

Calculation of the gravitational constant G using electromagnetic parameters

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Abstract

In this paper, we will derive the following formula for the value of the gravitational constant G:

$$G = \frac{\pi \alpha q^2}{m_e^2 \epsilon_0} e^{-\frac{1}{\sqrt{2}} \left(\frac{1}{\alpha} - \frac{\pi \alpha}{2} \right)} = 6,67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (1)$$

Being α the fine structure constant, q the elementary charge, m_e the mass of the electron, ϵ_0 the permittivity of the free space, e the exponential function and π the relation between a circumference and its diameter.

Values attached:

$$\alpha = \frac{q^2}{2 \epsilon_0 h c} = 7.297152568 \times 10^{-3} [1]$$

$$q = 1.60217662 \times 10^{-19} \text{ C} [2]$$

$$m_e = 9.10938356 \times 10^{-31} \text{ kg} [2]$$

$$\epsilon_0 = 8.85418782 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 [3]$$

$$\pi = 3.14159265359 [4]$$

$$e = 2.71828182845 [5]$$

$$h = 6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg} \text{ s}^{-1} \text{ (Planck constant)} [6]$$

$$c = 299792458 \text{ m s}^{-1} \text{ (Speed of light)} [7]$$

As it can be checked, all of them are electromagnetic or mathematical constants and properties of the electron. No constant related to gravity has been used to arrive to this value. This formula is ok numerically and in units. And it is the output of this paper.

To get to this formula, we will consider space as being composed by the particles that occupy it. These particles include mass particles and force carriers (as photons). Gravity

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will appear as an emergent phenomenon caused by this reason. Having in mind these considerations for a special case (an isolated electron emitting photons) we get to this formula that validates the assumptions.

1. Introduction

As commented, we will consider space as composed by the particles that occupy it. To be able to isolate G and calculate it, we will make the following study/approximation:

- We will consider an isolated electron emitting photons.
- We will consider that the vacuum space surrounding this electron is only composed by these photons.

Of course, this is not exactly true, as these space will be also receiving photons and particles from the rest of the universe. Also, the quantum vacuum will be creating particles constantly.

Anyhow, we will see that with this consideration we will get a very approximate calculation of G (less than 1% error).

2. Electron as an elementary particle

The calculations in this paper will be done always using electrons and their emitted photons.

The reason for using electrons is important. They are the most elementary particles that has a charge (the elementary charge in fact). This means:

- The mass of the electron can be considered as the total energy of the photons emitted by it (the total electrostatic energy created by them). [8] This is not exact [9] but is a good approximation. This is not true for a proton for example.
- The classical radius of an electron can be considered as the radius of a mass that is having the energy of all the photons emitted by it [8]. Again, not true for a proton for example.

3. Radius of the electron

We will use the concept of classical radius of an electron r_e . This concept is well defined and described in the literature [8]. Its value is:

$$r_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{m_e c^2} = 2.8179403267 \times 10^{-15} m \quad (2)$$

4. Size of a photon

In this study, we will consider a single photon as an elementary sine wave [10]. In a sine wave when the wavelength is 2π the peak value is 1 [10], so the peak value in an elementary sine wave with arbitrary wavelength will be $\lambda/2\pi$ [10].

$$peak\ value = \frac{\lambda}{2\pi} \quad (3)$$

The root mean square (rms) value in a sine wave is the peak value divided by $\sqrt{2}$ [11] so:

$$rms\ value = \frac{peak\ value}{\sqrt{2}} = \frac{\lambda}{2\pi\sqrt{2}} \quad (4)$$

From now on, when we want to represent the rms value of an elementary sine wave with a specific wavelength λ we will call it as λ_{rms} :

$$\lambda_{rms} = \frac{\lambda}{2\pi\sqrt{2}} \quad (5)$$

As we will see later, the calculations will lead us to consider the size of the photon (or more specific, its radius) as the rms value of the wave of the photon according equation (5).

