Einstein rebooted, Bell's theorem refuted, etc.

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Abstract: Rebooting Einstein's ideas about local-causality, an engineer brings local-causality to quantum theory via operators and variables in 3-space. Taking realism to be the view that external reality exists and has definite properties, his core principle is commonsense local realism (CLR): the union of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively). Endorsing Einstein-separability — system X is independent of what is done with system Y that is spatially separated from X — Bell's famous mission is advanced. That is, by means of parameters λ, a more complete specification of EPRB's physics is successful. A consequent locally-causal refutation of Bell's theorem allows EPRB correlations to be explained in a classical way, in line with Einstein's ideas, without reference to Hilbert space, quantum states, etc. Conclusion: Bell's theorem is based on a mathematical error; an error in reduction is inconsistent with Bell's opening assumptions.

Keywords: Bell's error, causality, CLR, EPRB, equivalence, locality, Malus, quon, realism

Notes to the Reader: (i) Please be critical and ask questions re this draft! To facilitate discussion, improvement, correction, all paragraphs and equations are numbered. (ii) Texts freely available online – see References – are taken as read. (iii) All results here accord with quantum theory and experiment. (iv) Negating wave/particle quantum/classical dichotomies, fundamental entities – eg, electrons, photons, protons – are elements of Q, the set of *quons* (identifier q). Accepting Einstein-separability (Laudisa 1995), quons are presumed separable, rejecting suggestions of *inseparable entanglement* (Feingold & Peres 1985) or each pair being a *single nonlocal indivisible entity* (Mermin 1985). In short: Q merges so-called quantum and classical entities into one elementary family.

1 Introduction

> #1.0. "Einstein argued that the EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way," Bell (2004:86). "In a complete physical theory of the type envisioned by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can be thought of as initial values of these variables at some suitable instant," Bell (1964:196).

#1.1. Following Bell's (1964) example – adopting his valid formalisms and using principles consistent with Einstein's ideas – we'll be working to account for EPRB correlations in a classical way.

#1.2. Taking realism to be the view that external reality exists and has definite properties, our core principle is commonsense local realism (CLR), the union of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively).

#1.3. CLR thus rejects the nonlocal mechanism identified in these Bellian conclusions:

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"In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant," Bell (1964:199). 'Detailed analysis [Bell's theorem] shows that any classical account of these correlations has to contain just such 'spooky action at a distance' as Einstein could not believe in ... rendering Einstein's conception of the world untenable,' after Bell (2004:86).

#1.4. Based on CLR – adding/subtracting parameters but free of instantaneous signals – our theory will be Lorentz invariant and pro-Einstein. For Bell missed this fact and its relationship to equivalence classes: Similar tests on similar things produce similar results, and correlated test on correlated things produce correlated results, without mystery. In this context we show that Bell makes a mathematical error (an error in reduction), that is inconsistent with his opening assumptions. The error equates his work to a bygone naive-realism, beyond the requirements of local causality where a weaker locally-causal deduction goes through. Let's see.

2 Analysis

$$A^{\pm} \equiv \pm 1 = [\mathbf{a}\cdot\mathbf{a}^{\pm}] \leftarrow q(\mathbf{a}^{\pm}) \leftarrow \delta_{\mathbf{a}} \leftarrow q(\lambda_i) \leftarrow S_E \rightarrow q(\mu_i) \rightarrow \delta_{\mathbf{b}} \rightarrow q(\mathbf{b}^{\pm}) \rightarrow [\mathbf{b}\cdot\mathbf{b}^{\pm}] = \pm 1 \equiv B^{\pm}$$

$$\llbracket \quad Alice's\ locale \quad \rrbracket \quad \llbracket Source \rrbracket \quad \llbracket \quad Bob's\ locale \quad \rrbracket$$

Figure 1: Experiment E, based on the EPRB experiment in Bell (1964). Every relevant element of the subject reality is shown in 3-space; nothing irrelevant is found.* With pristine spin-related properties λ_i and μ_i, spin-$\frac{1}{2}$ quons $q(\lambda_i)$ and $q(\mu_i)$ emerge from source S_E via a spin-conserving decay such that $\lambda_i + \mu_i = 0$. The quons interact with detectors (polarizer-analyzers) freely and independently operated by Alice and Bob. These interaction events are locally-causal and spacelike-separated. Thus, under Einstein causality – elements belonging to spacelike-separated sites commute – the respective elements are physically independent.

