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How The Special Relativity Doppler's Principle Results in its Refutation

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ABSTRACT

In his 1905 paper on special relativity,[1] Einstein derived the relativistic Doppler shift formula from the Lorentz transformation. The wave time-period equation deduced from the relativistic Doppler shift is shown to result, with the help of the Lorentz transformation, in another time equation in contradiction with the formerly mentioned one. The end result: the Lorentz contraction factor γ is reduced to unity; the whole theory of relativity is thus undermined. Furthermore, for the case of relative circular motion, a critical contradiction of the relativistic Doppler principle with the special relativity time dilation prediction is revealed.

INTRODUCTION

In his 1905 paper,[1] Einstein derived the Lorentz transformation (LT) equations for the space and time coordinates on the basis of the relativity principle and the constancy of the speed of light. He deduced the transformation equations for the electric and magnetic forces from the Maxwell-Hertz and his derived LT equations. The obtained electrodynamics transformations applied on the wave equations for light led him to the general relativistic Doppler shift formula.

Considering two relatively moving inertial reference frames K (stationary) and K' (traveling) with a uniform speed v, for every event coordinates (x, y, z, t) in K, there corresponds the coordinates (x', y', z', t') in K'. The LT equations relating the event coordinates in the two frames are: $t' = \gamma(t - vx/c^2)$, $x' = \gamma(x - vt)$, $y' = y$, $z' = z$, where c is the speed of light in the empty space, and $\gamma = 1/\sqrt{1 - v^2/c^2}$. The LT equations are as well applicable to event coordinate intervals (i.e., $\Delta x, ..., \Delta t$, and $\Delta x', ..., \Delta t'$). It has been shown in earlier studies[2,3] that the actual consequence of the constancy of the speed of light principle is the time transformation equation $\Delta t' = \gamma \Delta t (1 - v/c)$. The LT time equation can be obtained from the actual time transformation equation only if the relation $\Delta x = c\Delta t$ (with $\Delta t \neq 0$) is plugged in the second term of the actual time equation when it's written in the from $\Delta t' = \gamma(\Delta t - \Delta t.v/c)$, yielding the LT time equation $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$. The actual time transformation equation $\Delta t' = \gamma\Delta t(1 - v/c)$, or $\Delta t = \gamma\Delta t'(1 + v/c)$ is in line with the relativistic Doppler shift equation (in terms of the wave period) when the source is at rest in the traveling frame (i.e., $\Delta x' = 0$). However, it contradicts the Inverse Lorentz transformation (ILT) time equation, $\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$, since the actual time equation will reduce to the LT time equation only if $\Delta x'$ was equal to $c\Delta t'$ (with $\Delta t' \neq 0$) in the former equation's term $v\Delta t'/c$. Yet, the ILT time equation erroneously results in $\Delta t = \gamma\Delta t'$ for $\Delta x' = 0$ (source at rest in the moving frame K') – and not $\Delta t = \gamma\Delta t'(1 + v/c)$.

In this paper, the above contradiction will be reconfirmed through different, more elaborative approaches.

THE CONTRADICTORY WAVE PERIOD EQUATIONS

The relativistic Doppler shift formula is derived by Einstein based on the Lorentz transformation. The following is an excerpt from his 1905 paper.[1]

> *"Applying the equations of transformation found in § 6 for electric and magnetic forces, and those found in § 3 for the co-ordinates and the time, we obtain directly…*
>
> *From the equation for ω' it follows that if an observer is moving with velocity v relatively to an infinitely distant source of light of frequency ν, in such a way that the connecting line "source-observer" makes the angle ϕ with the velocity of the observer referred to a system of co-ordinates which is at rest relatively to the source of light, the frequency ν' of the light perceived by the observer is given by the equation*
>
> $$\nu' = \nu\frac{1 - \cos\phi . v/c}{\sqrt{1 - v^2/c^2}}. \qquad (1)$$
>
> *This is Doppler's principle for any velocities whatever. When $\phi = 0$ the equation assumes the perspicuous form*
>
> $$\nu' = \nu.\frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$
>
> *We see that, in contrast with the customary view, when $v = -c$, $\nu' = \infty$."*

The above relativistic Doppler equation, a special relativity result, can be written as

$$\nu' = \gamma\nu(1 - v/c),$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Or, in terms of the wave period, the inverse of frequency,

$$\Delta t' = \gamma \Delta t \left(1 + v / c\right) \tag{2}$$

From Einstein's setting: *"an observer is moving with velocity* v *relatively to an infinitely distant source of light of frequency* ν *... referred to a system of co-ordinates which is at rest relatively to the source of light",* we conclude that the observer is in the "traveling" primed frame, K' and the source is at rest in the "stationary" frame K; hence we have $\Delta x = 0$ for the source.

Writing Eq. (2) in the form

$$\Delta t' = \gamma \Delta t + \gamma \Delta t . v / c, \tag{3}$$

and using the Lorentz transformation spatial equation

$$\Delta x' = \gamma\left(\Delta x - v \Delta t\right)$$

under the condition $\Delta x = 0$ (i.e., $\Delta x' = -\gamma v \Delta t$), Eq. (3) becomes

$$\Delta t' = \gamma \Delta t - \Delta x' / c. \tag{4}$$

Now, using the inverse Lorentz transformation spatial equation

$$\Delta x = \gamma\left(\Delta x' + v \Delta t'\right)$$

under the same condition of $\Delta x = 0$ (i.e., $\Delta x' = -v \Delta t'$), Eq. (4) becomes

$$\Delta t' = \gamma \Delta t + v \Delta t' / c, \text{ or}$$

$$\Delta t' = \gamma \Delta t / \left(1 - v / c\right) \tag{5}$$

Obviously, the wave period Eqs. (2) and (5) lead to $\gamma = 1$. This result can be immediately deduced from Eq. (2) and the Lorentz transformation time equation

$$\Delta t' = \gamma(\Delta t - v \Delta x / c^2), \tag{6}$$

since the latter equation yields $\Delta t' = \gamma \Delta t$ under the above condition of $\Delta x = 0$.