$$r_f = \lambda_{rms} = \frac{\lambda}{2\pi\sqrt{2}} \quad (6)$$

This means, the total front surface of a photon will be πr_f^2 . And the linear dimension of the photon, perpendicular to its movement direction will be the diameter of this surface. This is, two times the rms value (value from rms to rms):

$$\text{Size of a photon perpendicular to its direction} = 2r_f = 2\lambda_{rms} = \frac{2\lambda}{2\pi\sqrt{2}} = \frac{\lambda}{\pi\sqrt{2}} \quad (7)$$

The size of a photon (or even if it has a size) has always been a controversial study. Anyhow, all the studies [12][13] conclude that if this size exists, it is somehow related to its wavelength, as we have considered before. As commented in [13], if I try to shoot a photon through a conducting tube much smaller than a wavelength, it does not go through.

Later calculations will confirm that the consideration of the rms value matches with the expected results.

5. Increment of space created by an electron

As commented, gravity is considered as an emergent phenomenon (a side effect) of space being composed by particles (including force carriers). This means, when a particle emits other particles to space, it is creating new space. And this new space is the one that is warping space, provoking gravity.

To be able to make a relation between space created by gravity and space created by new particles (in this case the photons emitted by an electron), we need to calculate the first one.

This will be the only time in the paper where we use a formula related to gravity.

We consider Schwarzschild equation [14]. This equation calculates the deformation of space in the different directions caused by a mass. It matches very well with our situation (the space surrounded by an isolated electron).

$$ds^2 = - \left(1 - \frac{2Gm}{c^2 r}\right) dt^2 + \left(\frac{1}{1 - \frac{2Gm}{c^2 r}}\right) dr^2 + r^2 \sin^2 \theta d\vartheta^2 + r^2 d\theta^2 \quad (8)$$

In an instant of time and in radial direction we have:

$$ds^2 = \left(\frac{1}{1 - \frac{2Gm}{c^2 r}}\right) dr^2 \quad \frac{ds}{dr} = \sqrt{\frac{1}{1 - \frac{2Gm}{c^2 r}}} \approx 1 + \frac{Gm}{c^2 r} \quad (9)$$

The first element 1 represents the no distortion (Euclidean space). So if we take the geometrical line that keep the same r (the circumference), we can calculate the space incremented space in that line due to gravity as:

$$\Delta circumference = 2\pi r \left(1 + \frac{Gm}{c^2 r}\right) - 2\pi r = \frac{2\pi Gm}{c^2} \quad (10)$$

This is the increment in of space in the circumferential direction. If we want to convert to the equivalent increment of space in radial direction we divide by 2π .

$$\Delta radius = \frac{Gm}{c^2} \quad (11)$$

And we see that this increment is kept constant whatever the radius. This is logical, as the space created by the particle is the same at a precise moment of time and it is transferred outwards. Its effects reduce with distance as this new space is dispersed more, the bigger the radius. But the total increment of space of a total shell is the same independently of the radius.

At this moment, we will make a little counter-intuitive trick. We will consider this increment of space also as a wave and we will apply the same considerations of chapter 4.

This means, we calculate the effective increment of space as two times the rms value (rms to rms value) of the $\Delta radius$:

$$\Delta increment\ of\ space = \frac{2Gm}{\sqrt{2}\pi c^2} = \frac{Gm}{\sqrt{2}\pi c^2} \quad (12)$$

With the same considerations as in chapter 4.

This strange trick is a forced move, needed in this theory and we will check later that it works. But of course, the implications and doubts regarding this step will be discussed (with some logical doubts) later in the paper (chapter 15).

At this stage, we will follow on and we will check that in the end everything fits.

6. Wavelength of the photons emitted by an electron

For this paper we have to consider a mean value of the wavelength of the photons emitted by an electron. Be careful here, the electron emits two type of photons: real photons and virtual photons.