The principal-axis of Alice's dichotomic linear-polarizer $\delta_{\mathbf{a}}$ is at unit-vector $\mathbf{a}$ in 3-space. The output of the interaction $\delta_{\mathbf{a}} q(\lambda_i)$ is $q(\mathbf{a}^{\pm})$, with $\mathbf{a}^{+} \equiv +\mathbf{a}$; etc. The polarized quon $q(\mathbf{a}^{\pm})$ goes to Alice's analyzer $[\mathbf{a}\cdot\mathbf{a}^{\pm}]$ which reports (via its inner-product function) the $\mathbf{a}$-related spin-projection $A^{\pm} = \pm 1$ in units of $s\hbar$; intrinsic spin $s = \frac{1}{2}$ here. $B^{\pm}$, $q(\mu_i)$, $\delta_{\mathbf{b}}$ and $\delta_{\mathbf{b}} q(\mu_i) \rightarrow q(\mathbf{b}^{\pm})$ similarly. Under E, to confirm related probabilities, we allow experiments like $\delta_{\mathbf{b}} q(\mathbf{a}^{+}) \rightarrow q(\mathbf{b}^{+}) \oplus q(\mathbf{b}^{-})$.

Since A and B are discrete, we replace Bell's integrals with sums, and Bell's 1964:(14) with discrete variables λ_i, etc; consistent with EPRB and Bell's (1964:195) indifference.

* "In Bohr's view, the characteristic new feature in quantum physics is merely the restricted divisibility of the phenomenon, which requires a specification of all parts of the experimental setup for unambiguous description," Mehra (1975:152).

#2.1. From Fig. 1: two spin-$\frac{1}{2}$ quons $q(\lambda_i)$ and $q(\mu_i)$ – with pristine spin-related properties λ_i and μ_i (from a set of multivectors uniformly distributed in 3-space; aka hidden-variables, beables); and probability zero that any two quon-pairs are the same – emerge from a spin-conserving decay such that

$$\lambda_i + \mu_i = 0; \text{ ie, } \mu_i = -\lambda_i \equiv \lambda_i^{-} \text{ for notational convenience.} \tag{1}$$

#2.2. (1) defines the interdependency of λ_i and μ_i, the variables we use to form a more complete specification of experiment E around causality and locality. Since one pristine property may be represented in terms of the other, let's first focus on λ_i; call it the primary random variable for now (for the choice and the name matter not). Then μ_i becomes the secondary variable (for now) with $\mu_i = \lambda_i^-$.

#2.3. From the well-known action of linear-polarizers on quons $q(\mathbf{a}^\pm)$, we can match the general laboratory operation $\delta_\mathbf{a} q(\mathbf{a}^\pm) \to q(\mathbf{a}^\pm)$ with the interaction $\delta_\mathbf{a} q(\lambda_i) \to q(\mathbf{a}^\pm)$ in Fig. 1. The following equivalence relations consequently hold for the i-th and j-th quons:

$$\text{If } \delta_\mathbf{a} q(\lambda_i) \to q(\mathbf{a}^+) \text{ then } \lambda_i \sim \mathbf{a}^+ \because \delta_\mathbf{a} q(\mathbf{a}^+) \to q(\mathbf{a}^+) \text{ exclusively.} \tag{2}$$

$$\text{If } \delta_\mathbf{a} q(\lambda_j) \to q(\mathbf{a}^-) \text{ then } \lambda_j \sim \mathbf{a}^- \because \delta_\mathbf{a} q(\mathbf{a}^-) \to q(\mathbf{a}^-) \text{ exclusively.} \tag{3}$$

