Massification of the spacetime

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Abstract: We proposed a description of the gravitational phenomena in a new medium, which merges the Minkowski four-dimensional spacetime and the bare mass density into the single idea. Under influence outer gravitational field the Minkowski four-dimensional spacetime does not change, while the bare mass density changing and becomes the effective mass density. This is an alternative attempt to describe gravitational phenomena, using a new idea of massification of the spacetime.

1. Introduction

In this paper we proposed a description of the gravitational phenomena in a new medium, which merges the Minkowski four-dimensional spacetime and the bare mass density into the single idea. Under influence outer gravitational field the bare medium becomes the effective medium, while the Minkowski four-dimensional spacetime does not change, in contrast to the General Relativity (GR), where under influence outer gravitational field the pseudo-Riemannian spacetime is dynamically curved.

The concept of the effective mass tensor of the body plays important role in the contemporary physics, e.g. is well-known in the solid-state physics. This concept is a very attractive because the equations of the motion includes full information about all existing fields (for example gravitational, electromagnetic, etc.) surrounding the body, without their exact analysis. The effective mass can be isotropic or anisotropic, positive or negative.

The concept of effective mass tensor to describe gravitational phenomena, instead of usual metric tensor, for the first time was discussed in [1, 2]. Our mathematical model we call the massification of the spacetime.

We define now the bare medium and the effective medium. Then we will compare the equation of motion in the effective medium with the equation of motion in the classical mechanics.

2. The bare medium

In the absence of any outer fields, homogeneous, isotropic and time independent medium, called the bare medium with the bare mass density \( \rho_{\text{bare}} \), is defined as follows

\[
\rho_{\mu\nu}^{\text{bare}} \overset{\text{def}}{=} \rho_{\text{bare}} \cdot \eta_{\mu\nu} = \text{diag}(\rho_{\text{bare}}, \rho_{\text{bare}}, \rho_{\text{bare}}, \rho_{\text{bare}}) \tag{1}
\]

where: \( \rho_{\mu\nu}^{\text{bare}} \) is the bare mass density tensor, \( \eta_{\mu\nu} \) is the Minkowski tensor, \( \mu, \nu = 0, 1, 2, 3 \).

This mathematical model merges the Minkowski four-dimensional spacetime and the bare mass density \( \rho_{\text{bare}} \) into the single idea. The bare mass density is no longer the scalar and becomes the tensor. Note that \( \rho_{\text{bare}} \) never reaches zero (\( \rho_{\text{bare}} \neq 0 \)), although it may be very close. In the contrast to
the vacuum, the bare medium is a never empty. So determined the spacetime with the bare mass density is equivalent to the field of inertia, which is a special case of the gravitational field. The inertial field is responsible for the inertia of the body and is described by the tensor $\rho_{\mu\nu}^{\text{bare}}$.

In the bare medium the metric is defined as the Minkowski metric

$$ds^2(\eta_{\mu\nu}) = \eta_{\mu\nu} \cdot dx^\mu dx^\nu.$$  

(2)

This metric is independent of the inertial frame of reference and well suited to describe all the physical phenomena occurring in Special Relativity (SR). The bare medium has influences on the physical processes. The presence of bodies and their motion has no influence on the bare medium. Particles behave in accordance with the principle of inertia, i.e. they are at rest or moving in a straight line at constant speed with respect to the bare medium (not with respect to the spacetime itself).

During the uniform motion, clocks and roots indicate the different time and length, than at the rest. This difference results from the change of the bare mass density during the uniform motion with respect to the bare medium.

3. The effective medium

Assume that under influence outer gravitational field the bare mass medium becomes the effective medium and the metric is defined as

$$ds^2(\rho_{\mu\nu}(x)) \overset{\text{def}}{=} \frac{\rho_{\mu\nu}(x)}{\rho_{\mu\nu}^{\text{bare}}} \cdot dx^\mu dx^\nu.$$  

(3)

where: the effective mass density tensor $\rho_{\mu\nu}(x)$ is a symmetric and position dependent. The effective mass can be positive or negative, so the metric (3) also may take positive or negative values. Tensor $\rho_{\mu\nu}(x)$ describes all the physical properties of the effective medium. Describes also the mathematical relationship between the effective medium and the bare medium under the influence the gravitational field and, in a some sense, is similar to the metric tensor $g_{\mu\nu}(x)$ with metric

$$ds^2(g_{\mu\nu}(x)) = g_{\mu\nu}(x) \cdot dx^\mu dx^\nu.$$  

(4)

in GR. Rest and motion of all bodies takes place with respect to the effective medium, which becomes a new reference frame. Additionally the presence of bodies and their motion has influence on the effective medium. In non-inertial systems the field of inertia passes into the gravitational field.

