where the value of $c = l_f/t_f$ is invariant. However, what happens if the expression is presented in the form $l_f=(2\theta_{si}/m_f)t_f$? Would a scaling of t_f to match the age of the universe then prescribe conditions regarding the size or expansion of the universe?

We next consider the challenge of applying Informativity expressions to physical phenomena. The universal expansion (the theoretical rate of expansion as a count of l_f with respect to the passage of time as a count of t_f) differs in definition from a measure of the stellar expansion (the rate at which galaxies are moving away from one another dz/dt). Whereas Informativity can be used to present an expression for universal expansion, to correlate this to a meaningful measure of stellar expansion requires a common frame of reference with an initial correlation.

To demonstrate the challenge, we begin by multiplying the universe's diameter $D_u=n_{TfD}$ (billion light-years) and age $A_u=n_{TfA}$ (billion years) as a count of fundamental measures times t_f. With $l_f=ct_f$, we have

$$(n_{TfA}n_{TfD}t_f)l_f m_f = 2\theta_{si}t_f(n_{TfA}n_{TfD}t_f)\,, \tag{40}$$

$$(n_{TfD}t_f)m_f\left(\frac{n_{TfA}t_f c}{n_{TfD}t_f}\right) = 2\theta_{si}(n_{TfA}t_f)\,, \tag{41}$$

$$D_U m_f\left(\frac{A_U c}{D_U}\right) = 2\theta_{si}A_U\,. \tag{42}$$

We find that a single solution exists for each moment in time D_U and A_U, where $(A_Uc/D_U)=1/m_f$ cancels m_f,

$$D_U = 2\theta_{si}A_U = 2*3.26239*13.799 = 90.0354 \ \textit{billion light years}\,, \tag{43}$$

$$m_f = \frac{D_U}{A_U c} = \frac{90.0354}{13.799*299792458} = 2.17643\ 10^{-8}\,kg\,. \tag{44}$$

The analysis is not a straight-forward extrapolation regarding how far light has traveled given the age of the universe, an interest aspect encountered in many Informativity expressions. If we are to recognize the passage of time where $(c)nt_f=nl_f$, then we must also recognize a similar process at work in determining length. We may further note that the magnitude of the effect is $2\theta_{si}$. Informativity tells us that for any given value in age, there is a corresponding value for diameter. Age, diameter, and the speed of light also prescribe an invariant value for a fundamental measure of mass. The galactic expansion, likewise, must advance at $(1/2\theta_{si})\times 100=15.326\%$ of the reference.

In the physics literature, measurements by Riess et al. [18] do show a stellar expansion of 10–15%. Measurements for D_U and A_U are also measured at 91 billion light-years and 13.799± 0.021 billion years, respectively [19]. In 2011, formulations by Barrow and Douglas [20] comparing the cosmological constant and the age of the universe had been worked out predicting a constant relationship. In 2015, the analysis of the WMAP data by Gasanalizade and Hasanalizade [21] also confirmed a constant correlation between the age of the universe and its expansion.

Unfortunately, while we may find the data complementary, when working with Informativity there is often a barrier in correlating physical events. It is, for instance, unclear how matter in the universe relates to the expansion and questions regarding how we might measure this expansion in the local frame.

On more certain grounds, we may resolve precise expressions for other measures such as energy. Using mass from Eq. (33) and Einstein's equation where n_{mf} is a count of the fundamental mass measure, then

$$E = mc^2 = n_{Mf}\left(\frac{\theta_{si}}{Q_{Lf}r_{Lf}c}\right)c^2 = n_{Mf}\left(\frac{\theta_{si}c}{Q_{Lf}r_{Lf}}\right). \tag{45}$$

