Schrödinger Equation for a System with Exponential-Type Restoring Force Function

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Abstract

In this paper the Schrödinger equation is derived for a dynamical system with exponential-type restoring force function.

1. Let us consider the exponential-type restoring force function arisen in a class of exactly integrable quadratic Liénard-type nonlinear dissipative oscillator equations which admit also a position-dependent mass dynamics as well as in a class of generalized Duffing-type equations [1-4]

$$F(x) = -\omega^2 x e^{2\gamma\varphi(x)} \tag{1}$$

where γ and ω are arbitrary parameters, and $\varphi(x)$ an arbitrary function of x. The associated potential reads

$$V(x) = \omega^2 \int x e^{2\gamma\varphi(x)} dx \tag{2}$$

2. Derivation of Schrödinger equation for $\varphi(x) = \frac{1}{2}x^2$

The potential (2) becomes

$$V(x) = \frac{\omega^2}{2\gamma} e^{\gamma x^2} \tag{3}$$

For $\gamma < 0$, this potential tends to zero as $x \to \pm\infty$, and becomes $V(x) = \frac{\omega^2}{2\gamma} < 0$, for $x = 0$, so that the time-independent Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \right]\psi = E\psi \tag{4}$$

where $\psi = \psi(x)$, takes the form

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{\omega^2}{2\gamma} e^{\gamma x^2} \right]\psi = E\psi \tag{5}$$

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After a few algebraic treatment, (5) may be written in the form

$$\frac{d^2\psi}{dx^2} - \frac{m\omega^2}{\gamma\hbar^2}\left(e^{\gamma x^2} - \frac{2\gamma E}{\omega^2}\right)\psi = 0 \tag{6}$$

With $\lambda^2 = -\frac{m\omega^2}{\gamma\hbar^2}$ and $\alpha = \frac{2\gamma E}{\omega^2}$

the equation (6) may be rewritten as

$$\frac{d^2\psi}{dx^2} + \lambda^2\left(e^{\gamma x^2} - \alpha\right)\psi = 0 \tag{7}$$

The potential, obtained for the restoring force function $F(x) = -\omega^2 x e^{2\gamma x}$

$$V(x) = \frac{\omega^2}{4\gamma^2}(2\gamma x - 1)e^{2\gamma x} \tag{8}$$

gives as Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{\omega^2}{4\gamma^2}(2\gamma x - 1)e^{2\gamma x}\right]\psi = E\psi \tag{9}$$

which becomes

$$\frac{d^2\psi}{dx^2} - \frac{m\omega^2}{2\hbar^2\gamma^2}\left[(2\gamma x - 1)e^{2\gamma x} - \frac{4\gamma^2 E}{\omega^2}\right]\psi = 0 \tag{10}$$

Considering here $\lambda^2 = \frac{m\omega^2}{2\gamma^2\hbar^2}$ and $\alpha = \frac{4\gamma^2 E}{\omega^2}$, yields

$$\frac{d^2\psi}{dx^2} - \lambda^2\left[(2\gamma x - 1)e^{2\gamma x} - \alpha\right]\psi = 0 \tag{11}$$

The obtained Schrödinger equations (7) and (11) are under investigation. However, it is interesting to note that other potentials generated from the exponential-type restoring force functions introduced in previous studies [1-4] may also be considered to investigate the Schrödinger equation. In this perspective as example, the potential, that is the Schrödinger equation, associated to the following restoring force function

$$F(x) = -\omega^2 h(x) e^{\mu\varphi(x)} \tag{12}$$

where μ and ω are arbitrary parameters, and $\varphi(x)$ and $h(x)$ are arbitrary functions of x,

will be studied in future works.

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