# From the General theory of Relativity to nonsingular theory of gravity

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**Abstract**

The article presents some results of a nonsingular theory of gravity (hereinafter - NTG) based on the axiomatics which differs somewhat from the axioms of the general theory of relativity (GR) and free from the internal problems inherent to GR. Nonsingular space kinematics is constructed on this basis. It is proved that from the condition of energy positivity follows: global isotropy of homogeneous space, existence of a change of the acceleration - deceleration eras and negative value of the scalar curvature of space. Nonsingular basic phenomenological model of evolution of the Universe is constructed, described by the smooth one-parametrical dependence from the moment of the beginning of evolution to an arbitrary point of time, coordinated with the observational astronomical data but without an involvement of the hypotheses of the existence of a dark energy, dark matter and inflatons. The particle-like solution of the NTG equations for the static isotropy metric is found. The behavior of the solution in the region of weak fields is specified on the basis of correspondence principle with GR. It is shown that in the certain region of space the distributions of fields can exist, for which the equality of inertial (defined according to Mach's principle) and gravitational mass is satisfied. The horizon characteristic for the solutions of GR equations in the isotropic case is absent in NTG.

Keywords: Gravitational Physics, Space Kinematics, Cosmology.

## 1. Introduction

One hundred years ago at the derivation of the gravitational equations from the variational principle D. Hilbert formulated an axiom of the general invariance of the action relative to arbitrary transformations of coordinates.

The success of the canonical theory of gravity ostensibly corroborated validity of such assumption and it has acquired the status of the fundamental principle eventually.

However Penrose's and Hawking's proof of the theorems on singularity of the solutions of GR equations is a sufficient cause to doubt a possibility to describe on its basis phenomena in the microcosm and in the scale of the Universe.

In the light of the new experimental data [1] GR doesn't seem as unshakeable as before any more. For an explanation of the derived results within this theory it was necessary to introduce certain hypothetical entities which haven't been found yet.

The Nobel lecture of B. P. Schmidt [2] comes to the end with the statement: «An enormous body of theoretical work has been undertaken in response to the discovery of the accelerating Universe. Unfortunately, no obvious breakthrough in our understanding has yet occurred – cosmic acceleration remains the same mystery that it was in 1998. The future will see bigger and better experiments that will increasingly test consistency of our Universe with the Flat Λ-CDM Model. If a difference were to emerge, thereby disproving a Cosmological Constant as the source of acceleration, it would provide theorists with a new observational signature of the source of the acceleration. Short of seeing an observational difference emerge, we will need to wait for a theoretical revelation that can explain the standard model, perhaps informed by a piece of information from an unexpected source».

In our opinion, just general covariance of the equations is a source of the troubles of GR. Detected on the stage of its formation, today these troubles have become the whole set of problems unresolved so far: the problem of energy, singularities, black holes, cosmological constant, cold dark matter, and finally the problem of description of the elementary particles which appears in the canonical theory of gravitation as "micro black holes".

An obvious way to construct the non-generally covariant theory of gravity without violating of the axiom of Hilbert (as we see it) is the introduction of a priori constraints that restrict the choice of coordinate system. Such attempts have repeatedly been made in last one hundred years; the example of it is the unimodular theory of gravity whose origins date back to Einstein. Generally an appearance of the edges of space-time manifold is a consequence of the constraints introduction. In the presence of the differential constraint there is an opportunity to choose a position of the edge so that to single out nonsingular region of the manifold.

The article presents some results of a theory of gravity free from the internal problems inherent to GR.

## 2. Gravitational field equations

Our basic assumption is that the components of the metric tensor $g_{\mu\nu}$ are constrained by the conservation law:

$$\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}g^{\mu\nu}\Gamma^{\rho}_{\nu\rho}\right)=0,\ \Gamma^{\rho}_{\nu\rho}=\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial x^{\nu}},\ g=\det\left(g_{\mu\nu}\right),\ (\mu,\nu=0,1,2,3). \tag{2.1}$$

To obtain the rest of the gravitational field equations on the mass shell, proceeding from the Hilbert action and introducing the Lagrange multiplier – the scalar field $\Phi$, write the action for the gravitational field in the presence of the constraint (2.1) as:

$$S_{gr}=-\frac{c^3}{16\pi G}\int\left(R+\Lambda\right)\sqrt{-g}d^4x,\ \Lambda=\Gamma^{\rho}_{\mu\rho}g^{\mu\nu}\frac{\partial\Phi}{\partial x^{\nu}}. \tag{2.2}$$

The presence in Lagrangian of the additional members besides to the scalar curvature leads to the occurrence of the energy density tensor of the gravitational field in the Hilbert-Einstein equations when the metric is varying.

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi G}{c^4}(\varepsilon_{gr})_{\mu\nu}+\frac{8\pi G}{c^4}(\varepsilon_{mat})_{\mu\nu}, \tag{2.3}$$

$$\frac{16\pi G}{c^4}(\varepsilon_{gr})_{\mu\nu}=-g_{\mu\nu}\frac{\partial}{\partial x^{\rho}}\left(g^{\rho\lambda}\frac{\partial\Phi}{\partial x^{\lambda}}\right)-\Gamma^{\rho}_{\mu\rho}\frac{\partial\Phi}{\partial x^{\nu}}-\Gamma^{\rho}_{\nu\rho}\frac{\partial\Phi}{\partial x^{\mu}}. \tag{2.4}$$

These equations together with (2.1) are sufficient to determine the components of the metric and the field $\Phi$. *The equations are covariant relative to the local unimodular and global scale transformations of coordinates.*

## 3. Space kinematics

Since now the gravitational field has certain energy that in contrast to GR the metric is nontrivial even in the absence of any matter. It is natural to consider all the components of the metric tensor in that initially empty space as not dependent on the space-like coordinates. If the spatial metric is non-degenerate then the most general expression for the space-time interval can be reduced to the form [3] by the unimodular coordinates transformation:

$$ds^2=g_{00}(x^0)(dx^0)^2+g_{mn}(x^0)dx^m dx^n,\ \gamma=-\det(g_{mn})>0,\ (m,n=1,2,3). \tag{3.1}$$

An absence of the general invariance of the action (3.2) doesn't allow us to eliminate the metric component $g_{00}$, therefore the expressions for the nonzero components of the curvature tensor are rather different from the expressions given in [3].

$$R^0_0=-\frac{1}{2\sqrt{g_{00}}}\frac{d}{dx^0}\left(\frac{1}{\gamma\sqrt{g_{00}}}\frac{d\gamma}{dx^0}\right)-\frac{1}{4g_{00}}g^{mk}\frac{dg_{kp}}{dx^0}g^{pn}\frac{dg_{nm}}{dx^0}, \tag{3.2}$$

$$R_k^p = -\frac{1}{2\sqrt{\gamma g_{00}}}\frac{d}{dx^0}\left(\sqrt{\frac{\gamma}{g_{00}}}g^{mp}\frac{dg_{km}}{dx^0}\right). \tag{3.3}$$

Nonzero components of the energy-momentum tensor density:

$$\left(\varepsilon_{gr}\right)_0^0 = -\frac{c^4}{16\pi G}\left[\frac{d}{dx^0}\left(\frac{1}{g_{00}}\frac{d\Phi}{dx^0}\right) + \frac{2}{g_{00}\sqrt{g_{00}\gamma}}\frac{d\sqrt{g_{00}\gamma}}{dx^0}\frac{d\Phi}{dx^0}\right], \tag{3.4}$$

$$\left(\varepsilon_{gr}\right)_k^p = -\frac{c^4}{16\pi G}\frac{d}{dx^0}\left(\frac{1}{g_{00}}\frac{d\Phi}{dx^0}\right)\delta_k^p . \tag{3.5}$$

The gravitational field equations will have the form:

$$\frac{d}{dx^0}\left(\frac{1}{g_{00}}\frac{d\sqrt{\gamma g_{00}}}{dx^0}\right) = 0, \tag{3.6}$$

$$-\frac{1}{2\sqrt{g_{00}}}\frac{d}{dx^0}\left(\frac{1}{\gamma\sqrt{g_{00}}}\frac{d\gamma}{dx^0}\right) - \frac{1}{4g_{00}}g^{mk}\frac{dg_{kp}}{dx^0}g^{pn}\frac{dg_{nm}}{dx^0} = \frac{\sqrt{\gamma g_{00}}}{2}\frac{d}{dx^0}\left(\frac{1}{g_{00}\sqrt{\gamma g_{00}}}\frac{d\Phi}{dx^0}\right), \tag{3.7}$$

$$-\frac{d}{dx^0}\left(\sqrt{\frac{\gamma}{g_{00}}}g^{mp}\frac{dg_{km}}{dx^0}\right) = \delta_k^p\frac{d}{dx^0}\left(\frac{\sqrt{\gamma g_{00}}}{g_{00}}\frac{d\Phi}{dx^0}\right). \tag{3.8}$$

Eq. (3.8) shows that:

$$g^{mp}\frac{dg_{km}}{dx^0} + \delta_k^p\frac{d\Phi}{dx^0} = \sqrt{\frac{g_{00}}{\gamma}}L_k^p . \tag{3.9}$$

The constant matrix $L_k^p$ is not arbitrary. Since eq. (3.9) shows

$$\frac{dg_{kn}}{dx^0} + g_{kn}\frac{d\Phi}{dx^0} = \sqrt{\frac{g_{00}}{\gamma}}g_{np}L_k^p , \tag{3.10}$$

that the matrix must satisfy the conditions:

$$g_{np}(x^0)L_k^p \equiv g_{kp}(x^0)L_n^p . \tag{3.11}$$

For the metric tensor of the general form this condition will be accomplished only in case when the matrix $L_k^p$ is proportional to the single matrix. Otherwise the matrix $L_k^p = diag(L_1,L_2,L_3)$ and the metric tensor must also be diagonal.

