Beal Conjecture Proved & Specialized to Prove Fermat's Last Theorem

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Beal conjecture has been proved on a single page; and the proof has been specialized to prove Fermat's last theorem, on half of a page. The approach used in the proof is exemplified by the following system. If a system functions properly and one wants to determine if the same system will function properly with changes in the system, one would first determine the necessary conditions which allow the system to function properly, and then guided by the necessary conditions, one will determine if the changes will allow the system to function properly. So also, if one wants to prove that there are no solutions for the equation $c^z = a^x + b^y$ when $x, y, z > 2$, one should first determine why there are solutions when $x,\ y,\ z = 2$, and note the necessary condition in the solutions for $x,\ y,\ x = 2$. The necessary condition in the solutions for $x,\ y,\ x = 2$ will guide one to determine if there are solutions when $x, y, z > 2$. The proof in this paper is based on the identity $(a^2 + b^2)/c^2 = 1$ for a primitive Pythagorean triple $(a,\ b, c)$. It is shown by contradiction that the uniqueness of the $x,\ y,\ x = 2$ identity excludes all other $x,\ y,\ z$–values, $x, y, z > 2$ from satisfying the equation $c^z = a^x + b^y$. One will first show that if $x,\ y,\ z = 2$, $c^z = a^x + b^y$ holds, noting the necessary condition in the solution; followed by showing that if $x, y, z > 2$ (x, y, z integers), $c^z = a^x + b^y$ has no solutions. Two proof versions are covered. The first version begins with only the terms in the given equation, but the second version begins with the introduction of ratio terms which are subsequently and "miraculously" eliminated to allow the introduction of a much needed term for the necessary condition for $c^z = a^x + b^y$ to have solutions or to be true. Each proof is very simple, and even high school students can learn it. The approach used in the proof has applications in science, engineering, medicine, research, business, and any properly working system when desired changes are to be made in the system.

Beal Conjecture Proved (Version 1 Proof)

Introduction

This paper proves the equivalent Beal conjecture that the equation $c^z = a^x + b^y$ has no solutions in positive integers a,b,c,x,y,z, where $a,b,c,$ are relatively prime and $x,y,z > 2$. Two simple proof versions are covered. The first version begins with only the terms in the given equation, but the second version begins with the introduction of ratio terms which are subsequently eliminated to allow the introduction of a much needed term for the necessary condition for $c^z = a^x + b^y$ to have solutions or to be true. For an application, the proof of Beal conjecture is specialized to prove Fermat's last theorem. A step-by-step procedure is used in the proof to facilitate easy reading.

Beal Conjecture Proved: Version 1

Given: $c^z = a^x + b^y$ (x,y,z integers; $a, b,$and c are relatively prime positive integers)

Required: To prove that $c^z = a^x + b^y$ does not have solutions if $x,y,z > 2$

Plan: One will first show that if $x,\ y,\ x = 2$, $c^z = a^x + b^y$ has solutions, followed by showing that if $x,y,z > 2$ ($x,y,z,$ integers) **,** $c^z = a^x + b^y$ has no solutions.

Proof:

Step 1: $c^z = a^x + b^y$;

$$\frac{a^x + b^y}{c^z} = \frac{c^z}{c^z};$$

$$\boxed{\frac{a^x + b^y}{c^z} = 1} \qquad \text{(A)}$$

(A) is the necessary condition for $c^z = a^x + b^y$ to be true. or to have solutions.
(The ratio $(a^x + b^y)$ to $c^z = 1$)

Step 2: If $x,\ y,\ x = 2$, $\dfrac{a^x + b^y}{c^z} = \dfrac{a^2 + b^2}{c^2} = 1$ is true for a primitive Pythagorean triple $(a,\ b,\ c)$. (Example: For the integers $3,\ 4,\ 5$, $\dfrac{a^2 + b^2}{c^2} = 1$ $(3^2 + 4^2)/5^2 = 25/25 = 1$)
Thus, if $x,\ y,\ x = 2$, the necessary condition $(a^n + b^n)/c^n = 1$ is satisfied and $c^z = a^x + b^y$ is true or has solutions..