Here $n_{mf}=1$ and $lim_{b\to\infty}f(Q_{Lf}r_{Lf})=1/2$; we may then write

$$E = 2\theta_{si}c\,. \tag{46}$$

In comparison, if we reduce $h=4\pi\theta_{si}l_f$ as expressed in Eq. (14), then Planck's formulation $E=nhv= h/t_f$ is

$$E = n_{Mf}hv = n_{Mf}\left(\frac{4\pi\theta_{si}l_f}{t_f}\right) = n_{Mf}4\pi\theta_{si}c\,. \tag{47}$$

The energy of one fundamental measure of mass is $E_m=mc^2=2\theta_{si}c$ from Eq. (46), and the energy of one photon is $E_l=hv=h/t_f$. Substituting $\theta_{si}=h/4\pi l_f$ from Eq. (14) and resolving for E_m, we have

$$\frac{E_m}{E_l} = \frac{2Sc}{h/t_f} = \frac{2l_f}{h}\frac{h}{4\pi l_f} = \frac{1}{2\pi}, \tag{48}$$

$$E_m = \frac{E_l}{2\pi} = \frac{hv}{2\pi} = \left(\frac{1}{2\pi}\right)hv\ kg\ m^2s^{-2}\,. \tag{49}$$

Whereas Planck associated the energy of quantum states with harmonic oscillators that modeled the atoms lining the cavity, the correlation of energy between a fundamental measure of mass and Planck's blackbody spectrum is precisely a product of 2π. While comparing the two is not a precise contextual match, the correlation does reinforce our prior observation that angular measure and momentum are one and the same and only as such can we fully appreciate a value of $n=1/2\pi$.

Note also in Eq. (47) that the *Informativity differential* is missing from the description of light. As expected, the energy of light is invariant in a gravitational field.

F. Quantum uncertainty

With the fundamental measures at hand, we turn our attention to Heisenberg's uncertainty principle [22] first presented in 1927. The principle may be described as an expression representing a suite of mathematical inequalities that prescribe a fundamental limit to the precision with which pairs of physical properties of a particle can be known. These pairs are known as complementary variables. Pertaining to the position and momentum of a particle, the uncertainty principle states that the more precisely the position is determined, the less precisely its momentum is known. A more formal inequality relating the standard deviation of position σ_x and the standard deviation of momentum σ_p,

$$\sigma_X \sigma_P \geq \frac{\hbar}{2}. \tag{50}$$

was derived by Kennard [23] later that year and Weyl [24] in 1928.

With respect to Informativity, we find that our understanding of this product remains unchanged, but the components may be further refined. With the standard deviations in position and momentum, denoted by $f(r_{Lf}l_f)$ and $f(m_f v)$, respectively, we may use mass as expressed in Eq. (33) and replace the arc length $\hbar/2$ with Eq. (28). With mass incorporating the *Informativity differential* $Q_{Lf}r_{Lf}$, we introduce the same in position,

$$f(r)f(mv) = (r_{Lf} l_f Q_{Lf} r_{Lf})\left(\frac{n_{Mf}\theta_{si}}{Q_{Lf} r_{Lf} c} v\right) \geq \theta_{si} l_f\,. \tag{51}$$

Here the *Informativity differential* in position and momentum cancel out; the differences are the individual uncertainties. This observation is predicated on our modified understanding of mass.

Another issue concerns what happens when we reduce this formulation. First, we cancel out the *Informativity differential*, θ_{si} and l_f. With $c=l_f/t_f$ and v a count of l_f traversed per t_f (denoted as n_{Lf}), n_{Mf} is a count of the mass measure and r_{Lf} a count of l_f between the observer and target, then

$$(r_{Lf})\left(\frac{n_{Mf} v}{c}\right) \geq 1, \tag{52}$$

$$(r_{Lf})\left(n_{Mf}\frac{n_{Lf}}{t_f}\frac{t_f}{l_f}\right) = \left(n_{Mf}\frac{r_{Lf} n_{Lf}}{l_f}\right) \geq 1, \tag{53}$$

$$n_{Mf} r_{Lf} n_{Lf} \geq l_f\,. \tag{54}$$

With this, we see that uncertainty is threefold: mass, position and velocity. There are several notable outcomes. Where $v=c$, the uncertainty is reduced to just mass. Second, note that time is not a term associated with uncertainty. Third, the boundary for these three terms is l_f which until now seemed only a convenient theoretical unit of measure. Therefore, where we find physical support for the Heisenberg uncertainty principle, we must also find l_f to be of physical significance, defining the threshold.