Simplifying eq. (3.9) on *p* and *k* indexes:

$$3\frac{d\Phi}{dx^0} = -\frac{1}{\gamma}\frac{d\gamma}{dx^0} + \sqrt{\frac{g_{00}}{\gamma}}L_k^k , \tag{3.12}$$

and the system of equations (3.9) takes the form

$$g^{pm}\frac{dg_{km}}{dx^0} = \frac{1}{3\gamma}\frac{d\gamma}{dx^0}\delta_k^p + \sqrt{\frac{g_{00}}{\gamma}}\left(L_k^p - \frac{1}{3}\delta_k^p L_n^n\right). \tag{3.13}$$

Eq. (3.13) shows that:

$$g^{mk}\frac{dg_{kp}}{dx^0}g^{pn}\frac{dg_{nm}}{dx^0} = \frac{1}{3}\left(\frac{1}{\gamma}\frac{d\gamma}{dx^0}\right)^2 + \frac{g_{00}}{\gamma}\left[L_k^p L_p^k - \frac{1}{3}\left(L_n^n\right)^2\right]. \tag{3.14}$$

Using this expression and eq. (3.12) it is possible to eliminate Φ and all spatial metric components from the equation (3.7) and after introduction of the proper time $cdt = \sqrt{g_{00}}dx^0$ to write it as:

$$3\frac{d}{dt}\left(\frac{1}{\gamma}\frac{d\gamma}{dt}\right) + \frac{1}{2}\left(\frac{1}{\gamma}\frac{d\gamma}{dt}\right)^2 + \frac{3c^2}{2\gamma}[L_k^p L_p^k - \frac{1}{3}(L_n^n)^2] = g_{00}\sqrt{\gamma}\frac{d}{dt}\frac{1}{\gamma g_{00}}\left(\frac{1}{\sqrt{\gamma}}\frac{d\gamma}{dt} - cL_n^n\right). \tag{3.15}$$

Eq. (3.6) implies

$$\frac{1}{g_{00}}\frac{dg_{00}}{dt}+\frac{1}{\gamma}\frac{d\gamma}{dt}=\frac{1}{T\sqrt{\gamma}}, T=const. \quad (3.16)$$

This equation allows to eliminate $g_{00}$ from (3.15) and to write the equation for the function γ:

$$2\frac{d}{d\tau}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right)+\frac{1}{\gamma\sqrt{\gamma}}\frac{d\gamma}{d\tau}-\frac{\sigma}{\gamma}=0, \sigma=B_n^n-\frac{3}{2}[B_k^pB_p^k-\frac{1}{3}(B_n^n)^2], \quad (3.17)$$

where τ=t/T dimensionless proper time, $B_k^p=cTL_k^p$ - matrix of the dimensionless constants. The order of the equation (3.17) can be lowered at the function u(γ) introduction - dimensionless rate of change of the volume factor - $\sqrt{\gamma}$

$$u=\frac{d\sqrt{\gamma}}{d\tau}. \quad (3.18)$$

The equation takes the form:

$$8\gamma u\frac{du}{d\gamma}=4u^2-2u+\sigma, \quad \frac{4udu}{4u^2-2u+\sigma}=\frac{d\sqrt{\gamma}}{\sqrt{\gamma}}. \quad (3.19)$$

It is remarkable that when **σ > 1/4** determinant of the spatial metrics isn't equal to zero anywhere. Therefore in this case there are no singularities.

Integrating the equation (3.19) we find that:

$$\sqrt{\frac{\gamma}{\gamma_{\min}}}=f(u), \quad f(u)=\sqrt{\frac{4u^2-2u+\sigma}{\sigma}}\exp\left[\frac{1}{\sqrt{4\sigma-1}}\left(arctg\frac{4u-1}{\sqrt{4\sigma-1}}+arctg\frac{1}{\sqrt{4\sigma-1}}\right)\right], \quad (3.20)$$

where $\sqrt{\gamma_{\min}}$ - the minimum value of $\sqrt{\gamma}$ at $u = 0$.

Differentiating (3.20) with respect to τ gives:

$$\frac{1}{\sqrt{\gamma_{\min}}}\frac{d\sqrt{\gamma}}{d\tau}=\frac{df(u)}{du}\frac{du}{d\tau}, \quad \frac{df}{du}=\frac{4u}{4u^2-2u+\sigma}f(u). \quad (3.21)$$

Hence we find the solution of the equation (3.17) in the parametric form in consideration of (3.18), (3.20).

$$\tau-\tau_{st}=\sqrt{\gamma_{\min}}\int_0^u\frac{4f(y)}{4y^2-2y+\sigma}dy, \sqrt{\gamma}=\sqrt{\gamma_{\min}}f(u). \quad (3.22)$$

*Evolution of space begins in the time point $\tau_{st}$ from a state of rest with the minimal volume factor.*

Consider the expression (3.4) for the energy density on the field equations. Using the relations (3.12) and (3.16), we can transform (3.4) as follows:

$$\left(\varepsilon_{gr}\right)_0^0=\rho_{gr}=\frac{c^2}{48\pi GT^2}\left[\frac{d}{d\tau}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right)+\frac{1}{2}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right)^2+\frac{1}{2\sqrt{\gamma}\gamma}\frac{d\gamma}{d\tau}-\frac{1}{2\gamma}B_k^k\right]. \quad (3.23)$$

Using the equation (3.17), we eliminate the second derivative, then

$$\rho_{gr}=\frac{c^2}{48\pi GT^2}\left[\frac{1}{2}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right)^2-\frac{3}{4\gamma}[B_k^pB_p^k-(B_k^k)^2]\right]=\frac{c^2}{48\pi GT^2\gamma}\left[2u^2-\frac{3}{4}[B_k^pB_p^k-(B_k^k)^2]\right] \quad (3.24)$$

The first term in the brackets vanishing at the small values of ***u***, the second term characterizing the global anisotropy of space is constant, positive and enters into the expression for the energy density with a minus sign. *The energy density will be positive only in case when homogeneous space is isotropic* ($Bmn \propto \delta mn$).

In this case the solution of the equations (3.13) can be presented in the form:

$$g_{kn}=-\gamma^{1/3}\delta_{kn}, \quad (3.25)$$

and the interval

$$ds^2 = c^2 (dt)^2 - \gamma^{1/3}(t) dx^m dx^n. \tag{3.26}$$

Introduce the Hubble parameter $H$ and the acceleration parameter $q$ (instead of the deceleration parameter [4]) according to the modern representations:

$$H \equiv \frac{1}{6T\gamma}\frac{d\gamma}{d\tau}, \quad q \equiv 1 + \frac{1}{6H^2T^2}\frac{d}{d\tau}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right). \tag{3.27}$$

The substitution of these expressions in (3.17) allows us to derive the equation describing change of the acceleration-deceleration eras.

$$q = \frac{3}{4}\left(\frac{\sqrt{\sigma}}{u(\gamma)} - \frac{1}{\sqrt{\sigma}}\right)^2 + 1 - \frac{3}{4\sigma}. \tag{3.28}$$

This implies that two scenarios are possible. When $\sigma > 3/4$ only acceleration ($q>0$) is possible. When $3/4 > \sigma > 1/4$ the change of the eras is possible: acceleration-deceleration-acceleration. The change of the eras happens at the values

$$u_1 = \frac{\sigma}{1+\sqrt{1-4\sigma/3}} > \frac{\sqrt{3}}{4(\sqrt{3}+\sqrt{2})} \approx 0.1376, \; u_2 = \frac{\sigma}{1-\sqrt{1-4\sigma/3}} < \frac{\sqrt{3}}{4(\sqrt{3}-\sqrt{2})} \approx 1.3624. \tag{3.29}$$

*Discovered recently* **[1]** *the change of the eras indicates that the second scenario takes place.* The maximum value of the deceleration is reached at $u=\sigma$

$$q_{\max} = 1 - \frac{3}{4\sigma} > -2. \tag{3.30}$$

After the onset of the second era of the acceleration, $q$ asymptotically approaches unity according to (3.28).

The energy density of the gravitational field (3.24) is related with the Hubble parameter as:

$$\rho_{gr} = \frac{3c^2H^2(\tau)}{8\pi G}. \tag{3.31}$$

Thus, *space is homogeneous and isotropic and has proper energy. And the density of the space energy is equal to the critical density at any moment of time*. The Hubble parameter reaches the maximum value during the era of the first acceleration at $u = \sigma/2 < u_1$

$$H_{\max} = \frac{\sqrt{\sigma}}{6T\sqrt{\gamma_{\min}}}\exp\left(-\frac{arctg\sqrt{4\sigma-1}}{\sqrt{4\sigma-1}}\right), \tag{3.32}$$

and then monotonously decreases, tending to the constant value

$$H_{\infty} = \frac{1}{6T\sqrt{\gamma_{\min}}}\exp\left(-\frac{1}{\sqrt{4\sigma-1}}\left(arctg\frac{1}{\sqrt{4\sigma-1}} + \frac{\pi}{2}\right)\right). \tag{3.33}$$

Determined by the relations (3.5) the spatial components of the energy-momentum tensor density are equal on the field equations to:

$$\left(\varepsilon_{gr}\right)^p_k = \frac{c^2}{48\pi GT^2}\left[\frac{d}{d\tau}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right) + \frac{1}{2}\left(\frac{1}{\gamma}\frac{d\gamma}{d\tau}\right)^2 - \frac{1}{2\sqrt{\gamma\gamma}}\frac{d\gamma}{d\tau} + \frac{1}{2\gamma}B^n_n\right]\delta^p_k, \tag{3.34}$$

and differ from expression for the energy density in the sign of the last two members. These components can possess both the positive and negative values during evolution. Eliminating the second derivate again by means of the equation (3.17) and assuming $(\varepsilon_{gr})^n_m = -p_{gr}\delta^n_m$ as it is accepted for macroscopic mediums, write the gravitational field pressure as:

$$p_{gr} = -\frac{c^2}{48\pi GT^2}\frac{2u^2 - 2u + \sigma}{\gamma}. \tag{3.35}$$

This implies when $0.25 < \sigma < 0.5$ there is a change of the pressure sign at the following $\boldsymbol{u}$ values:

$$u_3 = \frac{1-\sqrt{1-2\sigma}}{2} > \frac{\sqrt{2}-1}{2\sqrt{2}} \approx 0.146, \quad u_4 = \frac{1+\sqrt{1-2\sigma}}{2} < \frac{\sqrt{2}+1}{2\sqrt{2}} \approx 0.8536 \tag{3.36}$$