#2.4. That is, from (2)-(3): The polarizing-operator $\delta_\mathbf{a}$ delivers $q(\lambda_i)$ and $q(\mathbf{a}^+)$ to the same codomain – and it is impossible for $\delta_\mathbf{a}$ to deliver $q(\lambda_i)$ and $q(\mathbf{a}^+)$ to two different codomains – so equivalence relations hold between spin-related parameters λ_i and $\mathbf{a}^+$; etc. Thus, consistent with the validity of Bell's 1964:(1), the analyzer-functions and outputs in Fig. 1 (and their expectations) can be written:

$$A(\mathbf{a}, \lambda) = A^\pm = \cos(\mathbf{a}, \lambda \,|\, \lambda \sim \mathbf{a}^\pm) = \pm 1; \langle A \,|\, E\rangle = 0 \because P(\lambda \sim \mathbf{a}^+ \,|\, E) = P(\lambda \sim \mathbf{a}^- \,|\, E) = \tfrac{1}{2}. \tag{4}$$

$$B(\mathbf{b}, \mu) = B^\pm = \cos(\mathbf{b}, \mu \,|\, \mu \sim \mathbf{b}^\pm) = \pm 1; \langle B \,|\, E\rangle = 0 \because P(\mu \sim \mathbf{b}^+ \,|\, E) = P(\mu \sim \mathbf{b}^- \,|\, E) = \tfrac{1}{2}. \tag{5}$$

#2.5. In words: $\cos(\mathbf{a}, \lambda \,|\, \lambda \sim \mathbf{a}^+)$ denotes the cosine of the angle between $\mathbf{a}$ and $\mathbf{a}^+$, given λ is equivalent to $\mathbf{a}^+$; so the outcome is $A^+ = +1$; etc. Our theory is therefore locally-causal: from (4)-(5) and Fig. 1, $A^\pm$ and $B^\pm$ are locally-caused by precedent local events $\delta_\mathbf{a} q(\lambda_i)$ and $\delta_\mathbf{b} q(\mu_i)$, respectively, which are spacelike-separated. The expectations in (4) and (5) are zero because λ and μ are hidden (unknown) random variables. Nevertheless, $A_i^\pm$ and $B_i^\pm$ are pairwise correlated via the pairwise correlation of λ_i and μ_i in (1).

#2.6. We now move to derive $\langle AB \,|\, E\rangle$, the expectation for experiment E, via the probabilities for the conjunction of the outcomes in (4) and (5). Since primacy is arbitrary (#2.2), and given the correlation in (1), the following string of probability relations holds:

$$P(\lambda \sim \mathbf{a}^+ \,|\, E, \mu \sim \mathbf{b}^+) = P(\mu \sim \mathbf{b}^+ \,|\, E, \lambda \sim \mathbf{a}^+) = P(\lambda^- \sim \mathbf{b}^+ \,|\, E, \lambda \sim \mathbf{a}^+) = P(\lambda \sim \mathbf{b}^- \,|\, E, \lambda \sim \mathbf{a}^+) \tag{6}$$

$$= P(\delta_\mathbf{b} q(\lambda \sim \mathbf{a}^+) \to q(\lambda \sim \mathbf{b}^-) \,|\, E) = P(\delta_\mathbf{b} q(\mathbf{a}^+) \to q(\mathbf{b}^-) \,|\, E) = \cos^2 \tfrac{1}{2}(\mathbf{a}^+, \mathbf{b}^-) = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{7}$$

#2.7. The probability relation LHS (7) is, as shown, equivalent to a classical (local) test on spin-$\frac{1}{2}$ quons of known polarization. So, per RHS (7), this probability relation is given by Malus' $\cos^2$ Law for the relative intensity of beams of polarized spin-$\frac{1}{2}$ quons. Thus, since our equivalence relations hold within such probability functions, Malus' Law is generalized to entangled quons:

$$P(\delta_\mathbf{b} q(\lambda \sim \mathbf{a}^+) \to q(\mathbf{b}^+) \,|\, E) = P(\delta_\mathbf{b} q(\mathbf{a}^+) \to q(\mathbf{b}^+) \,|\, E) = \cos^2 \tfrac{1}{2}(\mathbf{a}^+, \mathbf{b}^+) = \cos^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}); \tag{8}$$

$$P(\delta_\mathbf{b} q(\lambda \sim \mathbf{a}^-) \to q(\mathbf{b}^+) \,|\, E) = P(\delta_\mathbf{b} q(\mathbf{a}^-) \to q(\mathbf{b}^+) \,|\, E) = \cos^2 \tfrac{1}{2}(\mathbf{a}^-, \mathbf{b}^+) = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}); \text{ etc.} \tag{9}$$