IV. DISCUSSION

Investigations of the scalar constant *S* are central to understanding Informativity. With a physical correlation to *S*, we may resolve a distance-specific expression for Planck's constant, build fundamental expressions for length, time, and mass and equate Informativity to Planck's formulations to realize a new unifying expression. The foundations of Planck's formulations and

Informativity have little in common, but each model may be used to resolve the fundamental measures. The defining difference and advantage the Informativity approach offers is that each term carries a grounded physical correlation. The fundamental measures of Informativity, as such, are not a theoretical construct.

Much of what has been presented strongly focuses on the ideal of fundamental measures as a means to expressing descriptions of nature. It is conjectured that fundamental measures are the best-suited means to understand nature, but it may equally be noted that the translation of measurement to such a unit system is no more significant than factoring or scaling an expression. The underlying structure that makes such a scaling uniformly convenient is expressly neither a phenomenon that may be proven to be a fundamental property of the universe nor a requirement to understanding the expressions of Informativity.

A likewise and equally notable construct of the presentation is the incorporation of non-dimensionalization. While an important step in exposing an understanding of θ_{si}, non-dimensionalization in itself is not a prerequisite to Informativity. There are other approaches that may be used to achieve the same results. The approach taken is intentional as a precursor to breaking the bonds of a straight-forward refactoring of the Planck expressions. The approach also exposes a novel way of understanding gravity from an alternative geometric perspective. While no physical evidence is specifically cited, if evidence were to be found pointing to an underlying space-time fabric that is both logical and quantized, then this approach would dictate that gravity is an inevitable byproduct of whole-unit quantization.

To all of this, we build a foundation with geometric interpretations that describes light and matter in simple terms of a radius and circumference of a circle. With Q_{Lf}, r, c^3, and G used to describe gravity, we find that the scalar constant S is $Q_{Lf}rc^3/G$, which is also $\hbar/2l_f$. Moreover, we find that $C=2\pi r=2\pi l_f$ compared with the arc length set off by θ_{si} precisely describes the minor arc of that circle in terms of angular measure or momentum. Where $E=mc^2$ and $E=nh\nu$, we also find that a fundamental measure of mass and a photon are separated in energy precisely by factor of 2π. And finally, in the analysis of Heisenberg's uncertainty principle, we resolve that certainty is defined relative to one significant measure, l_f.

In conclusion, note the following two tests of Informativity.

A. Measurements of predicted values.

The measure for the signal and idler θ_{si} is so prevalent in Informativity that it may have been taken to be a fundamental constant of nature. This is not a role that is necessarily established; θ_{si} is a prediction of this model that does not arise until after Eq. (19) where it is introduced. As a matter of clarification, θ_{si} is a derived value based on existing expressions that describe gravity and supporting evidence such as Planck's understanding of fundamental measures. In this light, θ_{si} is a predicted radian measure of particular importance to our understanding of light. The reverse argument may also be made, that our knowledge of θ_{si} allows us to derive an expression for gravity. However, both arguments cannot be made simultaneously. Where one phenomenon is understood, the other must be an outcome. For this reason, an argument is put forward that leads to verifiable expressions of physical phenomena.

Shwartz and Harris [12] reported angular measures needed to entangle photons in pure Bell states based on their measure of $\theta_{si}=\theta_s=\theta_i$ exactly equal to that predicted by Informativity. Their model conforms to their observational data from nonlinear X-ray optics experiments, which provides measures of relative angular precision to 10^{-5} radians. Measurements with accuracies of up to 10^{-6} radians will be possible in 2017 at the European X-ray Free Electron Laser facilities (XFEL) in Hamburg, Germany.