*The gravitational field has a positive pressure in the interval $u_3<u<u_4$, in other cases it has a negative pressure.*

Let us consider the curvature tensor. Substituting the relations (3.13), (3.14) in (3.2), (3.3) we will find the expressions for the curvature tensor on the field equations:

$$R_0^0 = -\frac{1}{2c^2}\frac{d}{dt}\left(\frac{1}{\gamma}\frac{d\gamma}{dt}\right) - \frac{1}{12c^2}\left(\frac{1}{\gamma}\frac{d\gamma}{dt}\right)^2,$$

$$R_k^k = -\frac{1}{2c^2\sqrt{\gamma}}\frac{d}{dt}\left(\frac{\sqrt{\gamma}}{\gamma}\frac{d\gamma}{dt}\right).$$

Excepting the second derivatives, write the expressions for the scalar curvature of space-time ${}^4R$ and space ${}^3R$.

$${}^3R = R_k^k = -\frac{1}{4c^2T^2\gamma}(4u^2 - 2u + \sigma) = -\frac{(4u-1)^2 + 4\sigma - 1}{16c^2T^2\gamma} < 0. \tag{3.37}$$

$${}^4R = R_0^0 + R_k^k = -\frac{1}{2c^2T^2\gamma}\left(\frac{8}{3}u^2 - 2u + \sigma\right). \tag{3.38}$$

(3.37) implies *the space curvature is always negative.* But the space-time curvature changes during evolution and possesses at first negative, then positive and at last again negative values.

According (3.31), (3.32) the maximum density of the gravitational field energy is equal

$$\rho_{gr\max} = \frac{c^2\sigma}{96\pi G T^2\gamma_{\min}}\exp\left(-\frac{2arctg\sqrt{4\sigma-1}}{\sqrt{4\sigma-1}}\right). \tag{3.39}$$

Assumed that σ=1/4 for definiteness, connect the constant value

$$T\sqrt{\gamma_{\min}} \approx \frac{1}{8e}\left(\frac{c^2}{6\pi\cdot G\cdot\rho_{gr\max}}\right)^{1/2} \tag{3.40}$$

with the maximum energy density.

Now (3.22) can be written as:

$$t - t_{st} = T\sqrt{\gamma_{\min}}\int_0^u \frac{4f(y)}{4y^2 - 2y + \sigma}dy,\ H(u) = \frac{1}{3T\sqrt{\gamma_{\min}}}\frac{u}{f(u)}. \tag{3.41}$$

According (3.20) f(u) depends on the constant σ only. Substituting in these relations the current values [5] of the time from the beginning of evolution till now ($t^0$-$t_{st}$ = 4.355•$10^{17}$ sec ) and Hubble parameter ($H^0$ = 2.181•$10^{-18}$sec$^{-1}$) gives, taking into account (3.40), couple of equations for two unknown – σ and the value of parameter $\boldsymbol{u^0}$ at the current time

$$t^0 - t_{st} = T\sqrt{\gamma_{\min}}\int_0^{u^0} \frac{4f(y)}{4y^2 - 2y + \sigma}dy,\ H^0 = \frac{1}{3T\sqrt{\gamma_{\min}}}\frac{u^0}{f(u^0)}.$$

It is considered to be the maximum energy density equal to the Planck's energy density in the standard cosmological model. Providing that $\rho_{gr\max} = \varepsilon_{Pl}$ the solution of this system of equations is:

$$\sigma = 0.250119943,\quad u^0 = 6.119897974. \tag{3.42}$$

The results of the calculations of other parameters for this case are presented in Table 1.

| $\rho_{gr\max} = \varepsilon_{Pl}$ ; $T\sqrt{\gamma_{\min}}$ =5.798463086·$10^{-46}$ sec; σ=0.250119943; $u^0$=6.119898285 | | | | | |
|---|---|---|---|---|---|
| u | q | z | $^3R$, $cm^{-2}$ | $t-t_{st}$ , sec | H, $sec^{-1}$ |
| 6.119898285 | 0.7599 | 0 | $-4.382\cdot10^{-56}$ | $4.358\cdot10^{17}$ | $2.181\cdot10^{-18}$ |
| 1.362298981 | 0 | 0.850004 | $-6.309\cdot10^{-56}$ | $1.876\cdot10^{17}$ | $3.074\cdot10^{-18}$ |
| 0.853468568 | -0.5 | 1.416151 | $-9.216\cdot10^{-56}$ | $1.129\cdot10^{17}$ | $4.290\cdot10^{-18}$ |
| 0.8 | -0.58189 | 1.525709 | $-9.989\cdot10^{-56}$ | $1.029\cdot10^{17}$ | $4.593\cdot10^{-18}$ |
| 0.7 | -0.76002 | 1.792870 | $-1.223\cdot10^{-55}$ | $8.275\cdot10^{16}$ | $5.435\cdot10^{-18}$ |
| 0.6 | -0.97891 | 2.201837 | $-1.679\cdot10^{-55}$ | $6.051\cdot10^{16}$ | $7.019\cdot10^{-18}$ |
| 0.5 | -1.24964 | 2.939474 | $-2.973\cdot10^{-55}$ | $3.650\cdot10^{16}$ | $1.089\cdot10^{-17}$ |
| 0.4 | -1.5776 | 4.831122 | $-1.126\cdot10^{-54}$ | $1.305\cdot10^{16}$ | $2.826\cdot10^{-17}$ |
| 0.35 | -1.7544 | 7.804654 | $-5.944\cdot10^{-54}$ | $4.160\cdot10^{15}$ | $8.514\cdot10^{-17}$ |
| 0.3 | -1.9157 | 24.3401 | $-8.520\cdot10^{-52}$ | $1.959\cdot10^{14}$ | $1.739\cdot10^{-15}$ |
| 0.250119943 | -1,9986 | $1.740781\cdot10^{11}$ | $-1.062\cdot10^{6}$ | $7.092\cdot10^{-16}$ | $4.702\cdot10^{14}$ |
| 0.146531432 | -0.5 | $9.818436\cdot10^{20}$ | $-1.223\cdot10^{67}$ | $4.071\cdot10^{-45}$ | $4.943\cdot10^{43}$ |
| 0.137701018 | 0 | $1.017771\cdot10^{21}$ | $-1.787\cdot10^{67}$ | $3.361\cdot10^{-45}$ | $5.174\cdot10^{43}$ |
| 0.125059971 | 1 | $1.058713\cdot10^{21}$ | $-2.801\cdot10^{67}$ | $2.610\cdot10^{-45}$ | $5.289\cdot10^{43}$ |
| 0 | ∞ | $1.172766\cdot10^{21}$ | $-2.069\cdot10^{68}$ | 0 | 0 |

| $\rho_{gr\max} = 10^{46}$ g·$cm^{-1}sec^{-2}$;$T\sqrt{\gamma_{\min}}$ =1.296575763·$10^{-11}$sec; σ=0.2505961314; $u^0$=6.117403956 | | | | | |
|---|---|---|---|---|---|
| u | q | z | $^3R$, $cm^{-2}$ | $t-t_{st}$ , sec | H, $sec^{-1}$ |
| 6.117403956 | 0.75982 | 0 | $-4.382\cdot10^{-56}$ | $4.358\cdot10^{17}$ | $2.181\cdot10^{-18}$ |
| 1.362007273 | 0 | 0.849896 | $-6.309\cdot10^{-56}$ | $1.876\cdot10^{17}$ | $3.074\cdot10^{-18}$ |
| 0.853131610 | -0.5 | 1.416264 | $-9.219\cdot10^{-56}$ | $1.129\cdot10^{17}$ | $4.291\cdot10^{-18}$ |
| 0.8 | -0.58133 | 1.525121 | $-9.987\cdot10^{-56}$ | $1.030\cdot10^{17}$ | $4.592\cdot10^{-18}$ |
| 0.7 | -0.75929 | 1.792084 | $-1.222\cdot10^{-55}$ | $8.282\cdot10^{16}$ | $5.432\cdot10^{-18}$ |
| 0.6 | -0.97792 | 2.200602 | $-1.678\cdot10^{-55}$ | $6.058\cdot10^{16}$ | $7.014\cdot10^{-18}$ |
| 0.5 | -1.2482 | 2.936815 | $-2.969\cdot10^{-55}$ | $3.656\cdot10^{16}$ | $1.088\cdot10^{-17}$ |
| 0.4 | -1.5753 | 4.819468 | $-1.120\cdot10^{-54}$ | $1.313\cdot10^{16}$ | $2.810\cdot10^{-17}$ |
| 0.35 | -1.7514 | 7.757364 | $-5.828\cdot10^{-54}$ | $4.230\cdot10^{15}$ | $8.381\cdot10^{-17}$ |
| 0.3 | -1.9117 | 23.5132 | $-7.317\cdot10^{-52}$ | $2.166\cdot10^{14}$ | $1.575\cdot10^{-15}$ |
| 0.250596131 | -1,9929 | $2.510051\cdot10^{5}$ | $-4.756\cdot10^{-29}$ | 236.74 | 0.001413 |
| 0.146868390 | -0.5 | $3.483573\cdot10^{9}$ | -0.0245 | $9.1035\cdot10^{-11}$ | $2.214\cdot10^{9}$ |
| 0.137992728 | 0 | $3.611425\cdot10^{9}$ | -0.0358 | $7.5143\cdot10^{-11}$ | $2.317\cdot10^{9}$ |
| 0.125298066 | 1 | $3.757034\cdot10^{9}$ | -0.0562 | $5.8328\cdot10^{-11}$ | $2.369\cdot10^{9}$ |
| 0 | ∞ | $4.162766\cdot10^{9}$ | -0.4146 | 0 | 0 |

*TABLE 1. Space kinematics at two different values of the maximum energy density.*

The results of similar calculation, but with the maximum energy density equal to that at which the electroweak phase transition occurs, are shown in the same table. The comparison of these data shows that the results of the calculation are in good agreement up to red shift at least,

$$z(u) = \left(\sqrt{\frac{\gamma(u^0)}{\gamma(u)}}\right)^{1/3} - 1 , \ z(0.3) \approx 24 . \tag{3.43}$$

despite the difference in the value of the maximum energy density on more than sixty orders. This circumstance excludes doubts in a possibility of the unambiguous description of space evolution in this range of the red shift variation. The relations (3.20), (3.16), (3.43) specify in implicit form the dependence of the Hubble parameter on red shift. At the present time there is a possibility to compare the calculated theoretical dependence with the experimental data.

