Proof of No Singularity in Schwarzschild Black Hole and Big Bang

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

In this paper, we propose that the center of Schwarzschild black hole (SBH) and the Big Bang are in the minimum entropy equal to the Boltzmann constant. Then we prove the uncertainty relation (UR) of SBH and Big Bang UR which suggest no singularity in SBH and Big Bang.

I. Introduction
S.W. Hawking and R. Penrose proved the theory of singularities [1]. It shows that the singularities are in the black holes and the universe originated the Big Bang singularity. Many literatures discussed no singularity in black holes and Big Bang with the quantum effect; please refer to [2]. Moreover, M. Planck considered the reduced Planck constant ℏ being the minimum action [3]. R. Penrose considered the Big Bang being in the minimum entropy equal to zero [4]. It is the initial condition of Big Bang probably. Similarly we propose the center of Schwarzschild black hole (SBH) and Big Bang being in the minimum entropy, but the minimum entropy doesn’t equal to zero. Then we can prove the uncertainty relation (UR) of SBH and Big Bang UR which proposed by Y. Bao [5].

This paper is organized as follows. In Sec. II, we prove the UR of SBH. In Sec. III, we prove the Big Bang UR. We conclude in Sec. IV.

II. No singularity in SBH
Y. Bao proposed the UR of SBH by the generalized relation [5].

\[ M_{\text{BH}} V_{\text{BH}} \sim M_{\text{P}} V_{\text{P}} = M_{\text{P}} L_{\text{P}}^3 = \hbar^2 G / c^4 \]  

(1)

where \( G, c, M_{\text{BH}}, V_{\text{BH}}, M_{\text{P}}, \) and \( L_{\text{P}} = \sqrt{\hbar G / c^3} \) denote the gravitational constant, speed of light in vacuum, SBH mass, volume of SBH center, Planck mass, Planck volume and Planck length, respectively. It is impossible to measure the SBH mass and volume simultaneously. Therefore it suggests no singularity in SBH with quantum effect.

We prove (1) now. For the Coulomb-like gravitational fields, from \( s_{\text{grav}} = 2\pi\alpha e^2 T_{\text{grav}} / \hbar^2 G \) [6] and \( S_{\text{grav}} = \int_V s_{\text{grav}} \) [7], we obtain

\[ S_{\text{grav}} = 2\pi\alpha e^2 T_{\text{BH}} V_{\text{BH}} / \hbar^2 G \]  

(2)

where \( s_{\text{grav}} \) is the entropic density, \( \alpha \) a constant, \( T_{\text{grav}} \) the effective temperature, \( S_{\text{grav}} \) the entropy of SBH center, and \( V \) the spatial volume. For the SBH center, \( V \rightarrow V_{\text{BH}} \) and \( T_{\text{grav}} \rightarrow T_{\text{BH}} \), where \( T_{\text{BH}} \) is the temperature of center.

We assume \( S_{\text{grav}} \sim \kappa \) that is the Boltzmann constant \( \kappa \) being the minimum entropy, resembling \( \hbar \). Then we find

\[ T_{\text{BH}} V_{\text{BH}} \sim \hbar^2 G / 2\alpha e c^2 \]  

(3)

Therefore the temperature of SBH center and its volume have the inversely-proportional relationship.

From the gravitational analogue of the fundamental law of thermodynamics in the form [7]

\[ T_{\text{grav}} dS_{\text{grav}} = dU_{\text{grav}} + p_{\text{grav}} dV \]  

(4)

where \( U_{\text{grav}} \) and \( p_{\text{grav}} \) denote the internal energy and isotropic pressure of the free gravitational field, respectively. Taking (3), \( p_{\text{grav}} = 0 \) [7], and \( dU_{\text{grav}} = d(M_{\text{BH}} c^2) \) to (4), we give

\[ M_{\text{BH}} V_{\text{BH}} \sim \hbar^2 G / 2\alpha e c^4 \]  

(5)

Similarly for the wave-like gravitational fields, \( p_{\text{grav}} = \rho_{\text{grav}} / 3 \) [7], we obtain

\[ M_{\text{BH}} V_{\text{BH}} \sim 3\hbar^2 G / 8\sqrt{\alpha} e c^4 \]  

(6)

where \( \beta \) is a constant. Then we prove (1).

