Elementary proof of The Fermat’s Last Theorem

(Complete Edition)

Mr. Sattawat Suntisurat

King’s Mongkut Institute of Technology Ladkrabang

Mechanical Engineering, Thailand

E-mail: ttoshibak@gmail.com

Sep 28, 2016

Fermat’s Last Theorem (FLT):

\[ a^n + b^n \neq c^n \quad , \quad \text{if} \quad n > 2 \quad \text{and} \quad a, b, c \quad \text{are the integers.} \]

Prove,

Draw the graph (Pic.1) as below, I consider only 1st quadrant.

Pic.1

From Pic.1: if \( n \) is more, the curve will be near the point (c, c)
Then I make the grid (square 1 x 1) as below,

**Pic. 2**

Now I can define the intersection point means the integers, and I will prove these curves will not pass the intersection point for $n > 2$. 
No intersection point area

There are no intersection point area (yellow area), all curves in this area follow FLT.

Pic. 3

Next, I will find the intersection point between the curves and the symmetry axis.

\[ \sqrt[n]{c^n - b^n} = b \]

\[ b = \frac{c}{\sqrt[2]{n}} \] \hspace{1cm} (1)

From (1), b can’t be the integer, the curves will not pass the symmetry axis at intersection point.
From the Pic. 3, I will find the relation between $b$ and $c$ at the point $(c-1, c-1)$,

$$\frac{c}{\sqrt{2}} = c - 1$$

$$n = \frac{\ln(2)}{\ln(c)}$$

---------- (2)

From (2), in the no intersection point area, it can be determined

$$n > \frac{\ln(2)}{\ln\left(\frac{c}{c-1}\right)}$$

---------- (3)

Next, consider the curves in the no intersection point area.

$$a^n + b^n = c^n$$, $a$ and $b$ are not the integers.

$a$ and $b$ may be the rational (fraction) or irrational numbers,

Assume $a$ and $b$ are the rational number, $a = \frac{d}{e}$ and $b = \frac{f}{e}$

$d, e, f$ are the integers.

$$\left(\frac{d}{e}\right)^n + \left(\frac{f}{e}\right)^n = c^n$$

----------- (4)
See pic. 4, I draw the line (L line) in the no intersection point area.

The line will pass all the curves for all degree of \( n \rightarrow \infty \).

Assume L line pass a-axis at \( \frac{d}{e} \), it can be written as below,

\[
\left( \frac{d}{e} \right)^{n_1} + \left( \frac{f_1}{e} \right)^{n_1} = c^{n_1} \quad \text{for} \quad n = n_1
\]

\[
\left( \frac{d}{e} \right)^{n_2} + \left( \frac{f_2}{e} \right)^{n_2} = c^{n_2} \quad \text{for} \quad n = n_2
\]

\[
\left( \frac{d}{e} \right)^{n_3} + \left( \frac{f_3}{e} \right)^{n_3} = c^{n_3} \quad \text{for} \quad n = n_3
\]

\[
\left( \frac{d}{e} \right)^{n} + \left( \frac{f}{e} \right)^{n} = c^{n} \quad \text{for} \quad n \rightarrow \infty
\]

\( f_1 < f_2 < f_3 < \ldots < f_n \) \quad \text{and} \quad n_1 < n_2 < n_3 < \ldots < n_n
Multiply the $e^n$ all of the equation,

$$d^{n_1} + f_1^{n_1} = (ce)^{n_1} \quad \text{for} \quad n = n_1$$

$$d^{n_2} + f_2^{n_2} = (ce)^{n_2} \quad \text{for} \quad n = n_2$$

$$d^{n_3} + f_3^{n_3} = (ce)^{n_3} \quad \text{for} \quad n = n_3$$

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$$d^{n_n} + f_n^{n_n} = (ce)^{n_n} \quad \text{for} \quad n \rightarrow \infty$$

All equations show it can be written in $a^n + b^n = c^n$ by $a, b, c$ can be the intergers.