Step 3: Proof for $x,y,z > 2$ **by contradiction**
If $x,y,z > 2$, and one assumes that $\dfrac{a^x + b^y}{c^z} = 1$, then $\dfrac{a^x + b^y}{c^z} = \dfrac{a^2 + b^2}{c^2}$ (B)
(By the transitive equality property, since $\dfrac{a^2 + b^2}{c^2} = 1$.)

Step 4: From (B) and equating the exponents, $x > 2 = 2$ is false, since an integer greater than 2 cannot be equal to 2; similarly, $y > 2 = 2$ is false; and $z > 2 = 2$ is false.
The above falsities imply contradiction, and $\dfrac{a^x + b^y}{c^z} \neq \dfrac{a^2 + b^2}{c^2}$ not as in (B); and hence, the assumption that $\dfrac{a^x + b^y}{c^z} = 1$, if $x,y,z > 2$, is false.

Step 5: Therefore, $\dfrac{a^x + b^y}{c^z}$ is not equal to 1. if $x,y,z > 2$. Since the necessary condition $\dfrac{a^x + b^y}{c^z} = 1$, is not satisfied if $x,y,z > 2$, the equation, $c^z = a^x + b^y$ has no solutions if $x,y,z > 2$. Therefore $c^z = a^x + b^y$ has solutions only if $x,\ y,\ x = 2$, and does not have solutions if $x,y,z > 2$. The proof is complete.

Beal Conjecture Proved: Version 2 (Using ratios)

Confirmation of Version 1 Proof

Given: $c^z = a^x + b^y$ (x,y,z integers; a, b,and c are relatively prime positive integers)

Required: To prove that $c^z = a^x + b^y$ does not have solutions if $x,y,z > 2$

Plan: One will first show that if x, y, $x = 2$, $c^z = a^x + b^y$ has solutions, followed by showing that if $x,y,z > 2$ (x,y,z, integers) **,** $c^z = a^x + b^y$ has no solutions. One begins by applying ratio terms.

$c^z = a^x + b^y$ (1) (Given)

$a^x + b^y = c^z$ (2) (rewriting)

$a^x = rc^z$ (3) (r is a ratio term)

$b^y = sc^z$ (4) (s is a ratio term) ($r + s = 1$)

$rc^z + sc^z = c^z$ (5) (substitute for a^x and b^y from (3) and (4)

$c^z(r + s) = c^z$ (6)

Now, by the substitution axiom, since $r + s = 1$, $r + s$ can be replaced by any quantity = 1. One can therefore replace $r + s$ by $\frac{a^2 + b^2}{c^2}$,

since $\frac{a^2 + b^2}{c^2} = 1$ for a primitive Pythagorean triple (a, b, c).

Then equation (6) becomes $c^z(\frac{a^2 + b^2}{c^2}) = c^z$ (7)

If $z = 2$, (7) becomes $c^2(\frac{a^2 + b^2}{c^2}) = c^2$ (8)

$c^2 = c^2(\frac{a^2 + b^2}{c^2})$ (8) (rewriting)

Equation (8) is true since $\frac{a^2 + b^2}{c^2} = 1$. Consequently, equations (8) and (1) hold. Therefore, if x, y, $x = 2$, $c^z = a^x + b^y$ has solutions.

Example on ratio terms

If $4 + 8 = 12$, and the ratio terms are $\frac{1}{3}$ and $\frac{2}{3}$, then $4 = \frac{1}{3} \bullet 12$, $8 = \frac{2}{3} \bullet 12$; and the sum of the ratio terms is $\frac{1}{3} + \frac{2}{3} = 1$

Elimination of the ratio terms r and s

The author was impressed and gratified by the substitution axiom which permitted the introduction of the much needed necessary condition $(a^x + b^y)/c^z = 1$.