#2.8. Using this generalization, we derive $\langle AB \,|\, E\rangle$, the expectation for experiment E. From (4)-(9):

$$\langle A^+B^+ \,|\, E\rangle = P(\lambda \sim \mathbf{a}^+ \,|\, E) \cos(\mathbf{a}, \lambda \,|\, \lambda \sim \mathbf{a}^+) P(\mu \sim \mathbf{b}^+ \,|\, E, \lambda \sim \mathbf{a}^+) \cos(\mathbf{b}, \mu \,|\, \mu \sim \mathbf{b}^+) \tag{10}$$

$$= \tfrac{1}{2} P(\mu \sim \mathbf{b}^+ \,|\, E, \lambda \sim \mathbf{a}^+) = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{11}$$

$$\text{Similarly: } \langle A^+B^- \,|\, E\rangle = \langle A^-B^+ \,|\, E\rangle = -\tfrac{1}{2} \cos^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}); \langle A^-B^- \,|\, E\rangle = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{12}$$

$$\therefore \langle AB \,|\, E\rangle = \langle A^+B^+ \,|\, E\rangle + \langle A^+B^- \,|\, E\rangle + \langle A^-B^+ \,|\, E\rangle + \langle A^-B^- \,|\, E\rangle = -\mathbf{a} \cdot \mathbf{b}. \, QED. \blacksquare \tag{13}$$

$$\text{Finally: } P(AB = +1 \,|\, E) = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \; P(AB = -1 \,|\, E) = \cos^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{14}$$

#2.9. (13), our locally-causal result, reproduces the results of quantum theory and contradicts Bell's theorem. But before turning to Bell (and revealing his error), we next demonstrate the utility and the validity of (1)-(14). To that end, beginning with our modification of EPR to fully accord with (1)-(14), we refute Mermin (1990). Mermin's Bell-based analysis – with its "always-vs-never refutation" of EPR's ideas – is (for us) an all-or-nothing test of Bell's interpretation of Einstein's ideas.

3 Mermin's "always-vs-never refutation of EPR" refuted

#3.0. "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible," EPR (1935:780).

"If, without in any way disturbing $q(\mu_i)$, Alice can predict with certainty that Bob's result will be $B_i = -1$ when he tests $q(\mu_i)$ with $\delta_{\mathbf{a}}$ [which may be a disturbance], then elements of reality $\delta_{\mathbf{a}}$ and $q(\mu_i \sim \mathbf{a}^-)$ mediate this result. The element of reality corresponding to Bob's $B_i = -1$ result will be $q(\mathbf{a}^-)$," after Watson (1998:417; 1999). For, after testing $q(\lambda_i)$ with $\delta_{\mathbf{a}}$, here's how Alice predicts Bob's result from her result $A_i = +1$ and (1)-(5):

$$A_i = +1 \because \delta_{\mathbf{a}} q(\lambda_i) \to q(\mathbf{a}^+) \to [\mathbf{a}{\cdot}\mathbf{a}^+] = +1. \therefore q(\lambda_i) = q(\lambda_i \sim \mathbf{a}^+). \therefore q(\mu_i) = q(\mu_i \sim \mathbf{a}^-). \quad (15)$$

$$\therefore \delta_{\mathbf{a}} q(\mu_i) = \delta_{\mathbf{a}} q(\mu_i \sim \mathbf{a}^-) \to q(\mathbf{a}^-) \to [\mathbf{a}{\cdot}\mathbf{a}^-] = -1 = B_i.\ QED. \blacksquare \text{ And vice-versa.} \quad (16)$$

#3.1. We now consider experiment M, Mermin's (1990; 1990a) 3-quon variant of GHZ (1989). Respectively: Three spin-$\frac{1}{2}$ quons with spin-related properties λ, μ, ν emerge from a spin-conserving decay such that (taking ν to be the tertiary variable; see #1.2),

$$\lambda + \mu + \nu = \pi. \therefore \nu = \pi - \lambda - \mu. \quad (17)$$

#3.2. Respectively throughout: the quons separate in the y-z plane and interact with spin-$\frac{1}{2}$ polarizers that are orthogonal to the related line of flight. Let a, b, c denote the azimuthal angles of each polarizer's principal-axis relative to the positive x-axis, and let the test results be A, B, C. Then, as in (4)-(5), let

$$A(a, \lambda) = A^{\pm} = \cos(a{-}\lambda\,|\,\lambda \sim a^{\pm}) = \pm 1, \quad (18)$$

