Appearance of Magic Numbers in the Unified Approach for the Nuclear Structure

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Abstract : Taking the clues from the recently observed phenomena like correlated neutron-proton ejection from nuclei by energetic e^-/protons, EMC effect and inconsistent assumptions of liquid drop and shell models for the same set of nuclei (for explanation of different nuclear properties), a unified approach for the nuclear structure was suggested by the author [1]. In the proposed approach, instead of freely moving individual neutrons/protons, nucleons exist in the form of tightly bound np and npn (and pnp also for proton rich nuclei) quasi-particles, which move in overall Coulomb well due to all other quasi-particles. The proposed approach was found to be very useful in understanding many unexplained phenomena [1]. Now, the existence of np and npn quasi-particles is found to be strongly confirmed by the presence of magic numbers across the periodic table where many stable nuclei with different atomic number are found to be have same number of np and npn. For example, there are 30 different stable (or most stable isotope of nuclei with no stable isotope) nuclei with different Z having the magic number of 38 np, 20 nuclei with the magic number of 32 np and 16 with magic number of 40 np.

Introduction

The evolution of nuclear physics has been strongly hampered by the poor understanding of nuclear forces. In spite of initial expectations, the nuclear force model proposed by Yukawa remained a qualitative reasoning only and can not be used for description of real nuclear structure or force [2]. The unsatisfactory situation in the nuclear physics is reflected by the fact that for the adequate description of different nuclear properties, one has to use different nuclear models with adjustable parameters for different nuclei regions. Moreover, the basic underlying assumptions of two main nuclear models (liquid drop model and shell model) which are used to account many of the nuclear properties are based on apparently contradictory assumptions [3].

The unsatisfactory state of our current understanding about the nuclear structure is further illuminated by many of unexpected experimental observations in recent years [1,3-7]. For example, direct measurements for nucleon-nucleon correlations have been carried out at Brookhaven National Laboratory [7] using proton beam and Thomas Jefferson National Accelerator Facility using about 5 GeV electron beam [8,9]. Interestingly, Brookhaven analysis demonstrated that the removal of a proton from the carbon nucleus is 92 % (+8 to -18%) of the time is accompanied by the emission of a correlated neutron that carries momentum roughly equal and opposite. This result is in a direct contradiction of independent particle motion in shell model and indicates a strong correlation of proton-neutron wavefunction. Another surprising effect comes from the observation of EMC effect which demonstrate that the structure function (and hence charge structure) of nucleon are strongly altered (compared to free nucleon) inside the nuclear medium.

These experimental observations along with many other arguments and well know fact that the nucleons (proton and neutron) are themselves composed of elementary charged particles invite us to consider the possible electromagnetic origin of the nuclear forces as done in [1]. It was assumed that instead of moving as free nucleons, wavefunction of neutrons and protons overlap to have correlation between their internal charge structures, thus forming np, npn and possibly pnp (in proton rich nuclei) quasi-particles in a similar way to the Van der Waals interaction in atomic/molecular systems. Here, Coulomb well helps in localization of wavefunction, leading to better overlapping and thus, higher binding energy for the nucleon. Major portion of the binding energy of the nucleon is coming from formation of these quasiparticles which is further modified by other factors like Coulomb energy, formation of shell structure etc.

The number of np, npn and pnp quasiparticle inside nucleus can be calculated in a straightforward manner [1] from the number of proton and neutrons. For example, ${}_8O^{16}$ with 8 proton and 8 neutron would have only 8 np, whereas ${}_8O^{18}$ with 8 proton and 10 neutron would have 2 npn (due to two additional neutron) and 6 np. ${}_9F^{19}$ will have 8 np and 1 npn.

As discussed in [1], this simple model can resolve many issues in nuclear physics, for example, it can be used to provide a straightforward explanation for the almost constant binding energy from He^4 to Pu^{244} as major portion of binding energy is due to the overlap of proton-neutron wavefunction. It can account the correlated ejection of the neutrons and protons in the deep elastic scattering experiments as they are actually in form of np/npn quasi-particles inside the nuclei. Similarly, EMC effect must be there as the binding between the neutrons and protons inside the nuclei is due to their internal charge structures, which will modify their structure factors inside the nuclei compared to the free proton/neutron state. It can provide a simple explanation for the observed magic number for 16 neutrons in case of ${}_8O^{24}$ (It is simply 8 npn). Further, present approach provides a straightforward explanation for the mass and charge distributions observed in spontaneous fission events [1,3]. It provides a direct explanation for the evolution of asymmetric to symmetric fission fragment mass distribution with number of neutrons by just considering the numbers of np and npn quasiparticles and without making any ad-hoc assumption. It has been exceptionally successful in explaining all the fission mass distributions even for the recently observed baffling Hg^{180} case. On the other hand, one can expect that if the present approach is correct it must be reflected by the appearance of extra stable magic number configurations for the np & npn. As discussed in the next section, this is actually the case.