Generalizing equation (7), one obtains $c^z(\frac{a^x + b^y}{c^z}) = c^z$ (9) in which the necessary condition for (9) to hold is $\frac{a^x + b^y}{c^z} = 1$. One will next show that if $x,y,z > 2$, the condition $(a^x + b^y)/c^z = 1$ is never satisfied and consequently $c^z = a^x + b^y$ has no solutions.

Proof for $x,y,z > 2$ by contradiction

If $x,y,z > 2$, and one assumes that $\frac{a^x + b^y}{c^z} = 1$, then $\frac{a^x + b^y}{c^z} = \frac{a^2 + b^2}{c^2}$ (B)

(By the transitive equality property, since $\frac{a^2 + b^2}{c^2} = 1$, for a primitive Pythagorean triple (a, b, c). From (B), $x > 2 = 2$ is false, since an integer greater than 2 cannot be equal 2.. Similarly, $y > 2 = 2$ is false; and $z > 2 = 2$ is false. The above falsities imply contradiction. Hence, the assumption that $\frac{a^x + b^y}{c^z} = 1$, when $x,y,z > 2$, is false. Therefore, $\frac{a^x + b^y}{c^z}$ is not equal to 1. $((a^x + b^y)/c^z \neq 1)$ if $x,y,z > 2$. Since the necessary condition, $\frac{a^x + b^y}{c^z} = 1$, is not satisfied if $x,y,z >$ equation $c^z = a^x + b^y$ has no solutions if $x,y,z > 2$. Therefore, $c^z = a^x + b^y$ has solutions only if x, y, $x = 2$ and does not have solutions if $x,y,z > 2$.The proof is complete.

Conclusion for Beal Conjecture

Beal conjecture has been proved on a single page. One first determined why there are solutions when $x,\ y,\ x = 2$. The necessary condition in the solutions for $x,\ y,\ x = 2$ guided one to determine if there are solutions when $x, y, z > 2$. The necessary condition is $(a^x + b^y)/c^z = 1$, where $a,\ b,$ and c are relatively prime positive integers. This necessary condition is satisfied only if $x,\ y,\ z = 2$, to produce $(a^2 + b^2)/c^2 = 1$, where $(a,\ b,\ c)$ is a primitive Pythagorean triple. If $x, y, z > 2$, the necessary $(a^x + b^y)/c^z = 1$ is never satisfied. It was shown by contradiction that the uniqueness of the $x,\ y,\ z = 2$ identity excludes all other $x,\ y,\ z$–values-values, $x, y, z > 2$, from satisfying the equation $c^z = a^x + b^y$. The proof is very simple, and even high school students can learn it. The proof is very simple, and even high school students can learn it.

One will next specialize the above proof to prove Fermat's last theorem. However, note that one can also generalize the proof of Fermat's Last Theorem to obtain the proof of Beal Conjecture.

Fermat's Last Theorem Proved

To obtain the proof of Fermat's Last Theorem from the proof of Beal Conjecture, let $x = n$, $y = n$. $z = n$ in Beal Conjecture proof and delete redundant repetitions.

Thus, $x, y, z > 2$ becomes $n > 2$; $x, y, z = 2$ becomes $n = 2$; $a^x + b^y$ becomes $a^n + b^n$**;** c^z becomes c^n., Then one obtains the following proof

Given: $c^n = a^n + b^n$ (n an integer; a, b, and c are relatively prime positive integers)

Required: To prove that $c^n = a^n + b^n$ does not hold if $n > 2$

Plan: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer) **,** $c^n = a^n + b^n$ does not hold.