The real photons and its wavelength depends on its level of energy in the atom and the necessary energy to emit them or change its level, that as we know is quantized.

But the virtual photons are a complete unknown black box. It is known that if we consider them as having all the possible wavelengths, this matches with some effects [15][16]. But the mean value of this wavelength is unknown.

For this paper we will consider mean value of the wavelength as the Compton wavelength of the particle that emits them. In the case of the electron this leads to:

$$\lambda_e = \frac{h}{m_e c} = 2.4263102 \times 10^{-12} \text{ m} \quad (13)$$

This consideration will be very probably only valid in elementary particles. As in this case we will use always use electrons (elementary particles) throughout the report, we can avoid the discussion.

If we calculate now the double of the root mean square value commented in chapter 4 would be:

$$2 \lambda_{rms} = \frac{2h}{\sqrt{2} 2 \pi m_e c} = \frac{h}{\sqrt{2} \pi m_e c} = 5.4611167 \times 10^{-12} \text{ m} \quad (14)$$

This would be the diameter occupied by the photon (in perpendicular direction of propagation).

7. Definition of the magnitude $y(r)$

We will define the following magnitude:

$$y(r) = \frac{\Delta S(r)}{S(r)} = \frac{\Delta S(r)}{4 \pi r^2} \quad (15)$$

In a given shell of theoretical surface S and radius r , ΔS is defined as the increment of surface created by the emitted photons.

This means, y is the ratio, in a given shell of radius r , between the increment of surface created by the photons and the theoretical surface if these photons did not exist.

8. Equation

Now, we have all the elements needed to perform the calculation. We will create an equation, defining first the left hand side and the right hand side separately. Afterwards, we will make them equal.

To get to the equation, we will calculate the effects on space provoked by the photons emitted by the electron, in two different ways. We will start talking about a general position r , but finally we will apply to the radius r_e (radius of the electron). This way, we will arrive to an equation where all the data are known (an “over defined” equation). We will isolate G and calculate it. We will discover that the calculated value of G will have less than 1% error compared to real value of G.

Later, we will proceed to some fine tuning to try to get to the exact value of G with no error.

9. Calculation of the left hand side

We know that the magnitude $y(r)$ is depending on the surface itself. So, if we want to calculate the total creation of surface created by the photons (in a shell of radius r), we have to integrate the variation of this quantity divided by the quantity itself from infinitum to the radius r .

In another way, the total increment of this magnitude depends on the size of the magnitude itself. This can be written as:

$$\int_{\infty}^r \frac{dy}{y(r)} \quad (16)$$

As can be checked, it will not be possible to solve all the definite integrals (with zeros or infinite in their limits) avoiding to have infinite result in return. This is because equation (16) and equation (18) in chapter 10 are mutually excluding. If one is defined to be convergent, the other one will be divergent and vice versa.

The way we will use to avoid this infinite result is the following. We will solve all the integrals as indefinite and will include the integration constant in the result. This way, the result is valid, as the integration constant will be calculated for whatever values of the variable we want to use.

$$\int \frac{dy}{y(r)} = \ln y(r) + C_1 = \ln\left(\frac{\Delta S(r)}{4\pi r^2}\right) + C_1 \quad (17)$$

Being C_1 the integration constant

10. Right hand side of the equation

At each shell of dr , the new space (in ratio) created by the photons will correspond to the size of the photon ($2\lambda_{rms}$ of the photon in diameter) (7), divided by the unit of space at that shell in diameter ($2r$) multiplied by the advance unit of integral (dr) divided by the unit of space at that shell (r).

So the total space created by the photons will correspond to:

$$\int_{\infty}^r \frac{2\lambda_{rms}}{2r} \frac{dr}{r} \quad (18)$$

As commented before, to avoid infinite results we will solve as indefinite integral with an integration constant.

$$\int \frac{\lambda_{rms}}{r} \frac{dr}{r} = \int \frac{\lambda dr}{\sqrt{(2)}2\pi r^2} = \frac{-\lambda}{\sqrt{(2)}2\pi r} + C_2 \quad (19)$$

Being C_2 the integration constant.