III. No singularity at Big Bang
The Big Bang UR is [5]

\[ T_{\text{BB}} V_{\text{BB}} \sim \hbar^2 G / \kappa c^2 \]  

(7)

where \( V_{\text{BB}} \) is the volume of Big Bang and \( T_{\text{BB}} \) its temperature. It suggests no singularity at Big Bang with quantum effect also.

For a spatially flat Robertson–Walker geometry with scalar perturbations in a longitudinal gauge, such that the line-element can be written [7] \( (\hbar = G = c = \kappa = 1) \)

\[ ds^2 = a^2(t)[- (1+2\Phi)dt^2 + (1-2\Phi)(dx^2 + dy^2 + dz^2)] \]  

(8)

\[ u^a = [(1-\Phi) / a; \ u^i], z^a = (0; \nabla_i \Phi) / a \ | \nabla \Phi| \]  

(9)

\[ S_{\text{grav}} \sim t^{5/3} \]  

(10)

where \( a \) is the scale factor, \( u^a \) the timelike unit vector, \( z^a \) a spacelike unit vector, and \( t = \int a(t)dt \) the proper time of comoving observers.

When \( t \rightarrow 0 \), \( S_{\text{grav}} \rightarrow 0 \), so R. Penrose considered the Big Bang being in the minimum entropy equal to zero [4]. But we propose

\[ S_{\text{grav}} \sim t^{5/3} + S_{\text{grav}0} \]  

(11)
where $S_{\text{grav}} \geq 0$ is the minimum entropy. When $t \to 0$, $S_{\text{grav}} \to S_{\text{grav}0}$, $V \to V_B$, and $T_{\text{grav}} \to T_B$. Substituting them, $dU_{\text{grav}} = d(\rho_{\text{grav}} L^3)$, and $\rho = \rho_{\text{grav}}$ into (4), we obtain

$$T_B dS_{\text{grav}0} - V_B d\rho_{\text{grav}} + (1 + \omega) \rho_{\text{grav}} dV_B$$

(12)

where $\omega$ is the coefficient of state.

From [7] ($h = G = c = \kappa = 1$)

$$8 \pi \rho_{\text{grav}} = a |(a^i u^j)_{i < j}| / a^3$$

(13)

$$T_{\text{grav}} = 1 / |2 \pi$$

(14)

where $i, j$ are spatial indices, we gain

$$\rho_{\text{grav}} \sim c^2 G / h^2$$

(15)

Taking (15) to (12), we give

$$dS_{\text{grav}0} \sim c^2 G / h^2$$

(16)

Also assuming $S_{\text{grav}0} \sim \kappa$ that is Big Bang being in the minimum entropy equal to Boltzmann constant, and ordering $\omega = -1$, we find

$$T_B V_B \sim \kappa G / 2 c e^2$$

(17)

Hence we prove (7). Note (3) and (17), they are analogous, but their physical meaning aren't the same.

IV. Conclusion

In this paper, we assumed the center of SBH being in the minimum entropy equal to the Boltzmann constant $\kappa$, found the temperature of center and its volume having the inversely-proportional relationship; proved the UR of SBH which suggests no singularity in it, whether the gravitational fields are Coulomb-like or wave-like; proposed the Big Bang being in the minimum entropy equal to $\kappa$ also, proved the Big Bang UR suggesting no singularity.

From the original definition of entropy $S = Q / T$, for the Big Bang, the heat quantity $Q$ is tremendous but finite, if $S \to 0$, the temperature $T \to \infty$, that is infinite. This is the classical solution. Considering the quantum effect, $T$ is impossibly infinite, so $S \neq 0$. Then we proposed the Big Bang being in the minimum entropy equal to $\kappa$. Similarity for the center of SBH, $Q \approx M_B c^2$, the temperature of center isn't infinite with quantum effect. Note here we only consider the center of SBH, not the total SBH, it isn't against principle of entropy increase of black holes.

References