$$B(b, \mu) = B^{\pm} = \cos(b{-}\mu\,|\,\mu \sim b^{\pm}) = \pm 1, \quad (19)$$

$$C(c, \nu) = C^{\pm} = \cos(c{-}\nu\,|\,\nu \sim c^{\pm}) = \pm 1. \quad (20)$$

#3.3. Via the principles in (1)-(14) – and nothing more – we now derive $\langle ABC\,|\,M\rangle$, the expectation for experiment M (with condition-identifier M suppressed in (21) to limit its length):

$$\big\langle A^+B^+C^+\,|\,M\big\rangle$$

$$= P(\lambda \sim a^+)\cos(a{-}\lambda\,|\,\lambda \sim a^+)P(\mu \sim b^+)\cos(b{-}\mu\,|\,\mu \sim b^+)P(\nu \sim c^+\,|\,\lambda \sim a^+, \mu \sim b^+)\cos(c{-}\nu\,|\,\nu \sim c^+) \quad (21)$$

$$= \tfrac{1}{4}P(\nu \sim c^+\,|\,M, \lambda \sim a^+, \mu \sim b^+) = \tfrac{1}{4}P(\pi - \lambda - \mu \sim c^+\,|\,M, \lambda \sim a^+, \mu \sim b^+) \quad (22)$$

$$= \tfrac{1}{4}P(\pi - a^+ - b^+ - c^+\,|\,M) = \tfrac{1}{4}\cos^2\tfrac{1}{2}(\pi - a^+ - b^+ - c^+) = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a+b+c). \quad (23)$$

$$\text{Similarly: } \big\langle A^+B^-C^-\,|\,M\big\rangle = \big\langle A^-B^+C^-\,|\,M\big\rangle = \big\langle A^-B^-C^+\,|\,M\big\rangle = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a+b+c), \text{ and} \quad (24)$$

$$\big\langle A^+B^+C^-\,|\,M\big\rangle = \big\langle A^+B^-C^+\,|\,M\big\rangle = \big\langle A^-B^+C^+\,|\,M\big\rangle = \big\langle A^-B^-C^-\,|\,M\big\rangle = -\tfrac{1}{4}\cos^2\tfrac{1}{2}(a+b+c). \quad (25)$$

$$\therefore \langle ABC\,|\,M\rangle \equiv \Sigma\,\big\langle A^{\pm}B^{\pm}C^{\pm}\,|\,M\big\rangle \quad (26)$$

$$= \sin^2\tfrac{1}{2}(a+b+c) - \cos^2\tfrac{1}{2}(a+b+c) = -\cos(a+b+c).\ QED. \blacksquare \quad (27)$$

$$\text{Finally: } P(ABC = +1\,|\,M) = \sin^2\tfrac{1}{2}(a+b+c).\ P(ABC = -1\,|\,M) = \cos^2\tfrac{1}{2}(a+b+c). \quad (28)$$

#3.4. (27) is the correct result for experiment M – delivering Mermin's (1990a:733) *crucial minus sign* – ie, from (28): $\langle ABC\,|\,M\rangle = -1$ when $a + b + c = 0$. Thus, consistent with the ordinary rules for operators and functions in 3-space, we again deliver classically-intelligible EPR correlations. Since our results are at odds with Bell's ideas, but consistent with his EPRB-based mission, we now turn to Bell's theorem to locate his error.

4 Bell's theorem refuted: his famous 1964:(15) is false

> #4.0. "It is a matter of indifference ... whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, [Bell writes] as if λ were a single continuous parameter," Bell (1964:195). λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

#4.1. At Bell 1964:(3) – relying on his 1964:(15) – Bell announces his 'impossibility' theorem. So according to Bell, (13) and (27) are impossible. Alas, as indicated above, and as we'll soon see: it's Bell's theorem and his famous inequality, Bell 1964:(15), that are impossible in the context of EPRB.

#4.2. In short: a mathematical-reduction error converts an unnumbered equation in Bell (1964) into a false and unrecognized (but later adopted) inequality. So Bell's 1964:(15) and 1964:(3) – his famous inequality and his famous impossibility theorem, respectively – are false under EPRB, and beyond.