B. Measurements of gravitational lensing.

There are several good examples of gravitational lensing within the universe, but for the purposes of Informativity, the best measure of this effect is relative to the Sun. The issue with other targets is the considerable uncertainty in distance in relation to the *Informativity differential* effect. In general, if accurate measures are needed to be resolved, our Sun would most likely be the backdrop to such measurements.

To provide context, we present the effect as a difference from GR in the deflection of light grazing our Sun [25]. With θ the angle of deflection, r and M the radius and mass, G the gravitational constant, and c the speed of light, then

$$\theta = \frac{4GM}{rc^2} = \frac{G}{r^2}\frac{4rM}{c^2} = 8.5\ 10^{-6}\, radians. \tag{55}$$

We see that measuring the effects of Informativity only requires that we are able to detect the difference between Newton's expression G/r^2 and the Informativity expression $Q_{Lf}c^3/r\theta_{si}$ and then use that to solve for the radian difference between GR and Informativity,

$$\Delta\theta = \left(\frac{G}{r^2} - \frac{Q_{Lf}c^3}{r\theta_{si}}\right)\frac{4rM}{c^2} = \frac{4M(G\theta_{si} - Q_{Lf}rc^3)}{rc^2\theta_{si}} = 6.6\ 10^{-12}\ \ radians\,. \tag{56}$$

The effect resolves to six orders in magnitude less than the effects of GR. A search through existing data does not show precision that would reveal this effect, but with future efforts the difference may be resolved.

V. APPENDICES

Appendix A: Numerical limits to $Q_{Lf}r_{Lf}$

Throughout the paper, we find the term $Q_{Lf}r_{Lf}$ repeatedly. This term is referred to as the *Informativity differential* in recognizing the central role it plays in describing how fractional values less than the theoretical limit reflect a distortion effect in distance measurement. Knowing the limits to $Q_{Lf}r_{Lf}$ is also essential in resolving the fundamental measures.

The product of $Q_{Lf}r_{Lf}$ is Eq. (5) multiplied by b.

$$Q_{Lf}r_{Lf} = \left(\sqrt{1+b^2} - b\right)b\,. \tag{A1}$$

Note, what is measured always equals a whole-unit count of a fundamental measure, and with $a=1$ we find that $b=r_{Lf}$ for all values. This is easily verified in that the highest value for Q_{Lf} is obtained for $b=1$ where $(1+1^2)^{0.5} - 1 = 0.414$ and the 'observed' distance of c presented as a count r_{Lf} is always rounded down to the highest integer value equal to the count b with $Q_{Lf}=0.414$ at its highest and quickly approaching 0 with increasing b. Therefore,

$$Q_{Lf}r_{Lf}=\left(\sqrt{1+r_{Lf}{}^{2}}-r_{Lf}\right)r_{Lf}\,. \tag{A2}$$

The lower limit where $r_{Lf}=1$ is easily produced, $lim_{r=1}f(Q_{Lf}r_{Lf})=\sqrt{2}-1$. Conversely, if we divide by r_{Lf}, then add r_{Lf}, square, subtract r_{Lf}^{2}, and divide by 2, we find that

$$\frac{Q_{Lf}{}^{2}}{2}+Q_{Lf}r_{Lf}=\frac{1}{2}\,. \tag{A3}$$

Q_{Lf} decreases with increasing r_{Lf} until the left term drops out. Distance does not need to be significant to reduce the *Informativity differential* to 0.5. At just $10^{4}\ l_f$, $Q_{Lf}r_{Lf}$ rounds to 0.5 to nine significant digits.

Appendix B: Fundamental transforms

On occasion, we find the need to translate from one measure to another. For instance, we may have an expression given in terms of time, but want to create an expression in terms of length. This may be accomplished by multiplying by c, the speed of light. In this paper, this process is referred to as applying a fundamental transform. Each of the transforms may be derived from the definitions of the fundamental measures presented in Eqs. (20–22).