11. Joining both hands of the equation

If we join the left hand and the right side of the equation we have:

$$\ln\left(\frac{\Delta S(r)}{4\pi r^2}\right) + C_1 = \frac{-\lambda}{\sqrt{(2)}2\pi r} + C_2 \quad (20)$$

As this equation (20) is valid for whatever radius, will calculate the equation for a specific radius in order to create an “over defined” equation. This way, we will be able to isolate G as if it were an unknown variable.

To create this “over defined” equation, we will calculate the equation for a specific radius, a radius that we know and we have specific data for it. This radius will be the radius of the electron $r=r_e$. Also, please take into account that the photons considered in this case are emitted by an electron so in this case $\lambda=\lambda_e$ (13):

$$\ln\left(\frac{\Delta S(r_e)}{4\pi r_e^2}\right) + C_1 = \frac{-\lambda_e}{\sqrt{(2)}2\pi r_e} + C_2 \quad (21)$$

Apart from C_1 and C_2 , the only unknown variable is $\Delta S(r_e)$. We will calculate this value in the following chapter to be able to go on with the equation.

12. Calculation of $\Delta S(r_e)$.

At the radius r_e , the ΔS is the multiplication of the created space in radial direction

$$\frac{Gm}{\sqrt{2}\pi c^2} \quad (12) \text{ multiplied by the height of the photon (we can consider it in axial or}$$

tangential direction) that is $\frac{h}{\sqrt{2}\pi m_e c}$ (14).

$$\Delta S(r_e) = \frac{Gm_e}{\sqrt{2}\pi c^2} \frac{h}{\sqrt{2}\pi m_e c} = \frac{Gh}{2\pi^2 c^3} \quad (22)$$

13. Calculating the equation

If we recover the equation (21):

$$\ln\left(\frac{\Delta S(r_e)}{4\pi r_e^2}\right) + C_1 = \frac{-\lambda_e}{\sqrt{(2)}2\pi r_e} + C_2 \quad (21)$$

What we will do at first stage is to consider the two integrations constants equal to zero. This will be the same as choosing the start of the corresponding definite integral in a way that we get no infinite result. We will see later that this consideration, surprisingly works, as we will get an error for G below 1%.

$$\ln\left(\frac{\Delta S(r_e)}{4\pi r_e^2}\right) = \frac{-\lambda_e}{\sqrt{(2)}2\pi r_e} \quad (23)$$

Using (22):

$$\ln\left(\frac{Gh}{2\pi^2 c^3}\right) = \frac{-\lambda_e}{\sqrt{(2)}2\pi r_e} \quad (24)$$

$$\ln\left(\frac{Gh}{8\pi^3 c^3 r_e^2}\right) = \frac{-\lambda_e}{\sqrt{(2)}2\pi r_e} \quad (25)$$

If we apply the exponential to both sides:

$$\frac{Gh}{8\pi^3 c^3 r_e^2} = \frac{e^{-\lambda_e}}{\sqrt{(2)}2\pi r_e} \quad (26)$$

If we substitute λ_e (13) and r_e (2) we get:

$$\frac{Gh}{8\pi^3 c^3 \left(\frac{q^2}{4\pi \epsilon_0 m_e c^2}\right)^2} = \frac{e^{-\lambda_e}}{\sqrt{(2)2} \frac{\pi}{4\pi \epsilon_0 m_e c^2} q^2} \quad (26)$$

If we isolate G and use the definition of fine structure constant α [1]:

$$G = \frac{\pi \alpha q^2}{m_e^2 \epsilon_0} e^{\frac{-1}{\sqrt{2}} \left(\frac{1}{\alpha}\right)} = 6.6202087 \times 10^{-11} m^3 k g^{-1} s^{-2} \quad (27)$$

This formula is very similar to (1) but not equal. In fact the value of G we get has an error of less than 1% compared with the commonly accepted value of 6.67408×10^{-11} [17]:

$$error = \frac{6.67408 \times 10^{-11} - 6.6202087 \times 10^{-11}}{6.67408 \times 10^{-11}} \times 100 = 0.81 \% \quad (28)$$

As we have checked, all the assumptions considered (including eliminating the integration constants) have given a calculation of G with less than 1% error!