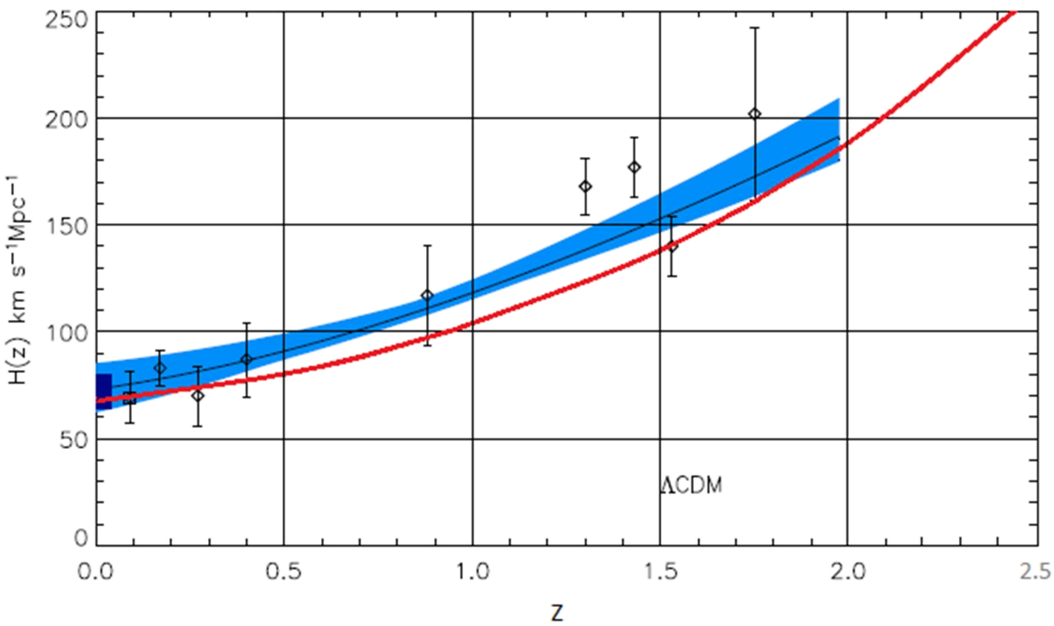


**Figure 1**

Figure 1 shows the results of the application of ΛCDM model to WMAP data for the construction of such dependence [7]. The red curve is plotted on the results of our calculations (time from the beginning of evolution and the current value of Hubble parameter correspond to [5]) and is in agreement with the experimental data. Significantly, this dependence has no free parameters, is defined only by the initial point at z=0 and stretches up to the initial moment of time as it can be seen from Table 1. In GR within ΛCDM model (black curve) the hypothesis of the existence of a dark energy and dark matter are introduced for agreement with experiment (black curve). And the hypothesis of the existence of inflatons is introduced to describe the dependence in the region of large z.

So, the following properties of space are a consequence of the assumption of the constraint (2.1) between the components of the metric tensor existence:

- is homogeneous;
- is globally isotropic;
- is material, possesses an energy and pressure that able to have both positive and negative values;
- has negative space curvature limited on an absolute value
- has the acceleration-deceleration-acceleration eras;
- is the only source of the Universe energy.

*An existing of this new law of nature doesn't contradict to the latest observational astronomical data on the Universe evolution.*

### 4. Phenomenological model of the Universe evolution.

So, we have shown that *Space* exists, unique material space.

From the speech delivered by A. Einstein in 1930, "The strange conclusion to which we have come is this – it now appears that space will have to be regarded as a primary thing and that matter is derived from it, so to speak, as a secondary result. Space is now turning around and eating up matter. We have always regarded matter as a primary thing and space as a secondary result. Space is now having its revenge, so to speak, and is eating up matter. But that is still a pious wish." [6].

Space is the main, but not the only form of existence of matter structures in the Universe. The gravitational field intensity increase will lead inevitably to appearance of new matter structures in process of evolution what in turn can significantly influence on its kinematics eventually.

Consider phenomenologically influences of matter on process of evolution of the Universe.

Let matter be born in some time point in Space described above. Owing to the homogeneity and isotropy of space the energy-momentum tensor of matter can be written as $(\varepsilon_{mat})_{\mu}^{\nu} = diag\,(\rho_{mat}, -p_{mat}, -p_{mat}, -p_{mat})$.

In the presence of matter the gravitational field equations (3.6-3.8) will take a form:

$$\frac{d}{dx^0}\left(\frac{1}{g_{00}}\frac{d\sqrt{\gamma g_{00}}}{dx^0}\right) = 0,$$

$$-\frac{1}{\sqrt{g_{00}}}\frac{d}{dx^0}\left(\frac{1}{\gamma\sqrt{g_{00}}}\frac{d\gamma}{dx^0}\right) - \frac{1}{6g_{00}}\left(\frac{1}{\gamma}\frac{d\gamma}{dx^0}\right)^2 = \sqrt{\gamma g_{00}}\frac{d}{dx^0}\left(\frac{1}{g_{00}\sqrt{\gamma g_{00}}}\frac{d\Phi}{dx^0}\right) + \frac{8\pi G}{c^4}(\rho + 3p)_{mat},$$

$$-\frac{1}{\sqrt{\gamma g_{00}}}\frac{d}{dx^0}\left(\sqrt{\frac{\gamma}{g_{00}}}g^{mp}\frac{dg_{km}}{dx^0}\right) = \delta_k^p\frac{1}{\sqrt{\gamma g_{00}}}\frac{d}{dx^0}\left(\frac{\sqrt{\gamma g_{00}}}{g_{00}}\frac{d\Phi}{dx^0}\right) - \frac{8\pi G}{c^4}(\rho - p)_{mat}\delta_k^p.$$

Repeating all the computation taking into account these additional members, we have the following integro-differential equation instead (3.19)

$$8\gamma u\frac{du}{d\gamma} = 4u^2 - 2u + \sigma + M(u,\gamma,\frac{d\gamma}{du}), \tag{4.1}$$

where

$$M(u,\gamma,\frac{d\gamma}{du}) = \frac{48\pi GT^2}{c^2}\left(-\gamma(\rho + p)_{mat} + \frac{1}{4}\int_0^u(\rho - p)_{mat}\left(\frac{d\gamma}{du}\right)\frac{du}{u}\right).$$

and it's supposed that the pressure and density of matter are equal to zero in the initial time.

The equations for cosmic acceleration, energy density, pressure and scalar curvature of space are also modified in this case; instead of (3.28), (3.31), (3.35) and (3.37) we have

$$q = 1 - \frac{3}{2u} + \frac{3\sigma}{4u^2} + \frac{3}{4u^2}M(u,\gamma,\frac{d\gamma}{du}), \tag{4.2}$$

$$\rho_{gr} + \rho_{mat} = \frac{c^2}{24\pi GT^2}\frac{u^2}{\gamma} = \frac{3c^2H^2(u)}{8\pi G} \equiv \rho_{cr}(u), \tag{4.3}$$

$$p_{gr} = -p_{mat} - \frac{c^2}{48\pi GT^2}\frac{1}{\gamma}\left[2u^2 - 2u + \sigma + M(u,\gamma,\frac{d\gamma}{du})\right]. \tag{4.4}$$

$${}^3R = R_k^k = -\frac{1}{4c^2T^2\gamma}[(4u^2 - 2u + \sigma) + M(u,\gamma,\frac{d\gamma}{du})]. \tag{4.5}$$

Hilbert's axiom suggests that the action of all kinds of matter is invariant relative to arbitrary transformations of coordinates. According to the observation data there is macroscopic matter, electromagnetic radiation, and neutrino in the Universe at the present time. These components weakly interact among themselves. In this case, owing to Hilbert's axiom the «conservation» laws for each type of matter are satisfied separately [3,4]

$$d\rho = -(\rho + p)\frac{d\sqrt{\gamma}}{\sqrt{\gamma}}. \qquad (4.6)$$

The pressure can be considered equal to zero for baryon matter, $p=\rho/3$ for an electromagnetic radiation, for neutrinos the similar relation will be valid until it is possible to neglect their mass. Eq. (4.6) shows that:

$$\rho_b = \rho_b^0 \frac{\sqrt{\gamma^0}}{\sqrt{\gamma}},\ \rho_r = \rho_r^0 \left(\frac{\sqrt{\gamma^0}}{\sqrt{\gamma}}\right)^{4/3},\rho_\nu = \rho_\nu^0 \left(\frac{\sqrt{\gamma^0}}{\sqrt{\gamma}}\right)^{4/3}. \qquad (4.7)$$

The values relating to the current time are marked by upper index. It is authentically known that the energy density of the two first components is respectively equal $\Omega_b = 0.0499$ and $\Omega_r = 5.46\cdot 10^{-5}$ of the critical energy density at the present time [5]. Data are less defined for neutrinos $\Omega\nu < 5.52\cdot 10^{-3}$. Then, to estimate the maximum degree of matter influence on the evolution process, it will be used exactly that value of the relative density of neutrinos. Thus, at times not too far from the present we have the following dependence of the energy density and pressure of matter from the bulk factor:

$$\rho_{mat} = \rho_{cr}^0\left[\Omega_b \frac{\sqrt{\gamma^0}}{\sqrt{\gamma}} + \Omega\left(\frac{\sqrt{\gamma^0}}{\sqrt{\gamma}}\right)^{4/3}\right], p_{mat} = \frac{\rho_{cr}^0}{3}\Omega\left(\frac{\sqrt{\gamma^0}}{\sqrt{\gamma}}\right)^{4/3}, \Omega = \Omega_\gamma + \Omega_\nu. \qquad (4.8)$$