To transform length to time $\Delta(l_f\rightarrow t_f)$, then compare Eqs. (20) and (21),

$$\Delta(l_f\rightarrow t_f)\frac{2G\theta_{si}}{c^{3}}=\frac{2G\theta_{si}}{c^{4}}\,. \tag{B1}$$

Therefore, $\Delta(l_f\rightarrow t_f)=1/c$ and as such, $l_f/c=t_f$. This transform is not typically mentioned as it is a definition of the model. To transform length to mass $\Delta(l_f\rightarrow m_f)$, then compare Eqs. (20–22),

$$\Delta(l_f\rightarrow m_f)\frac{2G\theta_{si}}{c^{3}}=\frac{2\theta_{si}}{c}\,. \tag{B2}$$

Therefore, $\Delta(l_f\rightarrow m_f)=c^{2}/G$ and as such, $l_fc^{2}/G=m_f$.

To transform time to mass $\Delta(t_f\rightarrow m_f)$, comparing Eqs. (21) and (22) gives

$$\Delta(t_f\rightarrow m_f)\frac{2G\theta_{si}}{c^{4}}=\frac{2\theta_{si}}{c}\,. \tag{B3}$$

Therefore, $\Delta(t_f\rightarrow m_f)=c^{3}/G$ giving $t_fc^{3}/G=m_f$.

Appendix C: Effective count of l_f in the measure of $\hbar$

The measure of Planck's constant requires a physical interaction at a specific relative distance. That distance may be resolved as a count of l_f using Eq. (5) where b_{Lf} rounds to r_{Lf} and Eq. (26) where we have substituted $\hbar/l_f^2$ from Planck's relation in Eq. (1). We have

$$Q_{Lf} = \left(1 + r_{Lf}^2\right)^{1/2} - r_{Lf} \,, \tag{C1}$$

$$\theta_{si} = \left(\frac{c^3}{G}\right) Q_{Lf} r_{Lf} l_f = \left(\frac{\hbar}{l_f^2}\right) Q_{Lf} r_{Lf} l_f \,, \tag{C2}$$

$$r_{Lf} = \frac{\theta_{si} l_f}{\hbar Q_{Lf}} = \frac{\theta_{si} l_f}{\hbar\left(\left(1 + r_{Lf}^2\right)^{1/2} - r_{Lf}\right)} \,, \tag{C3}$$

$$\left(r_{Lf}^2 + r_{Lf}^4\right)^{1/2} - r_{Lf}^2 = \frac{\theta_{si} l_f}{\hbar} \,, \tag{C4}$$

$$r_{Lf}^2 + r_{Lf}^4 = \left(\frac{\theta_{si} l_f}{\hbar} + r_{Lf}^2\right)^2 = \frac{\theta_{si}^2 l_f^2}{\hbar^2} + \frac{2\theta_{si} l_f r_{Lf}^2}{\hbar} + r_{Lf}^4 \,, \tag{C5}$$

$$r_{Lf}^2 \left(1 - \frac{2\theta_{si} l_f}{\hbar}\right) = \frac{\theta_{si}^2 l_f^2}{\hbar^2} \,, \tag{C6}$$

$$r_{Lf} = \sqrt{\frac{\theta_{si}^2 l_f^2}{\hbar^2\left(1 - \dfrac{2\theta_{si} l_f}{\hbar}\right)}} = \theta_{si} l_f \sqrt{\frac{1}{\hbar(\hbar - 2\theta_{si} l_f)}} \,. \tag{C7}$$

REFERENCES

[1] T. S. Kuhn, *Black–Body Theory and the Quantum Discontinuity*, pp. 253–254, (Oxford University Press, 1978, ISBN 0-19-502383-8).