#4.3. Here's Bell's critical error: To establish the inequality in Bell 1964:(3), Bell (1964:197) takes us to his proof – 'Contradiction: The main result will now be proved.' That is, we are taken to the crucial Bell 1964:(15) via Bell 1964:(14), unit-vector $\mathbf{c}$, and three unnumbered equations. Numbering them (14a)-(14c), Bell uses $(A(\mathbf{a},\lambda))^2 = 1$ to move from (14a) to (14b). In our discrete terms, Bell requires $A(\mathbf{a},\lambda_i)A(\mathbf{a},\lambda_{n+i}) = 1$, in line with naive realism. In the real world that we address in a classical way, but not naively (the same world that Bell addresses): $A(\mathbf{a},\lambda_i)A(\mathbf{a},\lambda_{n+i}) = \pm 1$.

#4.4. To see this in experiment E (ie, EPRB per Bell (1964) and Fig. 1 above), let $3n$ random quon-pairs be equally distributed (for convenience in presentation) over three randomized polarizer-settings $\mathbf{a}, \mathbf{b}, \mathbf{c}$. Then, for generality, with an index uniquely identifying each pair, let n be such that, to an adequate accuracy hereafter:

$$\text{Bell's (14a)} = \langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i) - A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i})] \tag{29}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)[A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) - 1]. \tag{30}$$

#4.5. Now (30) is the correct discrete form of Bell's (14a). And Bell's (14c) is a valid conclusion from his (14b). So, if Bell's (14b) = Bell's (14a), our (30) and Bell's (14c) should be equal. Let $\overset{?}{=}$ identify our suspicion of Bell's equality under these conditions; ie, we have our (30) = Bell's (14a) $\overset{?}{=}$ (14b) = (14c). So, in combination with Bell's (14b)-(14c)-(15), we have:

$$\langle BC\,|\,E\rangle \equiv -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_{2n+i})A(\mathbf{c},\lambda_{2n+i}) \tag{31}$$

$$\overset{?}{=} -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i});\ \text{from our (30)} = \text{from Bell's (14a)}. \tag{32}$$

#4.6. Alas, as foreshadowed at #4.3 above: to remove our **?** from (32) and justify his (14b) = (14a), Bell requires $\lambda_i = \lambda_{n+i}$: which is valid under the assumption of naive-realism but impossible under EPRB. Impossible because, under EPRB and in our discrete terms, λ_i and λ_{n+i} are random variables in 3-space, and $\lambda_i \neq \lambda_{n+i}$ in general. So before moving to Bell's explanation, here are two genuine EPRB-based inequalities, with (33) identified as Bell's error under EPRB:

$$\text{Bell 1964: (14b)} \neq \text{Bell 1964: (14a)}. \tag{33}$$

$$-\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) \neq -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_{2n+i})A(\mathbf{c},\lambda_{2n+i}) = \langle BC\,|\,E\rangle. \tag{34}$$

#4.7. To meet the requirement that $\lambda_i = \lambda_{n+i}$, while evidently missing (33) under EPRB, here's Bell (2004:147):

"To explain this dénouement [of Bell's theorem] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."

And here's d'Espagnat (1979:166), recast for EPRB: 'A physicist can infer that in every pair, one particle has the property A^+ and the other has the property A^-. Similarly, he can conclude that in every pair one particle has the property B^+ and one B^-, and one has property C^+ and one C^-. These conclusions require a subtle but important extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis A would give the definite result A^+, then that particle is said to have the property $A+$. In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times. ... This view is contrary to the conventional interpretation of quantum mechanics, but it is not contradicted by any fact that has yet been introduced.'

#4.8. We respond:

(i) λ was designed by Bell to denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242). Neither EPR nor Einstein proposed a return to naive-realism. We thus see, as a result of his error, mission-creep on Bell's part.

(ii) Against naive-realism we have Bohr's insight: "... the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'," Bell (2004: xi-xii). CLR's physical-realism – *some physical properties change interactively* – is consistent with Bohr's insight. "It seems to me that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [like Bell's *impossibility* theorem], and most of 'quantum logic'," Bell (2004: xi-xii). We agree.

(iii) In the context of factorization, Bell's later efforts fare no better when he begins to rely "for example, on a full specification of local beables in a given space-time region," Bell (2004:240). How would such be provided for our $q(\lambda_i)$? Even given $\lambda_i \sim \mathbf{a}^+$?