INCONSISTENCY BETWEEN THE RELATIVISTIC DOPPLER PRINCIPLE AND TIME DILATION PREDICTION

The Doppler principle Eq. (1) is applicable for any velocities. The particular case when $\phi = \pi / 2$, not discussed in the cited Einstein's paper, shall be considered. It corresponds to the observer moving in a circular motion around the source; or from the perspective of the observer, the source is rotating around the observer occupying the center of the source circular path. Using the general Doppler shift Eq. (1) for $\phi = \pi / 2$, we get

$$\nu' = \nu \frac{1}{\sqrt{1 - v^2 / c^2}}. \tag{7}$$

Or, in terms of the wave period

$$\Delta t' = \Delta t \sqrt{1 - v^2 / c^2}, \tag{8}$$

or

$$\Delta t' = \frac{1}{\gamma} \Delta t. \tag{9}$$

Since for the special case of $\phi = \pi / 2$, there would be no change in the relative distance between the source and the observer (circular motion), the only justification of the Doppler shift would be the relativistic time dilation.—Obviously, there would be no Doppler effect for this case in the classical approach where $\nu' = \nu(1 - \cos\phi \ .\ v / c)$, yielding $\nu' = \nu$.

However, the events under consideration here, separated by the wave period, are the emission of two consecutive light impulses, defining a wave cycle, from the source that is at rest in its reference system K; i.e. co-local events in the light source system K. Thus, this corresponds to the event coordinates $\Delta x = 0$. Hence, the wave period time observed from the moving system K' can be determined from the Lorentz transformation Eq. (6) to be

$$\Delta t' = \gamma \Delta t,$$

which is in contradiction with Eq. (9) resulting from the respective Doppler shift equation.

It is to be noted that for the considered duration of a light wave period, sufficiently small in terms of the relative motion effect, it can be considered with high degree of confidence that the respective relative distance traveled by the moving frame is infinitesimally small, so it can be approximated with a straight segment. Hence, the time transformation derived for inertial frames in relative motion (i.e., relatively moving in a straight line at a uniform speed) could be applied.

A NOTE ON THE GAMMA FACTOR

The shown contradiction emerging from the relativistic Doppler shift equation has been indirectly addressed by relativists by promoting the idea that the relativistic Doppler Effect is an observational result affected by the light signal travel time between the observer and the light source in relative motion with the observer. Accordingly, they contend the relativistic Doppler shift does not fall under the special relativity predictions alleged to be concerned only with the pure time dilation, without taking the observational effect of the light signal travel time into consideration. Apparently, this contention ignores the fact that Einstein derived the relativistic Doppler shift equation solely from the Lorentz transformation. They claim the relativistic Doppler equation to be the combination of the nonrelativistic observational time alteration due to the light signal having to travel to the relatively moving observer, and the relativistic time dilation.

The above notion has no substantiated evidence, since the special relativity as formulated by Einstein in his 1905 paaper[1] relies on light signal time of propagation from the different perspectives of the relatively moving observers. For instance, the basic equation used by Einstein in his 1905 paper[1] to derive the time transformation equation is "$1/2(\tau_0+\tau_2)=\tau_1$ ", where τ_0, τ_1 and τ_2 represent the initial time of a light signal emission in the relative motion direction, its arrival/ reflection time to a fixed mirror, and its return time to its initial position, all relative to the traveling observer. The respective round trip travel time relative to the "stationary" frame observer is affected by the relative motion and by the fact that the light signal has to travel an overall longer distance in its round trip journey at the same speed.

For instance, from the classical point of view, if c was the speed of light relative to the traveling frame K', the light signal travel time is $\Delta x'/c$ relative to K', where $\Delta x'$ is the travelled length in K', in either directions relative to the relative motion direction. However, with respect to K, if the speed of light was also c – according to the special relativity light speed constancy principle – the observed travel time is $\Delta x'/(c-v)=(\Delta x'/c)/(1-v/c)$ in the forward direction, and $\Delta x'/(c+v)=(\Delta x'/c)/(1+v/c)$ in the backward direction; dilated and contracted, with respect to the K' travel time, by a factor of $1/(1-v/c)$ and $1/(1+v/c)$, respectively.

If the beam travels back and forth along $\Delta x'$, the combined effect would be like a geometric average of the two factors, namely $1/\sqrt{1-v^2/c^2}$, which is the Gamma dilation factor, an observational factor!

Furthermore, the Lorentz transformation is shown to be fully derivable using an approach relying completely on light signal travel times between the frame origins.[3]

CONCLUSION

The relativistic Doppler shift equation is a special relativity result derived by Einstein from the Lorenz transformation. Yet, when this equation is analyzed in the frame of the special relativity, it leads to contradictory result manifested as the Lorentz contraction factor γ being equal to one, and as inconsistent time dilation equations in the case of observer-source circular relative motion.

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