Appearance of Magic Numbers for np and npn quasi-particles

The fully filled shells of np and npn should lead to some extra stability and it should be reflected with presence of more than few nuclei with these magic numbers (of np and npn). Though, the overall Coulomb potential due to all other quasi-particles is not equal to harmonic type potential, as approximation one can consider it. The solution of Schrodinger equation for the 3-D harmonic type potential leads to the magic numbers 2 (1s), 8 (1s, 1p), 20 (1s, 1p, 1d) and 40 (1s, 1p, 1d, 1f, 2s, 2p). Since the major portion of the binding energy is due to the proton/neutron wavefunction overlap due to the internal charge structure, 2p orbital may be filled before the 2s, leading to filled shell configuration for 38 or magic number 38. Similarly, fully filled 32 np would also be more stables as this is due to filled shells up to 1f. Hence, one can expect that not only 40, number 38 and 32 may also be more stable configurations or magic numbers for np.

This has been observed in reality also. There is exceptionally large number of stable nuclei with np equal to 40, 38 and 32 indicating that these numbers must be treated as magic numbers. Following stable (or comes with very-very long half life) even-even nuclei have number of np quasi-particles equal to 38.

1. $_{48}Cd^{106}$ (It is lightest Cd isotope)
2. $_{50}Sn^{112}$ (No stable lighter isotope for Sn)
3. $_{54}Xe^{124}$ (No stable lighter isotope for Xe also)
4. $_{56}Ba^{130}$ (No stable lighter isotop for Ba)
5. $_{58}Ce^{136}$ (Lightest stable isotope for Ce)
6. $_{60}Nd^{142}$ (Lightest stable isotope for Nd)
7. $_{62}Sm^{148}$ (Half life 7(3)10^{15} Years, lightest isotope is Sm^{144})
8. $_{64}Gd^{154}$
9. $_{66}Dy^{160}$
10. $_{68}Er^{166}$
11. $_{70}Yb^{172}$
12. $_{72}Hf^{178}$
13. $_{74}W^{184}$
14. $_{76}Os^{190}$
15. $_{78}Pt^{196}$
16. $_{80}Hg^{202}$
17. $_{82}Pb^{208}$ (heaviest stable isotope of Pb)

For the following heavy nuclei, having no stable isotope, longest living isotopes comes with magic number of 38 np.

18. $_{88}Ra^{226}$ (Half life of 1600 (7) Year)
19. $_{90}Th^{232}$ (Half life of $1.405 x 10^{10}$ Year, all heavier isotopes of Th are very short lived)
20. $_{92}U^{238}$ (Half life of $4.468 x 10^{9}$ Year, heavier isotopes of U are very short lived)
21. $_{94}Pu^{244}$ ($8.0 x 10^{7}$ Year, heavier isotopes of Pu are short lived)

For nuclei Cd, Sn, Xe, Ba, Ce and Nd magic number of np equal to 38 acts as cut-off for lower mass side. Further reducing the number of neutron would lead to the filling of next shell for np and hence it is avoided. The np number 38 defines the lightest isotopes for these nuclei. For heavier nuclei Th, U and Pu, magic number 38 define the most stable isotope of these nuclei and acts a cut-off for higher mass side as further increment of neutrons would lead to less than 38 available np quasi-particles leading to partially filled np shell. Out of total 24 even-even nuclei from $_{48}Cd$ to $_{94}Pu$, 21 nuclei have either stable isotope (here stable means isotope with half life of more than 10^{15} years) or their longest living isotope with magic number 38. Only three even-even nuclei Te, Po and Ra (Po and Ra do not have any isotope with half life more than a year) from Cd to Pu are not making any contribution to this list. This surprising observation is direct confirmation of existence of np and npn quasi-particles inside the nuclei. The successive even-even nuclei with configuration np 38 differ by 2 npn particles, in which each additional proton is accompanied with two neutrons.

From Cd^{106} to Pu^{244}, persistent appearance of magic number 38 for np is clearly verifying the validity of the present approach. The number of nuclei with magic number 38 is further increased if odd-even nuclei are also considered. List of nuclei with np equal to 38 (It is either stable isotope or naturally occurring isotope with very long half life or it makes the longest half life isotope for a given Z)

22. $_{61}Pm^{145}$ (Half life ~17.7 Year. It is the most stable isotope of Pm)
23. $_{63}Eu^{151}$ (Half life 4.62×10^{18}, makes about 47.8% of natural Eu)

24. $_{69}Tm^{169}$ (Only stable isotope of Tm)
25. $_{71}Lu^{175}$ (Only stable isotope of Lu)
26. $_{73}Ta^{181}$ (Only ground state stable isotope of Ta)
27. $_{75}Re^{187}$ (Half life of 41.2x10^{9} Year, makes about 62.2% of natural Re)
28. $_{77}Ir^{193}$ (One of two stable isotopes of Ir)
29. $_{81}Tl^{205}$ (one of two stable isotopes of Tl, makes about 70.5% of natural Tl)
30. $_{87}Fr^{223}$ (most stable isotope of Fr with half life of about 22 min.)