Only this would validate the theory but let's go further.

14. Calculating the integrals with integration constant (fine tuning of G)

Let's recover again equation (21):

$$\ln\left(\frac{\Delta S(r_e)}{4\pi r_e^2}\right) + C_1 = \frac{-\lambda_e}{\sqrt{(2)2} \pi r_e} + C_2 \quad (21)$$

Instead of having two constants, we can define a constant C:

$$C = C_2 - C_1 \quad (29)$$

So:

$$\ln\left(\frac{\Delta S(r_e)}{4\pi r_e^2}\right) = \frac{-\lambda_e}{\sqrt{(2)2} \pi r_e} + C \quad (30)$$

Using (22):

$$\ln\left(\frac{Gh}{2\pi^2 c^3}\right) = \frac{-\lambda_e}{\sqrt{(2)2} \pi r_e} + C \quad (31)$$

$$\ln\left(\frac{Gh}{8\pi^3 c^3 r_e^2}\right) = \frac{-\lambda_e}{\sqrt{(2)2} \pi r_e} + C \quad (32)$$

Now, we consider C the unknown variable (and we consider G known with the value of 6.67408×10^{-11} [17]). We isolate C, using (13), (2) and [17] and we get:

$$C = 8.1044677 \times 10^{-3} \quad (33)$$

If you look for what this number could be. I will spare you some time.

This number is exactly $\frac{\pi \alpha}{2\sqrt{2}}$ with an error of 0.01%!! as you can see below:

$$C = \frac{\pi \alpha}{2\sqrt{2}} = 8.1053208 \times 10^{-3} \quad (34)$$

$$\text{error} = \frac{\frac{\pi \alpha}{2\sqrt{2}} - 8.1044677 \times 10^{-3}}{\frac{\pi \alpha}{2\sqrt{2}}} \times 100 = 0.01\% \quad (35)$$

This means, if we substitute (34) in (32) we get:

$$\ln\left(\frac{Gh}{8\pi^3 c^3 r_e^2}\right) = \frac{-\lambda_e}{\sqrt{(2)2\pi r_e}} + \frac{\pi \alpha}{2\sqrt{2}} \quad (36)$$

If we apply the exponential and isolate G, we have:

$$G = \frac{\pi \alpha q^2}{m_e^2 \epsilon_0} e^{\frac{-1}{\sqrt{2}}\left(\frac{1}{\alpha} - \frac{\pi \alpha}{2}\right)} = 6,67408 \times 10^{-11} m^3 k g^{-1} s^{-2} (1)$$

Of course, this is what we expected as we have calculated C using the exact value of G. But what it is really surprising is that if we consider C=0 we get below 1% error as seen in chapter 13. And if we calculate C, instead of receiving a number with no meaning as we could expect, we get the number $\frac{\pi \alpha}{2\sqrt{2}}$ with a 0.01% error whatever this means.

Why this is like this is not known at this stage. We know $\frac{\pi}{2}$ is the fourth part of a circumference. It is also related to some trigonometric functions. It is two times the $\arctan(1)$ or two times the $\arcsen\left(\frac{1}{\sqrt{2}}\right)$ for example. But, no reason for this has been found so far.

It is to be noticed, as explained in the beginning. The approximation we have done, considers the new area of space as only created by the photons emitted by the electron. This is in a general situation not exact (other particles are hanging around from other parts of space) and should have their effect.

This could be one of the reasons that make our original calculation of G not exact (below 1% anyhow) but it is difficult to know what this $\frac{\pi \alpha}{2\sqrt{2}}$ really means.