[2] P. Mohr, B. Taylor, and D. Newell, *CODATA Recommended Values of the Fundamental Physical Constants: 2010*, p. 73 (2012), arXiv:1203.5425v1.

[3] M. Planck, *Über irreversible Strahlungsvorgänge*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu, Berlin, 5, p. 480 (1899).

[4] S. Hawking, *Gödel and the end of physics*, (Sept. 15, 2015) http://www.damtp.cam.ac.uk/events/strings02/dirac/hawking.

[5] E. Witten, *String Theory Dynamics in Various Dimensions*, Nucl. Phys. B 443, 1–2, 85–126, (1995), arXiv:hep-th/9503124.

[6] C. Rovelli, *Zakopane lectures on loop gravity*, arXiv: 1102.3660.

[7] Alain Connes, Michael Douglas, Albert Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, High Energy Phys., no. 2, Paper 3, 35 pp. (1998)

[8] S. Weinberg, *Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature* (Knopf Doubleday Publishing Group, 1992, ISBN 978-0-307-78786-6.)

[9] H. Langtangen and P. Geir, *Scaling of Differential Equations* (Springer, June 24, 2016, ISBN 978-3319327259).

[10] P. S. Wesson, *The application of dimensional analysis to cosmology*, Space Science Reviews, 27 (2): 117, Bibcode:1980SSRv.27.109W, doi:10.1007/bf00212237 (1980).

[11] T. Banks, W. Fischler, S. Schenker, L. Susskind, *M theory as a matrix model: A conjecture*, Phys. Rev. D. **55**, 5112 (1997)
[12] S. Shwartz and S. E. Harris, *Polarization Entangled Photons at X-Ray Energies*, Phys. Rev. Lett. **106**, 080501 (2011), arXiv: 1012.3499.
[13] L. Smolin, *The Case for Background Independence*, In D. Rickles, S. French, J. T. Saatsi, *The Structural Foundations of Quantum Gravity*, p. 196 (Clarendon Press, 2006).
[14] T. E. Glover *et al.*, *X-ray and optical wave mixing*, Nature, **488**, 7413, 603–608 (2012).
[15] J. D. Anderson, G. Schubert, V. Trimble and M. R. Feldman, *Measurements of Newton's gravitational constant and the length of day*, Europhysics Letters, **110**, 1, (2015).
[16] P. Mohr, B. Taylor, D. Newell, *CODATA Recommended Values of the Fundamental Physical Constants: 2014*, p. 3, (2015), arXiv: 1507.07956v1.
[17] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio *Tenth-Order QED Contributions to the Electron g-2 and an Improved Value of the Fine Structure Constant*, Phys. Rev. Lett. **109**, 111807 (2012), arXiv:hep-ph/1205.5368v2.
[18] A. G. Riess *et al.*, *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, The Astronomical Journal, **116**, 3 (1998), arXiv:astro-ph/9805201.
[19] R. Ciardullo, G. H. Jacoby, H. B. Dejonghe, *The radial velocities of planetary nebulae in NGC 3379*. Astrophys. J. **414**, 454–462 (1993).
[20] J. D. Barrow and D. J. Shaw, *The value of the cosmological constant,* General Relativity and Gravitation, 43 (10): 2555–2560 (2011), arXiv:1105.3105.
[21] A. Gasanalizade and R. Hasanalizade, *Determination of Dark Energy and Dark Matter from the values of Redshift for the present time, Planck and Trans-Planck epochs of the Big-Bang model*, (2015), arXiv:1111.2936v3.
[22] L. E. Ballentine, *The Statistical Interpretation of Quantum Mechanics*, Rev. Mod. Phys. **42** 4. 358–381, Section 3.2 (1970).
[23] E. H. Kennard, *Zur Quantemechanik einfacher Bewegungstypen*, Zeitschrift fur Physik (in German), **44** (4-5): 326–352 (1927).
[24] H. Weyl, *Gruppentheorie und Quantenmechanik* (Hirzel, Leipzig, 1928).