*Further when using the expressions which include $\rho_{mat}$, $p_{mat}$, taking into account the approximate character of the dependencies (4.7), we will consider that variation of the bulk factor and it's derivative is described by Space kinematics in a first approximation (relations (3.19), (3.20)), and the critical density is described by relation (4.3).* In this approximation

$$M(u,\gamma,\frac{d\gamma}{du}) \cong w(u),$$

$$w(u) = -2u^{02}\left[\Omega_b \frac{f(u)}{f(u^0)} + \frac{4}{3}\Omega\left(\frac{f(u)}{f(u^0)}\right)^{2/3}\right] + \int_0^u \left[\Omega_b \frac{f(u)}{f(u^0)} + \frac{2}{3}\Omega\left(\frac{f(u)}{f(u^0)}\right)^{2/3}\right]\frac{4u^{02}du}{4u^2 - 2u + \sigma} \qquad (4.9)$$

Substituting (4.9) in (4.1) we derive the equation describing the Universe evolution in consideration of the presence of matter. The solution of this equation can be written by a quadrature.

$$\sqrt{\frac{\gamma(u)}{\gamma_{\min}}} = \psi(u) = \exp\left(\int_0^u \frac{4udu}{4u^2 - 2u + \sigma + w(u)}\right). \qquad (4.10)$$

$$t - t_{st} = T\sqrt{\gamma_{\min}}\int_0^u \frac{4\psi(u)du}{4u^2 - 2u + \sigma + w(u)}. \qquad (4.11)$$

The constant σ in these relations, in the same way as it was done in the previous section, has to be defined together with the value of $\boldsymbol{u^0}$ from a condition of the equality of the evaluated time of the Universe existence and Hubble parameter to their values observed now.

$$t^0 - t_{st} = T\sqrt{\gamma_{\min}}\int_0^{u^0} \frac{4\psi(u)}{4u^2 - 2u + \sigma + w(u)}du,\ H^0 = \frac{1}{3T\sqrt{\gamma_{\min}}}\frac{u^0}{\psi(u^0)}. \qquad (4.12)$$

| |
|---|
| $\rho_{gr\max} = \varepsilon_{Pl}$; $T\sqrt{\gamma_{\min}}$ =5.798463086·$10^{-46}$ sec; $\Omega_b$=0.0499; $\Omega_\gamma$=5.46·$10^{-5}$ ; $\Omega\nu$=5.52·$10^{-3}$ |
| $u^0$=7.027 ; σ=0.25011930 |
| $\rho_{gr\max} = 10^{46}$ g·cm $^{-1}$sec$^{-2}$; $T\sqrt{\gamma_{\min}}$ =1.296575763·$10^{-11}$sec; $\Omega_b$=0.0499; $\Omega_\gamma$=5.46·$10^{-5}$ ; $\Omega\nu$=5.52·$10^{-3}$ |
| $u^0$=7.024 ; σ=0.25058907 |

*TABLE2. Solutions of the equations (4.12) ($t^0$-$t_{st}$ = 4.355·$10^{17}$sec, $H^0$ = 2.181·$10^{-18}$sec$^{-1}$) at two values of the maximum energy density.*

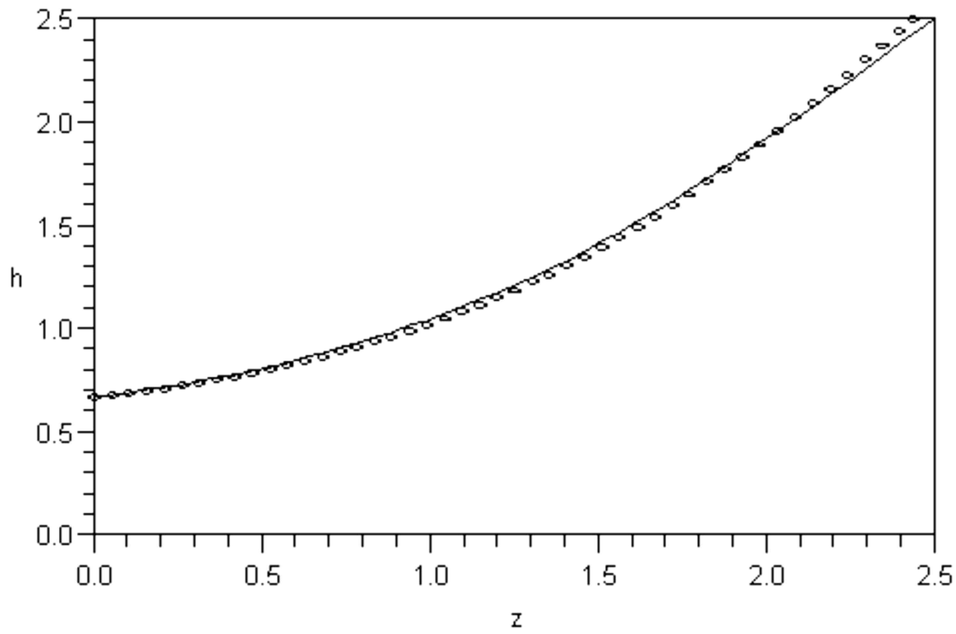


**Figure 2**

Figure 2 shows the results of calculation of the dependence of the Hubble parameter (H=100h km·sec$^{-1}$·Mps$^{-1}$) on red shift taking into account (full line) and without taking into account (points) the presence of matter. Figure 3 shows the results of calculation of the object age (in billions of years) depending on its observed redshift. In view of the data provided in the previous section, it is possible to conclude that prehistory effect on the further course of the given dependences is insignificant in the range of red shifts less than 2.3. Within this range, the course of the dependence can be reconstructed using one reliable value.

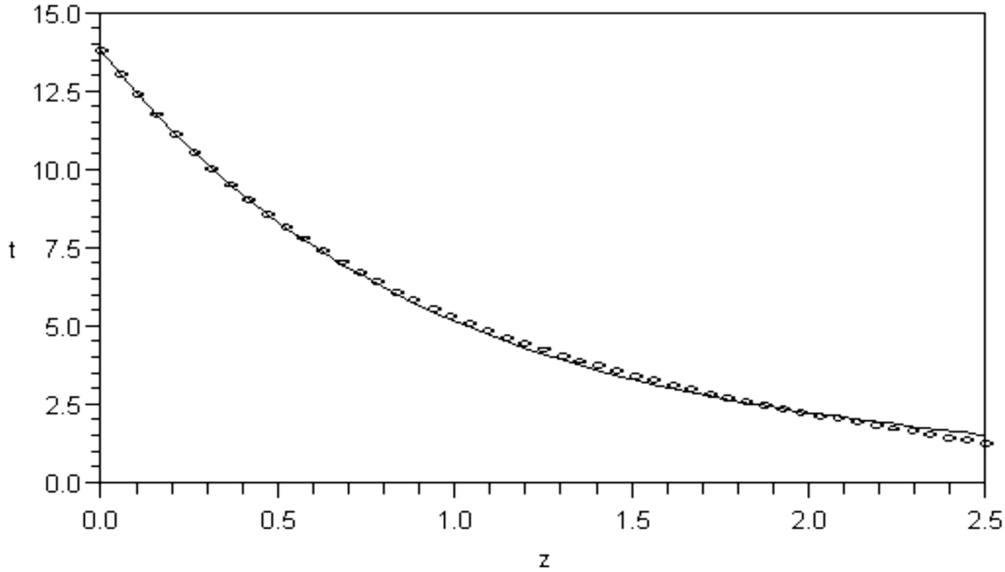


**Figure 3**

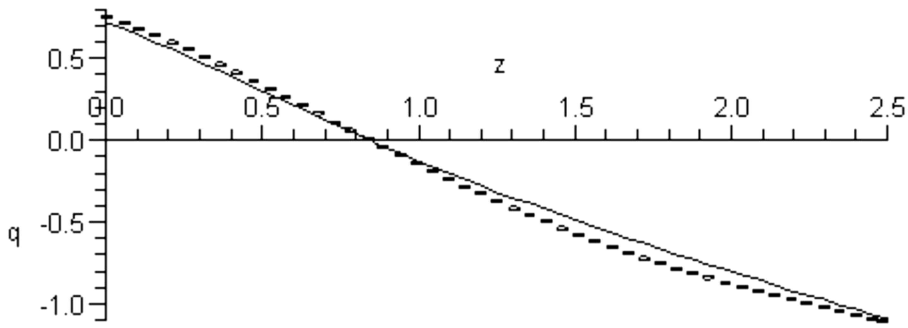


**Figure 4**

Figure 4 shows the results of calculation of the dependence of cosmic acceleration on red shift taking into account (full line) and without taking into account (points) the presence of matter. The birth of matter does not lead to a noticeable time shift of change of the deceleration-acceleration eras. Such behavior of the mentioned dependences is related to a small fraction of the energy of matter in its general quantity.

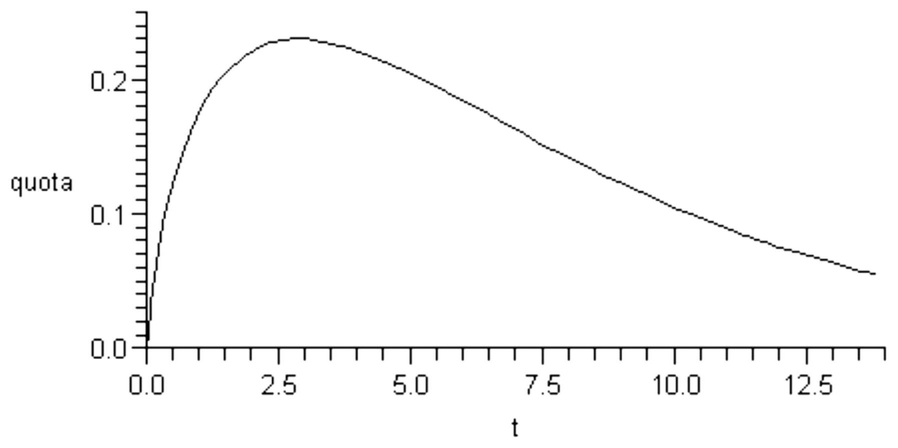


**Figure 5**

Figure 5 shows time dependence of a ratio of the average energy density of matter to the energy density in the Universe (in billions of years). The maximum fraction of the energy of matter does not exceed 0.232, at the present time this value is less than 0.055, and continues to decrease with time. In contrast to GR, where the energy density of matter increases indefinitely at time decrease, in NTG, it reaches a maximum and then decreases, tending to zero at the approach to the initial moment of time.