Two other nuclei with np equal to 38 are $_{65}Tb^{157}$ (Half life 76 Year), $_{67}Ho^{163}$ (Half life of 4570 (25) Years).

Similar to the case of np equal to 38, np equal to 32 should also form stable configuration. This has been confirmed by the presence of large number of stable nuclei with np equal to 32 as listed below,

1. $_{42}Mo^{94}$
2. $_{43}Tc^{97}$ (Half life 4.21x10^{6} Year, longest living isotope of Tc)
3. $_{44}Ru^{100}$
4. $_{45}Rh^{103}$ (only stable isotope of Rh)
5. $_{46}Pd^{106}$
6. $_{47}Ag^{109}$ (One of two stable isotope of Ag)
7. $_{48}Cd^{112}$
8. $_{49}In^{115}$ (Half life of 441 (25) x10^{12} Years. It constitutes 97.5% of natural Indium)
9. $_{50}Sn^{118}$
10. $_{51}Sb^{121}$ (One of two stable isotopes of Sb)
11. $_{52}Te^{124}$
12. $_{53}I^{127}$ (Only stable isotope of Iodine)
13. $_{54}Xe^{130}$
14. $_{55}Cs^{133}$ (Only stable isotope of Cs)
15. $_{56}Ba^{136}$
16. $_{57}La^{139}$ (Only stable isotope of La and constitutes 99.91% of natural La)
17. $_{58}Ce^{142}$
18. $_{60}Nd^{148}$
19. $_{62}Sm^{154}$
20. $_{64}Gd^{160}$

For the last four nuclei in the above list (i.e. Gd^{160}, Sm^{154}, Nd^{148} and Ce^{142}), np 32 define the heavies stable isotope.

Similarly, magic number of 40 np is found in exceptionally large number of stable (or very longed lived) isotopes as listed below.

1. $_{62}Sm^{146}$ (Half life of 6.7x10^{7} Year)
2. $_{64}Gd^{152}$ (Half life of 1.08x10^{12} Year)
3. $_{66}Dy^{158}$
4. $_{68}Re^{164}$
5. $_{70}Yb^{170}$
6. $_{72}Hf^{176}$
7. $_{74}W^{182}$

8. ${}_{75}Re^{185}$ (only stable isotope of Re)
9. ${}_{76}Os^{188}$
10. ${}_{77}Ir^{191}$
11. ${}_{78}Pt^{194}$
12. ${}_{79}Au^{197}$ (Only stable isotope of Au)
13. ${}_{80}Hg^{200}$
14. ${}_{81}Tl^{203}$
15. ${}_{82}Pb^{206}$
16. ${}_{83}Bi^{209}$ (Half life of $1.9x10^{19}$ Year)

Once extra neutrons are added to nuclei beyond Ca, npn quasiparticles may be filled in 1s, 1p or 1d, resulting in extra stability for 2, 6 or 10 npn quasiparticles. This is reflected by the table given below for list of stable even-even nuclei with npn equal to 6 (just p shell filling) and 10 (1d filled shell of npn). There are 10 stable even-even isotopes from Cd^{106} to Zn^{70}. It can be seen that beyond ${}_{48}Cd^{106}$, ${}_{50}Sn^{110}$ is not favoured as it would result in np number beyond 38. As discussed earlier, np number more than 38 is not allowed for nuclei ${}^{48}Cd$ to ${}^{60}Nd$.

1. ${}_{48}Cd^{106}$ (This nuclei is doubly magic with npn equal to 38)
2. ${}_{46}Pd^{102}$
3. ${}_{44}Ru^{98}$
4. ${}_{42}Mo^{94}$
5. ${}_{40}Zr^{90}$
6. ${}_{38}Sr^{86}$
7. ${}_{36}Kr^{82}$
8. ${}_{34}Se^{78}$
9. ${}_{32}Ge^{74}$
10. ${}_{30}Zn^{70}$

Similarly, stable isotopes with npn equal to 6 are,

1. ${}_{20}Ca^{46}$
2. ${}_{22}Ti^{50}$
3. ${}_{24}Cr^{54}$
4. ${}_{26}Fe^{58}$
5. ${}_{28}Ni^{62}$
6. ${}_{30}Zn^{66}$
7. ${}_{32}Ge^{70}$
8. ${}_{34}Se^{74}$
9. ${}_{36}Kr^{78}$

Beyond, N^{15}, the nuclei with unpaired np are particularly unstable and odd proton stable isotopes always have even number of neutron forming unpaired npn with last proton. That has been the case for example with F^{19}, Na^{23}, Al^{27}, P^{31}, Cl^{35}, K^{39} etc.

Summery and Conclusion

Exceptionally large number of stable isotopes of nuclei comes with np equal to 32, 38 and 40 indicating the shell structure for np quasi-particles. Similarly, large number of stable nuclei comes with npn equal to 6 and 10 indication filling of p and d shells for them. The presence of these magic nuclei validates the current approach [1] for nuclear structure and provides a direct signature of presence of np and npn quasi-particles.

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