(iv) "Very often factorizability is taken as the starting point of the analysis. Here we preferred to see it not as the *formulation* of 'local causality', but as a consequence thereof," Bell (2014:243). Our (4)-(9) show the way through Bell's factorization dilemma. We thus confirm Bell's (2004:239) 'utmost suspicion' regarding his own work toward a locally causal theory: Bell threw the baby out with the bathwater.

(v) Where Bell works with $q(\lambda) = q(\lambda = \mathbf{a}^+)$ – via a false inference to a fallacy and confusion about action at a distance – we arrive at the weaker $q(\lambda) = q(\lambda \sim \mathbf{a}^+)$ by deduction.

(vi) "I cannot say that action at a distance is required in physics. But I can say that you cannot get away with no action at a distance. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly. Well, that's just the fact of the situation; the Einstein program fails, that's too bad for Einstein, but should we worry about that?" (pp.5-6). "And it might be that we have to learn to accept not so much action at a distance, but [the] inadequacy of no action at a distance," (p.6). "And that is the dilemma. We are led by analyzing this situation to admit that in somehow distant things are connected, or at least not disconnected," (p.7). "I don't know any conception of locality which works with quantum mechanics. So I think we're stuck with nonlocality," (p.12). "There is no energy transfer and there is no information transfer either. That's why I am always embarrassed by the word action, and so I step back from asserting that there is action at a distance, and **I say only that you cannot get away with locality**. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is action at a distance," (p.13); from Bell (1990); emphasis added.

"Now, it's my feeling that all this action at a distance and no action at a distance business will go the same way [eg, as the ether]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved," Bell (1990:9). Given such confusion about action at a distance: we rest our case against Bell and many others.

#4.9. Finding no prior indication that Bell's 'additional supplementary variables' were to restore more to quantum theory than causality and locality, we show that Bell's famous inequality, Bell 1964:(15), is based on an error: his (14a) $\neq$ his (14b). In maintaining Bell's theorem, Bell and d'Espagnat return to naive-realism; and the same error infects the CHSH family of inequalities (see Bell 1980:14).

5 Conclusions

#5.1. Bell's theorem is refuted; Bell's rejection of Einstein's conception of the world is quashed; Bell's ambivalence re action-at-a-distance is resolved. Einstein separability holds.

#5.2. Given no hint that Bell (1964) is based on the likes of d'Espagnat's inferences to naive-realism, we conclude that Bell's theorem and its many variants are based on a mathematical error; an error in reduction: Bell 1964:(14a) $\neq$ Bell 1964:(14b). We remain open to evidence that supports an alternative proposition: that Bell, contrary to Einstein's views, began with d'Espagnat-style inferences.

#5.3. We find no evidence that EPR or Einstein had such a primitive notion as naive-realism in view. On the contrary, endorsing Einstein-separability – system X is independent of what is done with system Y that is spatially separated from X – we have advanced Bell's mission. That is, by means of parameters λ, a more complete specification of EPRB's physics has succeeded.

#5.4. We began on the right track: starting with (1), an ironclad fact, then adding a function to Bell's 1964:(1) to give (4)-(5). We thus arrived at (13), (16) and (27) via facts associated with equivalence relations and probability theory. In that (13) delivers the same result as quantum theory and the correct analysis of EPRB correlations in a classical way, we meet Einstein's case for the same.

#5.5. We were right, contrary to Bell's approach, to allow that polarizer/quon interaction may perturb a quon. Bypassing such perturbation in line with d'Espagnat's analysis, Bell limits the validity of his theorem to systems consistent with his error. Under CLR, the consequent strong classicality in Bell's theorem is replaced by the weaker reality of equivalence relations. Our theory thus reaches beyond the classical.

#5.6. Based on the rightness of CLR and equivalence relations, our theory readily refutes the all-or-nothing test of Bell's ideas in Mermin (1990a). And our theory is Lorentz invariant, for Bell missed the following fact and its association with equivalence relations: Similar tests on similar things produce similar results, and correlated test on correlated things produce correlated results, without mystery. We are thus able to correctly analyze multi-quon experiments via real operators in 3-space; without recourse to action-at-a-distance, collapse, Hilbert-space, non-locality, or the impossible requirement to fully specify a hidden variable in a given spacetime region, etc. Indeed — contra Bell (1990:13), "you cannot get away with locality" — we do get away with just that.

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