Introduction

It was demonstrated in a previous article, that the Schwarzschild weak fields approximation (SWFA) could be deduced from Newton's theory of gravitation with some supplementary axioms either: "the speed of light as speed limit", "the mass energy equivalence", "the relativistic length contraction", "the relativistic weak equivalence principle" and "the principle of the self-induction of the mass". Given that it has been demonstrated long ago that Newton's gravitation is derivable from the weak fields approximation of the general relativity, the inverse demonstration should not surprise anyone. What is interesting in this case, is that the resulting theorem is actually a generalization of the weak fields approximation, while general relativity only speaks to the distortion of time and space, the GEST (Gravitational Entropic Self-inductive Theory) deals with the potential energy of the system in the form of mass. Indeed, the principle of the self-induction of mass, which implies that the potential energy that generates mass must produce in turn its own mass, leads to an energy based definition of the elasticity of the space.

The GEST leads us to the conclusion that classical equation derived from the SWFA should rather be written as:

\[ \frac{m_x}{m_0} = \frac{t_x}{t_0} = \frac{l_0}{l_x} = k\sqrt{1-R_s/x} \]

The gravitational field, in consequence, not only distorts space and time in an infinitesimal point \( p \), it also stores field energy as mass, thus explaining the phenomenon of galactic dark mass (matter). In addition, \( k \) is a renormalization factor so that if \( x \) tends to infinity then \( m \), approaches \( km_0 \), \( t \) approaches \( kt_0 \) and \( l \) approaches \( l_0/k \). That is to say, that the units of time and length of a gravitational system are renormalized by a common factor \( k \), therefore, the system is at the energy level or potential \( k \). On the other hand, the unit of mass is an invariant in all reference frames, and the total mass of the system is the same for all observers.

Thus, in a system of total mass \( M_0 \) composed of \( n \) particles of mass \( m_0 \), the mass of each of these particles is modified by the relationship \( \frac{m_x}{m_0} = k\sqrt{1-R_s/x} \). However, as in special relativity, the spatial distribution of this extra mass is not indicated. It is here that the relationship \( m_x/m_0 \) really differs from the field relationships \( t_x/t_0 = l_0/l_x \). Indeed, the simplest approach is to assume that this extra mass is exactly in the same location as the mass \( m_0 \) but such distribution would go against all empirical evidences regarding the galactic dark mass. Empirical evidences forces us to consider that this extra mass be found in a spherical halo that may exceed the system's range. This extra mass does not belong to the component \( m_0 \) but to the system \( M_0 \).

The renormalization factor is equivalent to consider that a gravitational system is an inertial system such as a body at a constant speed and has, for this reason, its proper time and its proper length. Therefore, a measure of length or time of a gravitational system from another distant system should be subjected to a similar transformation to that of Lorenz. The renormalization of galactic systems leads us to postulate that this phenomenon greatly affects the measurement of the frequency shift of the light emitted by a galaxy with a certain energy level from an another galaxy at another energy level.

The objective of this article is to draw some important consequences of the GEST, for instance the derivation of the Tully-Fisher relation, the derivation of the calculation function of the intrinsic frequency shift and the shape of the distribution curve of dark mass. These theoretical extensions may help to understand the inexplicable experimental errors in the calculation of the Hubble constant and the problem of the creation and stabilization of galactic disks.
The Baryonic Tully-Fisher Relation

It was demonstrated\(^1\) that if the spherical halo of the dark mass perfectly follows the density distribution of the baryonic matter of mass \(M_0\) and moves at the same speed it follows that the dark mass coefficient \(\gamma = M/M_0\) [1] is given by the relation \(\gamma^4 - \gamma^2 = (RVc/8GM)^2\). It is possible to simplify this relationship by posing \(\gamma^4 - \gamma^2 \approx \gamma^2/2 - 1/7\) which allows to solve a simple quadratic equation and get \(\gamma \approx [1/4 + \sqrt{(23/112 + RVc/8GM)}]\). This equation gives a maximum error of 0.4 % when \(\gamma = 2\), of 0.05 % when \(\gamma = 3\) and remains less than 0.01 % after \(\gamma = 4\), which is a good approximation. It is possible to further simplify more by retaining an error lower than 1 % by \(\gamma \approx \sqrt{(RVc/8GM)} + 0.3\). To facilitate the calculations we will rather use \(\gamma \approx \sqrt{(RVc/8GM)}\) [2] which produces a systematic error of up to 15 %. The radius \(R\) being the maximum radius, \(V\) the speed and \(M\) the total galactic mass, it is possible to apply the virial theorem\(^3\) \(M = 2V^2R/G\) [3].