But it is conflict with (3), if $n > \frac{\ln(2)}{\ln(\frac{ce}{ce-1})}$ the curves will not pass the intersection point.

it can’t be written in the form $a^n + b^n = c^n \quad \text{for} \quad n \rightarrow \infty$

So I can judge $a$ and $b$ aren’t the rational numbers. But they are the irrational numbers in the no intersection point area.

$$a^n + b^n = c^n \quad , a \text{ and } b \text{ are the irrational numbers in the no intersection point area.}$$

Next, I will prove the FLT ,

$$a^n + b^n \neq c^n \quad , \text{if} \quad n > 2 \text{ and } a, b, c \text{ are the integers.}$$

Assume there is a equation $a^n + b^n = c^n \quad \text{and} \quad a, b, c, n \text{ are the integers.}$
Divided $k^n$ into equation \[ \left(\frac{a}{k}\right)^n + \left(\frac{b}{k}\right)^n = \left(\frac{c}{k}\right)^n \], $k$ is an integer \[ (5) \]

Then let $k$ to \[ n > \frac{\ln(2)}{\ln\left(\frac{c}{k}\right) - 1} \], the curve will be in the no intersection point area.

\[ \frac{c}{k} \] may be integer or fraction.

**Assume case#1** \[ \frac{c}{k} \] is an integer, let \[ \frac{c}{k} = m \]

From (5), \[ \left(\frac{a}{k}\right)^n + \left(\frac{b}{k}\right)^n = (m)^n \] \[ (6) \]

From (6), the equation is wrong, because it is conflict with the no intersection point area. 

\[ \frac{a}{k} \] and \[ \frac{b}{k} \] mustn’t be the rational numbers, **so the assumption case#1 is wrong.**

**Assume case#2** \[ \frac{c}{k} \] is fraction. I can apply the plotting graph method as below

Pic. 5
From Pic. 5, if \( n > \frac{\ln(2)}{\ln\left(\frac{c/k}{-1}\right)} \), the curve will be in the no intersection point area.

From (5), I will prove \( \frac{a}{k} \) and \( \frac{b}{k} \) mustn’t be the rational numbers for \( \frac{c}{k} \) too.

I draw the line (L line) in the no intersection point area. The line will pass all the curves for all degree of \( n \to \infty \).

Assume L line pass a-axis at \( \frac{a}{k} \), it can be written as below,

\[
\left(\frac{a}{k}\right)^{n_1} + \left(\frac{b}{k}\right)^{n_1} = \left(\frac{c}{k}\right)^{n_1} \quad \text{for } n = n_1
\]
\[
\left(\frac{a}{k}\right)^{n_2} + \left(\frac{b}{k}\right)^{n_2} = \left(\frac{c}{k}\right)^{n_2} \quad \text{for } n = n_2
\]
\[
\left(\frac{a}{k}\right)^{n_3} + \left(\frac{b}{k}\right)^{n_3} = \left(\frac{c}{k}\right)^{n_3} \quad \text{for } n = n_3
\]

\[
\left(\frac{a}{k}\right)^{n_\infty} + \left(\frac{b}{k}\right)^{n_\infty} = \left(\frac{c}{k}\right)^{n_\infty} \quad \text{for } n \to \infty
\]

\[b_1 < b_2 < b_3 < \ldots < b_{n_\infty} \quad \text{and} \quad n_1 < n_2 < n_3 < \ldots < n_\infty\]

Multiply the \( k^n \) all of the equation,

\[
a^{n_1} + b_{1}^{n_1} = c^{n_1} \quad \text{for } n = n_1
\]
\[
a^{n_2} + b_{2}^{n_2} = c^{n_2} \quad \text{for } n = n_2
\]
\[
a^{n_3} + b_{3}^{n_3} = c^{n_3} \quad \text{for } n = n_3
\]

\[
a^{n_\infty} + b_{n_\infty}^{n_\infty} = c^{n_\infty} \quad \text{for } n \to \infty
\]
All equations show it can be written in \( a^n + b^n = c^n \) by \( a, b, c \) can be the integers.

But it is conflict with (3), if \( n > \frac{\ln(2)}{\ln\left(\frac{c}{c-1}\right)} \) the curves will not pass the intersection point.

it can’t be written in the form \( a^n + b^n = c^n \) for \( n \to \infty \)

So I can judge \( \frac{a}{k} \) and \( \frac{b}{k} \) aren’t the rational numbers. But they are the irrational numbers in the no intersection point area. so the assumption case#2 is wrong.

From proof of case#1 and case#2, I can say...

No any integer \( a, b, c \) for \( a^n + b^n = c^n \) if \( n > 2 \)

The Fermat’s last Theorem is proved completely !!!