The rest and the main part of the energy is the energy of the gravitational field. It is this energy, evenly distributed in space, but not dark matter, is reflected in the character of dependences of the rotation curves of gravitation-coupled objects. Figure 6 shows the dependence of the energy density of the gravitational field (erg • cm-3) on red shift in the location of the observed gravitation-coupled objects.

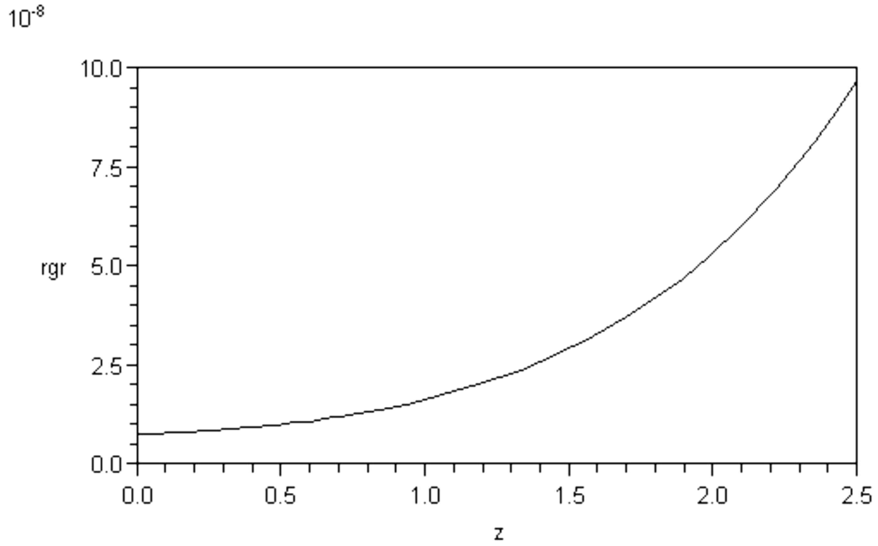


**Figure 6**

Thus, it is not required to enter any new forms of matter besides the already known forms to describe features of evolution of the Universe in NTG in contrast to GR.

## 5. Static isotropic metrics

Consider static spherically-symmetric metrics. The most general expression for space-time interval can be reduced to the form by the unimodular coordinates transformation [4]:

$$ds^2 = F(r)(dx^0)^2 - \frac{G(r)}{r^2}(\bar{x}\cdot d\bar{x})^2 - C(r)(d\bar{x}\cdot d\bar{x})$$

The constraint (2.1) is invariant relative to such transformations, but now in contrast to GR its existence doesn't allow to reduce quantity of the required metrics components till two.

Using the Kronecker symbols $\delta_{mn}$ , write the metric tensor $g_{\mu\nu}$ as:

$$g_{00} = F(r), \quad g_{0m} = 0, \quad g_{mn} = -C(r)\cdot\delta_{mn} - G(r)\frac{x_m x_n}{r^2}, \quad x_m = x^m, \tag{5.1}$$

$$g(r) = \det g_{\mu\nu} = -FC^2(C+G).$$

The tensor $g^{\mu\nu}$ (inverse to the metric tensor):

$$g^{00} = \frac{1}{F(r)}, \quad g^{0m} = 0, \quad g^{mn} = -\frac{1}{C(r)}\delta^{mn} + \frac{G(r)}{C(C+G)}\frac{x^m x^n}{r^2}. \tag{5.2}$$

$$g_{mn}g^{nk} = \delta^k_m.$$

In the presence of the constraint (2.1) it is more convenient to proceed not from the equations derived at the action variation on the metrics components, but to choose as one of the varied functions $\Delta(r) = \sqrt{-g(r)}$ .

The constraint gives the following contribution to the action:

$$\Lambda = \frac{\partial\Phi}{\partial x^\mu} g^{\mu\nu}\frac{1}{2g}\frac{\partial g}{\partial x^\nu} = -\frac{\Phi'(r)g'(r)}{2(C+G)g} = -\frac{\Phi'(r)\Delta'(r)}{\Delta^3}FC^2 \tag{5.3}$$

(The stroke hereinafter denotes differentiating with respect to $r$)

Other terms can be found using the known results of calculations [3,4]. The scalar curvature and volume element are generally covariant, therefore they can be found using "spherical" coordinates.

In "spherical" coordinates space-time interval is:

$$ds^2 = F(r)(dx^0)^2 - G(r)dr^2 - C(r)\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right).$$

By analogy to the "standard" form [4] write it as follows:

$$ds^2 = F(r)(dx^0)^2 - A(r)dr^2 - r*^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{5.4}$$

where $A(r)=G(r)+C(r)$, $r^*(r)=rC^{1/2}(r)$.

For this metrics the nonvanishing components of the connection differ slightly from the corresponding components of the "standard" form [4]:

$$\Gamma^t_{tr} = \Gamma^t_{rt} = \frac{F'}{2F}, \quad \Gamma^r_{rr} = \frac{A'}{2A}, \quad \Gamma^r_{\theta\theta} = -\frac{r*r*'}{A}, \quad \Gamma^r_{\varphi\varphi} = -\frac{r*r*'\sin^2\theta}{A}, \quad \Gamma^r_{tt} = \frac{F'}{2A},$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{r*'}{r*}, \; \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta, \; \Gamma^\varphi_{\varphi r} = \Gamma^\varphi_{r\varphi} = \frac{r*'}{r*}, \quad \Gamma^\varphi_{\varphi\theta} = \Gamma^\varphi_{\theta\varphi} = ctg\theta.$$

The curvature tensor changes according to this.

Using the expressions for the components of the connection, find the scalar curvature:

$$R = \frac{1}{2F}\left(\frac{F'}{A}\right)' + \frac{1}{2A}\left(\frac{F'}{F}\right)' + \frac{2}{r*^2}\left(\frac{r*r*'}{A}\right)' + \frac{2}{A}\left(\frac{r*'}{r*}\right)' - \frac{2}{r*^2} + \frac{2}{A}\left[\left(\frac{r*'}{r*}\right)^2 + \frac{r*'F'}{r*F}\right].$$

Singling out the divergent term, it can be written in the form:

$$R = \frac{1}{r*^2\sqrt{AF}}\frac{d}{dr}\left[r*^2\sqrt{AF}\left(\frac{F'}{AF} + \frac{4r*'}{r*A}\right)\right] - 2\left[\frac{r*'F'}{r*AF} + \frac{1}{A}\left(\frac{r*'}{r*}\right)^2 + \frac{1}{r*^2}\right]. \tag{5.5}$$

The action for the gravitational field:

$$S_{gr} = -\frac{c^3}{16\pi G}\int (R+\Lambda)\sqrt{AF}r*^2 \sin\theta\, drd\theta d\varphi dx^0 .$$

Substituting here the expression (5.5) for $R$ and (5.3) for $\Lambda$, omitting the divergent term and taking into account that $A=\Delta^2/FC^2$ we have:

$$S_{gr} = \frac{c^3}{8\pi G}\int\left(\frac{\Delta}{r*^2} + \frac{r*^2\, F}{\Delta r^4}(r*')^2 + \frac{1}{\Delta r^4}r*^3\, r*'\, F' + \frac{\Phi'\Delta' r*^4\, F}{2\Delta^2 r^4}\right) r^2 \sin\theta\, drd\theta d\varphi dx^0 .$$

Introduce a variable $\xi=r^3$ instead of $r$, then the action takes the form:

$$S_{gr} = \frac{3c^3}{8\pi G}\int\left(\frac{\Delta}{9r*^2} + \frac{Fr*^2}{\Delta}\left(\frac{dr*}{d\xi}\right)^2 + \frac{1}{\Delta}r*^3\frac{dr*}{d\xi}\frac{dF}{d\xi} + \frac{Fr*^4}{2\Delta^2}\frac{d\Phi}{d\xi}\frac{d\Delta}{d\xi}\right) d\xi \sin\theta\, d\theta d\varphi dx^0 .$$

From a principle of least action find the gravitational field equations in space free from matter:

$$\frac{d}{d\xi}\left(\frac{r*^4\, F}{\Delta^2}\frac{d\Delta}{d\xi}\right) = 0, \tag{5.6}$$

$$\frac{1}{9r*^2} - \frac{r*^2}{\Delta^2}\left(\frac{dr*}{d\xi}\right)^2 F - \frac{r*^3}{\Delta^2}\frac{dr*}{d\xi}\frac{dF}{d\xi} - \frac{1}{2\Delta^2}\frac{d}{d\xi}\left(r*^4\, F\frac{d\Phi}{d\xi}\right) = 0, \tag{5.7}$$

$$\frac{r*^2}{\Delta}\left(\frac{dr*}{d\xi}\right)^2 - \frac{d}{d\xi}\left(\frac{r*^3}{\Delta}\frac{dr*}{d\xi}\right) + \frac{r*^4}{2\Delta^2}\frac{d\Delta}{d\xi}\frac{d\Phi}{d\xi} = 0, \tag{5.8}$$

$$-\frac{2\Delta}{9r*^3} - 2r*\frac{d}{d\xi}\left(\frac{r*F}{\Delta}\frac{dr*}{d\xi}\right) - r*^3\frac{d}{d\xi}\left(\frac{1}{\Delta}\frac{dF}{d\xi}\right) + 2\frac{r*^3\, F}{\Delta^2}\frac{d\Delta}{d\xi}\frac{d\Phi}{d\xi} = 0. \tag{5.9}$$

Equation (5.6) implies:

$$\frac{r*^4\, F}{\Delta^2}\frac{d\Delta}{d\xi} = \alpha, \tag{5.6'}$$

where $\alpha$ is a constant with the dimension of length.