It is thus possible to obtain \((M/M_0)^4 \approx (RVc/8GM)^2\) by [1] and [2], then \(V^2R^2/M_0^2 \approx (cG^2/64)\) by applying the virial [3] and thus \(V^2 \approx (cG^2/64)(M_0^2/R^2)\). By simply modeling the galaxy as a disk with a thickness \(e\) and a homogeneous density \(d\), we obtain \(M_0 = \pi edR^2\) so \(R^2 = M_0/\pi ed\) therefore \(V^2 \approx M_0 (\pi de)(cG^2/64)\) which implies a relation of the type \(M_0 \propto V^2\). However, this calculation ignores the coefficient \((\pi de)\) which lacks only the multiplication by \(R^2\) to get \(M_0\). Let \(e = R/a\) thus \(M_0 = \pi d(R^2/a)\) [4] and so \(\ln(M_0) = \ln(\pi d/a) + 3 \ln(R)\) [5], therefore, by posing \(M_0 = \pi d(R^2/a)\) so \(\alpha = [\ln(\pi d/a) + \ln(R)]/\ln(M_0)\) which allows to obtain \(\alpha = (\ln(\pi d/a) + \ln(R))/[\ln(\pi d/a) + 3 \ln(R)]\) by [4] and [5]. Since \(\pi de/a\) is on the order of \(10^{-2}\) and \(R\) on the order of \(10^{2}\), we can approximate \(\alpha \approx 1/3\) and therefore \(V^2 \approx M_0^{3/5} (cG^2/64)\), which implies a relation of the type \(M_0 \propto V^{3.75}\). This relationship is in perfect agreement with the baryonic Tully-Fisher law\(^4\) \(V^{0.5} \propto M_0 \propto V^4\) and it is therefore possible to derive this law without changing Newton's gravitation\(^5\).

This perfect match to the Tully-Fisher relationship reveals that the calculation of the dark mass by \(\gamma^4 - \gamma^2 \approx (RVc/8GM)^2\) is probably valid only for spiral galaxies. Indeed, the development of this equation requires that the spherical halo of dark mass perfectly follows the density distribution of baryonic matter and has a nonzero homogeneous angular momentum, some very specific conditions, that are probably only fully realized in spiral galaxies.

The Intrinsic Frequency Shift

By the "relativistic weak equivalence principle" the gravitational dark mass is a Lorentz invariant equivalent to the inertial relativistic mass. Therefore, it is necessary that the factor \(\gamma = m_0/m_\gamma = t_0/t_\gamma = l_0/l_\gamma = k\sqrt{(1-R/x)}\) be strictly equivalent to the Lorentz factor of a relativistic speed. Thus, the classical Lorentz transformation of the velocities composition must consequently be used. This intrinsic frequency shift is distinct from the gravitational shift. Indeed, the gravitational frequency shift measures a difference within the same reference frame (renormalized frame) while the intrinsic shift measures a frequency difference between two distinct reference frames (renormalized frames).

It is possible to use here the original equation of the GEST \(1/\gamma = m_0/m_\gamma = t_0/t_\gamma = l_0/l_\gamma = k\sqrt{(1-R/x)}\) because \(R/x << 1\) and in both cases, by summing all the contributions of all the masses \(\gamma = M_0/M_\gamma\). Thus, from a radiation emitting galaxy \(1/\gamma = M_0/M_\gamma = 1/k_\gamma\) to a galaxy receiving the radiation \(1/\gamma = M_0/M_\gamma = 1/k_\gamma\), it is possible to get the velocities equivalents \(v_\gamma = c\sqrt{(1-1/\gamma^2)}\) and \(v_\gamma = c\sqrt{(1-1/\gamma^2)}\) which may be composed as \(v = (v_\gamma - v_\gamma)(1+(v_\gamma/v_x/c^2))\) which gives a shift \(z = \sqrt{((1+v/c)(1+v/c^2))} - 1\).