Just also to be noticed the complete number in the exponential $-\frac{1}{\sqrt{2}}\left(\frac{1}{\alpha}-\frac{\pi\alpha}{2}\right)$ could be also written as an approximation as:

$$-\sqrt{\frac{1-\pi\alpha^2}{2\alpha}}$$

Just in case, this could mean something in the future.

15. Why this theory could be totally wrong

The surprising result of the calculation of G with an error below 1%, at first sight could be stunning but of course has its tricks.

-Root mean square of photon wavelength (14). It seems logical to get root mean square value for a wave as a magnitude to represent it.

-But root mean square of the increment of length of space in radius direction (12). This value is clearly forced by the subsequent result as it is counter-intuitive to get the rms of a distance (even with the slight justification of not being linear, but a linearization of a circumference with its possible effects or considering space as composed by radiation and its corresponding waves).

-The magnitude $y(r)=\frac{\Delta S(r)}{S(r)}$ (16) and its definition to be integrated $\int_{\infty}^r \frac{dy}{y(r)}$ (17)

seems also forced to get to the ln and later the necessary exponential.

-In equation (18) is used the value of $2r$ to be coherent with the value of $2\lambda_{rms}$, but this could seem forced and generate doubts.

-The same can be argued with the hand right integral adding the value $\frac{dr}{r}$ (counter intuitive also) as a way to get the necessary result.

-It could be also that the equations (1) and (27) are ok, but the way to arrive to them is wrong. As these equations resemble statistic equations (as Bose-Einstein [21] or Weibull distribution [22] for example) could be that they are the result of a statistical study of the number of photons and their size in the different positions of space.

16. Why this theory could be ok

Even with the not intuitive assumptions or forced ways of doing the calculations, the result is surprising anyhow. Not only to get a result of the order of magnitude needed but to get only an error below 1% is really surprising.

In whatever theory, to arrive to a calculation of a constant of the order of magnitude of 10^{-11} with the correct measurement units and below 1% error in the representative digits, should make us at least think seriously that this theory could at least go in the good direction.

Also including the integration constant to get to the 100% precision and getting a value that corresponds with 0.01 % to a known constant (35), again should at least compel us to investigate further in this theory. Of course any number can be forced multiplying by different constants to get to something more or less known but 0.01% and only using constants related to the theory (α, π and powers of 2) at least is also surprising ($\frac{\pi \alpha}{2\sqrt{2}}$).

Forgetting about the numbers, I do not want to leave without mentioning other issue regarding the theory itself.

The idea of space being composed or at least affected by the particles that occupy it (not only masses but whatever particles that occupy it) is a theory that has been proposed by several scientists in different papers as [18][19][20].

The added value of this paper is that the theory leads to a calculation of G using only electromagnetic parameters that is a way of validating or at least giving some push to the theory.

17. Your comments and conclusions

No much more to be added here. We have proposed a theory that considers that space is composed by the particles (including mass particles and force carriers) that occupy it. Gravitation effects are nothing but a result of the composition of space and its irregularities causing it to warp in the different areas.

This theory could be one more in the internet universe but it happens to calculate very precisely (below 1% error with no arbitrary constants, and 0% error with directed fine tuning) the value of G.

$$G = 6.6202087 \times 10^{-11} m^3 k g^{-1} s^{-2} \quad (27)$$

$$G = 6.67408 \times 10^{-11} \times 10^{-11} m^3 k g^{-1} s^{-2} \quad (1)$$

$$error = 0.81\% \quad (28)$$

This view of space has been proposed in different papers [18][19][20]. Being this, the first time that has led to a calculation of G based only in electromagnetic parameters (as the majority of the particles of space are photons and alike).

Hope you enjoy it, do not hesitate to send me any comments to my email in the footer of the paper.

Bilbao, 14th September 2016

18. Acknowledgements

To my family and friends.

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