Multiply the equation (5.7) by $2\Delta$, subtract from result - (5.8), multiplied by $2F$, and add the result to the equation (5.9), multiplied by $r*$, after simple transformations reduce the equation to the form:

$$\frac{d}{d\xi}\left[\frac{r*^4}{\Delta}\left(\frac{dF}{d\xi} + F\frac{d\Phi}{d\xi}\right)\right] = 0.$$

This implies:

$$\frac{r*^4\, F}{\Delta}\left(\frac{1}{F}\frac{dF}{d\xi} + \frac{d\Phi}{d\xi}\right) = \beta,$$

where $\beta$ is one more constant with the dimension of length. Using (5.6') this equation can be written in the form:

$$\frac{1}{F}\frac{dF}{d\xi} + \frac{d\Phi}{d\xi} = \sigma\frac{1}{\Delta}\frac{d\Delta}{d\xi}, \quad \sigma = \frac{\beta}{\alpha}.$$

Taking into account that the function $\Phi(r)$ is defined accurate within a constant, find:

$$\Phi = -\ln(F\Delta^{-\sigma}). \tag{5.7'}$$

Rewrite the equation (5.8) as follows:

$$\frac{1}{\Delta}\left(\frac{r*dr*}{d\xi}\right)^2 + r*^2\frac{d}{d\xi}\left(\frac{r*}{\Delta}\frac{dr*}{d\xi}\right) = \frac{r*^4}{2\Delta^2}\frac{d\Delta}{d\xi}\frac{d\Phi}{d\xi}.$$

After the substitution of this expression in the equation (5.9) it takes the form:

$$r^{*4}\frac{d}{d\xi}\left(\frac{1}{\Delta}\frac{dF}{d\xi}\right)+2r^{*2}\frac{d}{d\xi}\left(\frac{Fr^*}{\Delta}\frac{dr^*}{d\xi}\right)-4\left[\frac{1}{\Delta}\left(\frac{r^*dr^*}{d\xi}\right)^2+r^{*2}\frac{d}{d\xi}\left(\frac{r^*}{\Delta}\frac{dr^*}{d\xi}\right)\right]F+\frac{2\Delta}{9r^{*2}}=0$$

This equation is equivalent to the following:

$$\frac{d}{d\xi}\left[\frac{r^{*6}}{\Delta}\frac{d}{d\xi}\left(\frac{F}{r^{*2}}\right)\right]+\frac{2\Delta}{9r^{*2}}=0\,.$$

Integrating this equation over ξ we have:

$$\frac{d}{d\xi}\left(\frac{F}{r^{*2}}\right)-\beta_1\frac{\Delta}{r^{*6}}+\frac{2}{9}\frac{\Delta}{r^{*6}}\int_0^\xi\frac{\Delta}{r^{*2}}d\xi=0\,,$$

where $\beta_1=\left[\frac{r^{*6}}{\Delta}\frac{d}{d\xi}\left(\frac{F}{r^{*2}}\right)\right]_{\xi=0}$ is one more constant with the dimension of length. This constant is equal to zero for the Minkowski metric. Let us assume further $\beta_1=0$ in order that the Minkowski metric could be the solution of this system of equations (in case when the constant $\alpha$ is equal to zero).

$$\frac{d}{d\xi}\left(\frac{F}{r^{*2}}\right)+\frac{2}{9}\frac{\Delta}{r^{*6}}\int_0^\xi\frac{\Delta}{r^{*2}}d\xi=0\,. \tag{5.9'}$$

Integrating one more time, represent the function $F(r)$ in the form:

$$F=\frac{2}{9}r^{*2}\int_\xi^\infty\left(\int_0^\xi\frac{\Delta}{r^{*2}}d\xi\right)\frac{\Delta}{r^{*6}}d\xi\,.$$

Transform the equation (5.8). Introduce a notation

$$U=\frac{r^*}{\Delta}\frac{dr^*}{d\xi}\,,$$

and substitute the expressions for derivatives of $\Delta$ and $\Phi$ from the equations (5.6') and (5.7'), then the equation (5.8) can be put in the form:

$$U^2+r^*U\frac{dU}{dr^*}=\frac{\alpha U}{2r^*F}\frac{d\Phi}{dr^*}\,.$$

$$V=\frac{1}{3r^*U},\quad \frac{d}{dr^*}\frac{1}{V}=\frac{3\alpha}{2r^*F}\frac{d\Phi}{dr^*}\,. \tag{5.8'}$$

Passing from the derivatives with respect to $\xi=r^3$ to the derivatives with respect to $r^*$ in all relations and introducing the dimensionless coordinate's $r/\alpha$ and $r^*/\alpha$ (keeping the previous notation $r$ and $r^*$ for them), we can write the initial system of equations as follows:

$$\frac{1}{\Delta}\frac{d\Delta}{dr^*}=\frac{3V(r^*)}{Fr^{*2}}\,, \tag{5.10}$$

$$V(r^*)=\frac{1}{1-\frac{3}{2}\int_{r^*}^\infty\frac{1}{r^*F}\frac{d\Phi}{dr^*}dr^*},\ \Phi=-\ln(F\Delta^{-\sigma})\,, \tag{5.11}$$

$$F(r^*)=2r^{*2}\int_{r^*}^\infty\left(\int_{r^*_{\min}}^{r^*}V(r^*)dr^*\right)\frac{1}{r^{*4}}V(r^*)dr^*\,. \tag{5.12}$$

$$\frac{\Delta(r^*)r^2}{r^{*2}}\frac{dr}{dr^*}=V(r^*)\,. \tag{5.13}$$

Generally speaking, the nonzero value $r^*_{min}=r^*(0)$ means a presence of *an edge* of space-time manifold.

Consider behavior of the metrics at $r^*_{min}=0$ and the small values $r^*$. If the integral

$$2\int_0^\infty\left(\int_0^{r^*} V(r^*)dr^*\right)\frac{V(r^*)}{r^{*4}}dr^* = b > 0 \tag{5.14}$$

exists, eq.(5.12) implies that the function $F(r^*)\approx b\cdot r^{*2}$ at the small $r^*$. Then assumed that $V(r^*)\approx b_1\cdot r^{*\nu}\geq 0$, $\Delta(r^*)\approx b_2\cdot r^{*\delta}\geq 0$ and substituting these expressions in (5.8', 5.10) we have:

$$\nu = 3,\ b_1 = \frac{2b}{2-\sigma\delta},\ \delta = \frac{6}{2-\sigma\delta}. \tag{5.15}$$

From the last relation follows:

$$\delta = \frac{1\pm\sqrt{1-6\sigma}}{\sigma}.$$

therefore $\sigma \leq 1/6$.
Integrating the equation (5.13) find at the small values $r$, $r^*$:

$$r^3(r^*) = 3\int_0^{r^*}\frac{V(r^*)}{\Delta(r^*)}r^{*2}dr^* \approx 3\frac{b_1}{b_2}\int_0^{r^*} r^{*(5-\delta)}dr^*. \tag{5.16}$$

The last integral exists only at ***δ** < 6*. In this case

$$\delta = \frac{1-\sqrt{1-6\sigma}}{\sigma},\quad \sigma < \frac{1}{6}. \tag{5.17}$$

Consider now the expression for the energy of the static isotropic gravitational field (Appendix I). In this case

$$E = \frac{c^4\alpha}{4G}\left[\frac{r^{*2}}{V(r^*)}\frac{F(r^*)}{}\frac{d\ln(F\Delta^{-\sigma})}{dr^*}\Bigg|_{r^*_{\min}}^{r^*\to\infty} - 3\ln F\Delta^{-\sigma}(r^*_{\min})\right]. \tag{A.8}$$

The last term in this relation has a logarithmic singularity at $r^*_{min}=0$.
The energy will have the finite value only at $r^*_{min} \neq 0$, that is *in the presence of the edge*. It is possible only at the value $\sigma \geq 1/6$.

The quantity $r^*_{min}$ is an independent parameter and for its definition the additive considerations are necessary. First, suppose that according to Mach's principle inertial mass $M_{in}$ is related to the total gravitational field energy $E$ out of the edge by Einstein's formula $E=M_{in}c^2$. Secondly, in accordance with Etvesh's experiment, we assume the equality of the quantity of this inertial and gravitational mass $M_{in}= M_{gr}$. And at last, based on correspondence principle with GR we assume that at the large values of $r^*$ the first term coefficient of the function $F(r^*)$ expansion in powers of $1/r^*$ is equal to the gravitational radius-to-α ratio.

$$F(r^*) = 1 - \frac{r_{gr}}{\alpha}\frac{1}{r^*} + .. = 1 - \frac{2M_{gr}G}{c^2\alpha}\frac{1}{r^*} + ... \tag{5.18}$$

In this case the relation (A.8) passes into the equation defining a quantity $r^*_{min}$.

$$\frac{r_{gr}}{\alpha} = \frac{2r^*_{\min}F(r^*_{\min})}{3V(r^*_{\min})} - \ln\frac{F(r^*_{\min})}{\Delta^{1/6}(r^*_{\min})}. \tag{5.19}$$

The solution of the system of equations (5.10) - (5.13), (5.19) can be found by a successive approximation method. Starting from the trial function $V^{(0)}(r^*)$ at the chosen initial value $r^*_{min}$ it is possible to find the function $F^{(0)}(r^*)$ as a first approximation from (5.12), and then to find $\Delta^{(0)}(r^*)$ from (5.10) and - new value $V^{(1)}(r^*)$ from (5.11). Continue this process before deriving on N step the values of the desired functions with the required accuracy. Find the value of $r^*_{min}$ from the equation (5.19). And then find the function $r(r^*)$ from the equation (5.13).