Since for the Milky Way \(\gamma_\gamma \approx 6\) and the minimum for a galaxy is \(\gamma_\gamma = 2\), then there exists a maximal redshift of \(z = 0.67\%\) which is not negligible. Between Andromeda (\(\gamma_\gamma = 12\)) and the Milky-Way, the blueshift is \(z = -0.53\%\), this shift maxes quickly because with \(\gamma_\gamma = 1000\) then \(z = -0.70\%\). It is therefore possible to see that the intrinsic shift could cause a problem with speeds equivalent from 19000 km/s for \(\gamma_\gamma = 2\), 6700 km/s for \(\gamma_\gamma = 3\), 2700 km/s for \(\gamma_\gamma = 4\), 940 km/s for \(\gamma_\gamma = 5\), -560 km/s for \(\gamma_\gamma = 7\), -930 km/s for \(\gamma_\gamma = 8\), -1200 km/s for \(\gamma_\gamma = 9\), -1300 km/s for \(\gamma_\gamma = 10\), -1500 km/s for \(\gamma_\gamma = 11\), -2000 km/s for \(\gamma_\gamma = 12\), -2100 km/s for \(\gamma_\gamma = \infty\).
It is important to note that the time dilation is also not negligible. For example, in the worst case, a phenomenon observed in a galaxy with \( \gamma = 2 \), gives a speed equivalent \( v = 19000 \) km/s which produces a time dilation of \( 1/(1-v/c^2) = 1.002 \). Thus, the pulsars in these galaxies appear rotate slightly faster by 0.2 %.

Astronomers and astrophysicists discussed the existence of sources of errors for some time \( ^{6,7,8,9,10,11} \) in the calculation of frequency shifts as they try to correct them \( ^{12,13} \). The Tully-Fisher relation is the most important secondary measure of the distance measurements of a broad set of spiral galaxies, it has a significant influence on the conventional calculation (through the creation of distances scales) of the Hubble constant, yet the errors persist inexplicably \( ^{14} \). Some authors have analyzed the existence of an intrinsic shift factor that could explain these errors. Russell \( (2015)^{15} \) after a comprehensive analysis concluded at the existence of an intrinsic redshift which can exceeding 5000 km/s and a clear tendency for the intrinsic redshifts to be more important than the intrinsic blueshifts. The result of this analysis is in perfect agreement with our theory. In light of this, it is imperative that astronomers and astrophysicists carry out the necessary tests to establish if this intrinsic shift is indeed produced by the dark mass as previously calculated.

The Distribution of the Dark Mass

The structure of the dark mass halo was widely studied and several empirical models had been proposed \( ^{16,17} \). The GEST, by using the principle of self-induction, allows us to apply a rigorous constraint on the mathematical structure of the halo. Indeed, the principle of self-induction is the only original premise to pass from Newton to SWFA and it is postulated that the induced mass is found at a position \( x \) identical to that of the original mass for the self-induction may logically occur. Indeed, self-induction implies that an inert mass \( m \) at a distance \( x \) from the center of mass induces a mass \( m' \) by the function \( m' = Rm/2R - Rm/2x \). This function is applied recursively for producing a mass \( m'' \) in exactly the same way, so \( m'' = Rm'/2R - Rm'/2x \) and so on. Therefore, for that self-induction to occur, it seems necessary that the induced mass be at exactly the same position \( x \), this can be achieved in three ways:

1. The mass generated is found exactly at the position \( x \) and is, in consequence, a form of intensification (renormalization) of the gravitational field of the inert mass \( m \). This distribution would perfectly follow that of matter and does not form a halo, this concept is refuted by empirical evidence.
2. The mass generated is found uniformly distributed in a spherical shell at position \( x \), and an inert mass \( m \) produces a sort of dark mass hem. This distribution also perfectly follows that of matter but forms a discontinuous halo, which would be physically difficult to explain.
3. The mass generated is found in a distribution of \( m' \) as the sum of the parts \( dm \) of this distribution applied to the recursive function \( Rdm'/2R - Rdm'/2x \) produce the same total mass \( m'' \). In this case, by induction, this distribution will generate the same total mass as in the cases (1) and (2).