Proof

Step 1: $c^n = a^n + b^n$;

$$\frac{a^n + b^n}{c^n} = \frac{c^n}{c^n};$$

$$\boxed{\frac{a^n + b^n}{c^n} = 1} \quad \text{(A)}$$

(A) is the necessary condition for $c^n = a^n + b^n$ to be true. or to have solutions.
(The ratio $(a^n + b^n)$ to $c^n = 1$)

Step 2: If $n = 2$, $\frac{a^n + b^n}{c^n} = \frac{a^2 + b^2}{c^2} = 1$ is true for a Pythagorean triple a, b, c, (Example: For the integers 3, 4, 5, $\frac{a^2 + b^2}{c^2} = 1$ $(3^2 + 4^2)/5^2 = 25/25 = 1$)
Thus, if $n = 2$,the necessary condition $(a^n + b^n)/c^n = 1$ is satisfied and $c^n = a^n + b^n$ is true or has solutions.

Step 3: One will next show that if $n > 2$, the necessary condition , $\frac{a^n + b^n}{c^n} = 1$, is never satisfied.

Step 4: Proof for $n > 2$ by contradiction

If $n > 2$, and one assumes that $\frac{a^n + b^n}{c^n} = 1$,

then $$\frac{a^n + b^n}{c^n} = \frac{a^2 + b^2}{c^2} \quad \text{(B)}$$

(By the transitive equality property, since $\frac{a^2 + b^2}{c^2} = 1$). From (B), and equating the exponents, $n > 2 = 2$ is false, since an integer greater than 2 cannot be equal to 2. Hence, the assumption that $\frac{a^n + b^n}{c^n} = 1$, if $n > 2$, is false.

Step 5: Therefore, $\frac{a^n + b^n}{c^n}$ is not equal to 1. $(\frac{a^n + b^n}{c^n} \neq 1)$ if $n > 2$. Since the necessary condition $\frac{a^n + b^n}{c^n} = 1$, is not satisfied if $n > 2$, the equation $c^n = a^n + b^n$ has no solutions if $n > 2$.
Therefore $c^n = a^n + b^n$ has solutions only if $n = 2$ and does not have solutions if $n > 2$.
The proof is complete.

Note: Fermat's Last Theorem can also be proved using the identity $\boxed{\sin^2 x + \cos^2 x = 1}$ or its equivalents, instead of $\frac{a^2 + b^2}{c^2} = 1$ (see viXra:1605.0195

Overall Conclusion

Two versions of the proof of Beal Conjecture have been presented in this paper. The first version began with only the terms of the given equation; but the second version began with the introduction of ratio terms which were later on "miraculously" eliminated to permit the introduction of a much needed necessary condition term for the equations to have solutions. The Beal proof was specialized to prove Fermat's last theorem. The necessary condition for the relevant equations involved to be true is that $(a^x + b^y)/c^z = 1$ (for Beal proof); and $(a^n + b^n)/c^n = 1$ for Fermat's proof. It was determined that the Beal equation, $c^z = a^x + b^y$ is true only if $x, y, x = 2$; and the Fermat's equation $c^n = a^n + b^n$ is true only if $n = 2$. Therefore, $c^z = a^x + b^y$ has solutions only if $x, y, z = 2$, and does not have solutions if $x, y, z > 2$. Similarly, the Fermat equation $c^n = a^n + b^n$ has solutions only if $n = 2$, and does not have solutions if $n > 2$. One should note above that Version 2 of Beal proof confirmed Version 1 of Beal proof.

PS

Application of the approach used in proving Beal Conjecture

If a 3-ton non-portable machine functions properly in environment number 2 and one wants to determine if the same machine will function properly in environments 3, 4, and, 5 up to 1000 different environments, one option is to dismantle the machine in environment number 2, and reassemble it in each of the new environments, up 1000 environments and test the machine. Another option, the better option, is to determine the necessary conditions which allow the machine to function properly in environment 2. If the necessary conditions are not available in environments 3, 4, 5, etc, the machine will not function properly in the new environments, and no efforts should be wasted in carrying the machine to the environments and be tested.

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