Construct a trial function. If eq. (5.18) is valid at large values of $r^*$, then eq. (5.10, 5.11) implies that $V(r^*) \approx 1-\nu/r^{*2}+...$ As in the presence of the edge the behavior of the desired functions is not determined at small values of $r^*$, it is natural to assume that the relative size of $r^*_{min}$ is more than unit. Providing that $r^*_{min} \geq 1$, specify a trial function as follows:

$$V^{(0)}(r^*) = 1 - \nu / r^{*2}. \tag{5.20}$$

Substituting this expression in eq. (5.12) we find

$$F^{(0)}(r^*) = 1 - \frac{2}{3}(r^*_{\min} + \frac{v}{r^*_{\min}})\frac{1}{r^*} + \frac{2}{5}(r^*_{\min} + \frac{v}{r^*_{\min}})\frac{v}{r^{*3}} - \frac{v^2}{3}\frac{1}{r^{*4}} . \tag{5.21}$$

Based on correspondence principle, in this approximation we have

$$\frac{r_{gr}}{\alpha} = \frac{2}{3}(r^*_{\min} + \frac{v}{r^*_{\min}}) . \tag{5.22}$$

A constant $v$ can be chosen so that the values of a trial function and first approximation coincide $V^{(0)}(r^*_{\min}) = V^{(1)}(r^*_{\min})$ in the point $r^*=r^*_{min}$. Substituting (5.20), (5.21) in (5.10) we find

$$\ln\Delta^{(0)}(r^*) = -3\int_{r^*}^{\infty} \frac{V^{(0)}(r^*)}{(r^*)^2 F^{(0)}(r^*)} dr^* , \tag{5.23}$$

and then from (5.11) we have

$$V^{(1)}(r^*) = \left(1 + \frac{3}{2}\frac{1}{r^* F^{(0)}(r^*)} - \frac{3}{2}\int_{r^*}^{\infty}\left(1 + \frac{V^{(0)}(r^*)}{2r^* F^{(0)}(r^*)}\right)\frac{dr^*}{(r^*)^2 F^{(0)}(r^*)}\right)^{-1} . \tag{5.24}$$

In this case

$$v = (1 - V^{(1)}(r^*_{\min}))r^{*2}_{\min} . \tag{5.25}$$

This equation defines $v$ as a function of $r^*_{min}$.

Spline approximations were used for the calculations in the higher approximations. After five successive approximations, solving the equation (5.19), we find (using six intervals in the calculations) with an error equal to fractions of a percent

$$r^*_{min} \approx 1.74.$$

This value is more than unit, as it was supposed. In a dimensional form

$$r^*_{min} \approx 0.935 r_{gr} .$$

The results of the calculations are presented in Table 3.

| $\sigma=1/6$; $x_{max}=0.575$ ; $r_{gr}/\alpha=1.859$ | | | | |
|---|---|---|---|---|
| $x=\alpha/r^*$ | $V(x)$ | $F(x)$ | $\Delta(x)$ | $C^{-1/2}(x)=r(x)/r^*$ |
| 0 | 1 | 1 | 1 | 1 |
| 0.1 | 0.9875 | 0.8160 | 0.7184 | 1.1792 |
| 0.2 | 0.9346 | 0.6381 | 0.4814 | 1.3523 |
| 0.3 | 0.8202 | 0.4746 | 0.2980 | 1.4556 |
| 0.4 | 0.6471 | 0.3386 | 0.1721 | 1.4378 |
| 0.5 | 0.4596 | 0.2366 | 0.0962 | 1.2288 |
| 0.575 | 0.3413 | 0.1813 | 0.0543 | 0 |

*TABLE 3. Solution of the system of equations (5.10..5.13) at the value σ=1/6.*

The value of one of the metric functions - $C(r)$ increases indefinitely at approaching to the edge, however the determinant of the metric tensor and all invariants of the Riemann tensor are limited at the same time. Indeed the Riemann tensor is generally covariant and the metrics has no singularities in the spherical coordinate system (5.4).

Thus, *at the presence of the constraint (2.1) there is a nonsingular stationary particle-like distribution of the centrosymmetrical gravitational field for which the equality of inertial (defined according to Mach's principle) and gravitational mass is satisfied.* A horizon (existed in the solution of GR equations for centrosymmetrical empty space) is absent in this case.

The calculations were carried out at **σ**=1/6. Generally the solution will exist also at the values **σ** lying in some interval adjacent to this value. The parameter **σ** can be chosen arbitrarily in the range of the acceptable values, therefore the distribution of fields in the region of about the gravitational radius will differ among themselves at the identical values of the total energy. Arising in this regard uncertainty generally cannot be eliminated.

## 6. Conclusion

A fundamental principle of the equivalence of all reference systems compatible with the Riemannian metric lies in the basis of NTG also as in GR. We don't put in doubt a firmness of the principle of the action invariance relative to arbitrary transformations of coordinates and the general covariance of all equations of matter motion respectively. At the same time, the covariance of the equations of gravitation is limited by the constraint between the components of the metric tensor in NTG in contrast to GR. Thus, only "medium-strong principle" of the equivalence is met a priori in NTG [4]. However, Section 5 shows for the static isotropic metrics that not only the strong equivalence principle is satisfied in NTG, but also Mach's principle.

The existence of the constraint allows to determine the energy-momentum tensor of the gravitational field, and in this case space-time itself is provided with the material properties and is the primary source of all energy of the Universe. It is proved that in such Universe the cosmological principle (the hypothesis of homogeneity and isotropy of space) will be strictly accomplished up to the beginning of the gravitational field structuring. Proceeding only from the existence of the constraint without any other assumptions, the equations describing all evolution of the Universe are derived, beginning from the natural initial conditions through a change of the acceleration-deceleration eras and up to the present time.

Comparison of the solutions given above with the solutions of the similar in a statement problems in GR, shows that NTG is technically much more difficult than GR. If we consider that correction data are relatively small in the transition from Newton's theory to GR, we can conclude taking into account correspondence principle that at the solving of practical tasks it will not be possible to find any relevant differences in solutions of NTG and GR. However, the situation becomes quite different when considering the predictions of these theories near the edge of the manifold for NTG or space-time singularity for GR. The behavior of Fridmann model near the initial singularity or the prediction of the alternation of Kasner epochs in this region based on GR puts in doubt an adequacy of such predictions. The same can be said about the spatial singularity, where GR predicts the existence of a horizon and black hole. Preliminary analysis shows that the singularities related to the gauge interaction are eliminated in NTG at nonperturbative approach already in the framework of the classical theory.

## Appendix I. Energy of the static isotropic gravitational field

By the Bianchi identity in the absence of matter the energy density of the gravitational field $T_\mu^{\ \nu}$ must satisfy to the relation:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}T_\mu^\nu\right)-\frac{1}{2}\frac{\partial g_{\lambda\rho}}{\partial x^\mu}T^{\lambda\rho}=0\,.$$

In case of a static field the energy of the gravitational field is conserved:

$$E=\int\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}T_0^\nu\right)d^4x=\int T_0^\nu\sqrt{-g}dS_\nu\,, \tag{A.1}$$

where according to (2.4)

$$T_0^\lambda=-\frac{c^4}{16\pi G}\left[\delta_0^\lambda\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right)-g^{\lambda\eta}\Gamma_{0\rho}^\rho\frac{\partial\Phi}{\partial x^\eta}-g^{\lambda\eta}\Gamma_{\eta\rho}^\rho\frac{\partial\Phi}{\partial x^0}\right],\Gamma_{\lambda\rho}^\rho=\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial x^\lambda}\,. \tag{A.2}$$

In a static field the last two terms in this relation are equal to zero and (A.1) (taking into account (A.2)) takes the form:

$$E = -\frac{c^4}{16\pi G}\int \sqrt{-g}\,\frac{\partial}{\partial x^{\mu}}\left(g^{\mu\nu}\frac{\partial \Phi}{\partial x^{\nu}}\right)dV . \tag{A.3}$$

Substituting here the expressions for the components of the metric tensor we derive from (5.2):

$$E = \frac{c^4}{16\pi G}\int \sqrt{-g}\,\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{C+G}\frac{d\Phi}{dr}\right)dV = \frac{c^4}{4G}\left[\int_0^{\infty}\sqrt{-g}\,\frac{d}{dr}\left(\frac{r^2}{(C+G)}\frac{d\Phi}{dr}\right)dr\right]. \tag{A.4}$$

Let's consider now that by definition and by the relation (5.13) also:

$$C(r^*) + G(r^*) = \frac{r^4(\sqrt{-g})^2}{r^{*4}\,F(r^*)},\quad r^2 dr = \frac{V(r^*)}{\sqrt{-g}}\,r^{*2}\,dr^* . \tag{A.5}$$

Substituting these expressions in (A.4) and passing to the dimensionless coordinate $r^*/\alpha$, we have:

$$E = \frac{c^4\alpha}{4G}\left[\frac{r^{*2}\,F(r^*)}{V(r^*)}\frac{d\Phi}{dr^*}\Bigg|_{r^*_{\min}}^{r^*\to\infty} - \int_{r^*_{\min}}^{\infty}\frac{r^{*2}\,F(r^*)}{V(r^*)\sqrt{-g}}\frac{d\Phi}{dr^*}\frac{d\sqrt{-g}}{dr^*}dr^*\right]. \tag{A.6}$$

By the relations (5.7'), (5.10)

$$\Phi = -\ln(F\Delta^{-\sigma}),\quad \frac{1}{\sqrt{-g}}\frac{d\sqrt{-g}}{dr^*} = \frac{3V}{r^{*2}\,F},\ \Delta = \sqrt{-g} . \tag{A.7}$$

Taking into account these relations

$$E = \frac{c^4\alpha}{4G}\left[-\frac{r^{*2}}{V(r^*)}\frac{dF}{dr^*}\Bigg|_{r^*_{\min}}^{r^*\to\infty} - 3\ln F\Delta^{-\sigma}(r^*_{\min})\right]. \tag{A.8}$$

Boundary values of the derivative of the function $F(r^*)$ appear in the relation.

Considering fields behavior at infinity and fact that $dF/dr^* = 2F/r^*_{min}$ by the relation (5.12) at $r^* = r^*_{min}$ we find:

$$-\frac{r^{*2}}{V(r^*)}\frac{dF}{dr^*}\Bigg|_{r^*_{\min}}^{r^*\to\infty} = -\frac{r_{gr}}{\alpha} + \frac{2F(r^*_{\min})r^*_{\min}}{V(r^*_{\min})} . \tag{A.9}$$

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