Let \( \sigma(x) \) a mass distribution function such that the integral from 0 to infinity gives \( m = \int R/2R \sigma(x) \, dx \). It is then possible to write: \( m' = R/R \sigma(x) \, dx \), \( m'' = R/R \sigma(x) \, dx \), \( m''' = R/R \sigma(x) \, dx \). Thus, the principle of self-induction is observed for the first term regardless of the distribution function which implies that it is necessary and sufficient that \( R/2 \in \sigma(x) \, dx = Rm/2R \) such as \( r \) is the mass position \( m \). Therefore, it's merely required to find a function \( f(x,r) \) so that the integral of \( f(x,r) \, dx \) from 0 to infinity is 1 and the integral of \( f(x,r) \, dx \) from 0 to infinity gives 1/r. Such functions exist, for example, the polynomial family \{ \( 2 \cdot r x(r+x)^3 \), \( 12 \cdot r x^2/(r+x)^3 \), \( 60 \cdot r x^3/(r+x)^5 \), \( 280 \cdot r x^5/(r+x)^7 \), \( 1260 \cdot r x^7/(r+x)^9 \), \( \ldots , k \cdot r x^{k+1}/(r+x)^{2k+1} \) \} seems to be valid for any value of \( i \), this property is checked up to \( i = 10 \). Just as the exponential family \{ \( e^{-i/2} \), \( 2 \cdot e^{i/2} \), \( 3 \cdot e^{3i/2} \), \( 4 \cdot e^{4i/2} \), \( 5 \cdot e^{5i/2} \), \( \ldots , i \cdot e^{i(i+1)/2} \) \} which also appears valid for any value of \( i \), this property was checked up to \( i = 10 \). There are probably many other families of functions that have this property, and we are only considering the spherical halos. There are also likely functions of six variables \( x, y, z, a, b, c \) generating ellipsoidal halos.

All these functions, despite their different natures, have the same form of left-shifted bell curve (see Figure 1 and 2), which seems to be a characteristic of functions such as \( f(x,r) \, dx = 1 \) and \( f(x,r)/x \, dx = 1/r \). Another remarkable property, \( \int f(x,r) \, dr = 1 \) and \( \int f(x,r)/r \, dr = 1/x \), that is to say, the property is symmetrical if we integrate over \( r \).
instead of \( x \). Although this property is obvious for the polynomial family which remains the same by exchanging \( x \) and \( r \), however it is far less evident for the exponential family. A remarkable property of the exponential family is that the apex of the curve (point of zero derivative) is \( x = r \). This property has a profound physical meaning: the dark mass generated by the part \( m_0 \) has its highest concentration at the location \( r \) of \( m_0 \) and would be distributed from the system's mass center to infinity. This property would reconcile the phenomenon of the dark mass as both as a component of the whole and the part.

The dark mass distribution curves (see Figure 1 and 2) are those produced by a single particle (star) \( m_0 \) at position \( r \); the function \( g(x) \) of resulting distribution for \( n \) stars, of identical mass, is \( g(x) = \sum_j f(x,r) \). If we have a distribution function of these stars \( D(r) \) then the resulting dark mass distribution function is \( g(x) = \int f(x,r) \, D(r) \, dr \). Another remarkable property is that if we assume that the dark mass follows one of these curves, for example, \( f(x,r) = 2 \, r \, x \, (r+x)^3 \) then it is possible to determine a similar curve \( D(r) = 30 \, r^2 \, x^5 / (r+x)^4 \) in a way that \( g(x) \) is also a valid distribution, here \( g(x) = \int [60 \, r^3 \, x^5 / (r+x)^4] \, dr \). So whatever the real function of distribution of dark mass, there is a distribution of baryonic matter such that the total dark mass distribution generated is similar to that of the baryonic matter, which in addition is identical to that produced if the entire dark mass was produced by a single particle (star) at the position \( R \). We call "existence of a homogeneous attractor" the existence of a distribution of baryonic matter, similar to the distribution of dark mass, which is a valid solution to the dark mass production equation.

The resemblance of the homogeneous attractor and the distribution models of the baryonic matter created ad hoc to model the distribution of matter in spiral galaxies is not likely coincidental. Let us note that one of the most commonly used models is the exponential model \( \sigma(x) = e^{x/2\pi r^2} \) which correctly gives the mass distribution \( dm(x) = x e^{x/2\pi r^2} \, dx \), precisely the simplest exponential distribution of dark mass we have found. That the baryonic matter and the dark mass mysteriously follow the same distribution law was already noticed\(^{18}\), the existence of a homogeneous attractor provides a solid theoretical justification for this phenomenon.

It is imperative that the galactic evolution simulations that take into account dark mass production by the stars following the distribution curves discussed herein be carried out to confirm or refute this phenomenon. It would be sufficient to slightly alter the galactic simulation models\(^{19,20,21}\) of cold dark matter (LCDM) so that the dark mass is divided into "clouds" being distributed according to one of the curves shown which constantly follows the movement of the masses that generates its.
Discussion

The results presented in this paper confirm the explanatory power of the GEST. This theory has a clear theoretical advantage that of being constructed from three generally accepted induced axioms. Indeed, the GEST uses as induced axioms Newton’s gravitation, the speed of light as a limit speed and the relativistic weak equivalence principle. It also uses as deducted axioms special relativity and two original axioms so the existence of a minimal compact state which is the associated black hole and the principle of self-induction of the mass; which are logically necessary axioms and therefore do not constitute real inductions but constrained deductions.

The question of whether the GEST is a modification of GR or is just an extension of it remains open but the fact that the distribution of the dark mass is completely separate from the GR field suggests that this is another field, most likely the BEHHGK field. According to the GEST, the BEHHGK field produces dark mass by coupling itself via gravity. The principle of weak equivalence suggests that the type of distribution curve that we presented is in fact the spatial distribution of the mass generated by the BEHHGK field. Thus, an inertial system moving at uniform speed would see its dark mass expand according to the same type of curve and an observer, within this system, could by gravitational measurements, detect that mass and conclude that he has a speed relative to the absolute space. Indeed, it would easily distinguish that fact of a dark mass produced by gravitational energy by knowing the internal properties of his system, such as its own baryonic mass.

We must consider the GEST as a theoretical bridge between GR indicating how space is deformed depending on the energy and the BEHHGK field, that indicates how the potential energy can produce mass that in turn deforms space. Thus, according to the GEST, the BEHHGK field is a pure relativistic product as the gravitational field and the two fields appear inextricably linked. In fact, they are so closely related that it may well be that these collectively represent a single field. The dark mass would then be the well of gravitational potential at the bottom of which lies the galaxy. The fact that the dark mass is not detected and it is represented by the BEHHGK field suggests that this field behaviour is indeed that of a condensate in which case, it may well be that it is also responsible for gravitation.

Conclusion

If only one of the original consequences described in this article proves true, the GEST will demonstrate its relevance. For now, we must admit that its explanatory value is interesting. In fact, it has already been demonstrated1 that the GEST enables simply and naturally: 1) The prediction of a minimal dark mass ratio of two (2) for every galaxy, which answers to one of the great shortcomings of the MOND theory. 2) The generation of a simple model of the amount of dark mass in spiral galaxies such as $M = kM_\odot$. It is unlikely that such an equation giving the precise values of 6 for the Milky Way and 12 for Andromeda is the result of chance. 3) The prediction of an accelerated expansion of the universe in the accepted order of magnitude.

The current contribution adds: 1) A simple calculation of the baryonic Tully-Fisher relation. 2) The derivation of an intrinsic frequency shift factor that could explain the experimental errors of the current measures of the Tully-Fisher relation. 3) The contraction and expansion of the galactic time. 4) A derivation of the dark mass distribution curves in perfect agreement with the empirical currently accepted distribution curves of the matter and the dark mass in the spiral galaxies.

This article, by predicting three extremely accurate new phenomena: the intrinsic frequency shift, the expansion and contraction of galactic time and the shape of the distribution curves of the dark mass generated by the stars, brings a possibility of confirmation or refutation of the GEST which is most likely the most important implication.