In the absence of any outer gravitational fields the effective mass becomes the bare mass and the metric (3) becomes the metric (2).

Let’s analyze the motion of the body in an effective medium and let’s compare the equation of motion with the classical Newtonian equation.

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1 Let us remember that the Minkowski four-dimensional spacetime does not change.
4. The equation of motion in the effective medium

The Lagrangian function for the body in the effective medium has form

\[ L = \frac{1}{2} \rho_{\mu\nu}(x) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \]

The equation of motion

\[ \frac{dp^\mu}{d\tau} = \frac{1}{2} \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \]  \( (5) \)

where: \( p^\mu(x) = \rho_{\mu\nu}(x) \frac{dx^\mu}{d\tau} \) is the effective density of the four-momentum, \( \tau \) is the proper time.

The equation of motion (5) explicitly refers to the effective medium, which is described by the effective mass density tensor \( \rho_{\mu\nu}(x) \). So the motion of the body takes place only in relation to the effective medium, not to the relation of the spacetime itself or all bodies in the Universe (Mach’s Principle\( ^2 \)). The new quality of the understanding has been reached.

When \( \rho_{\gamma\nu}(x) \) does not depends explicitly on \( \tau \), the equation (5) takes the form

\[ \rho_{\mu\nu}(x) \frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\mu\nu\rho}(\rho_{\mu\nu}(x)) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \]  \( (6) \)

where:

\[ \Gamma_{\mu\nu\rho}(\rho_{\mu\nu}(x)) = \frac{1}{2} \left( \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\gamma} + \frac{\partial \rho_{\gamma\nu}(x)}{\partial x^\mu} - \frac{\partial \rho_{\mu\gamma}(x)}{\partial x^\nu} \right) \]  \( (7) \)

assuming that the condition

\[ \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \frac{1}{2} \left( \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\gamma} + \frac{\partial \rho_{\gamma\nu}(x)}{\partial x^\mu} \right) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \]

is satisfied.

The equation (7) is very similar to the Christoffel symbols of the first kind, where instead metric tensor \( g_{\mu\nu}(x) \), we have the effective mass density tensor \( \rho_{\mu\nu}(x) \). This is an interesting result because the Christoffel symbols describing the metric connection, while the equation

\[ \frac{d}{d\tau} \left( g_{\gamma\nu}(x) \frac{dx^\nu}{d\tau} \right) - \frac{1}{2} \frac{\partial g_{\mu\nu}(x)}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \]  \( (8) \)

\( ^2 \) This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo [3].
is the geodesic equation in GR.

If the surrounding bodies consist only with the bare masses, i.e. \( \rho_{\mu\nu}(x) = \rho_{\mu\nu}^\text{bare} \), \( \Gamma_{\mu\nu}^\alpha(\rho_{\mu\nu}^\text{bare}) = 0 \) then the equation of motion (6) takes the form:

\[
\rho_{\mu\nu} \frac{d^2 x^\nu}{d\tau^2} = 0.
\]

The body with the bare mass density \( \rho_{\mu\nu}^\text{bare} \) is in the rest or moves in a straight line with the constant speed in the respect to the bare medium. The principle of inertia has gained a new meaning and the equation (9) determines the new inertial reference frame – the bare medium reference frame. This reference frame is determined by the bare medium property only.

During any change in state of motion of the body appears the inertia, which source is the spacetime with the effective mass density. The inertia becomes an intrinsic property of the massification of spacetime. The magnitude of the inertia of any body is also determined by the massification of spacetime. This is the opposite of that, than previously thought. Until now it was thought that inertia is determined by the masses of the Universe and by their distribution [4]. In our model an isolated object in the Universe always has of the inertial properties, because the spacetime and the bare mass density \( \rho_{\mu\nu}^\text{bare} \) formed an inseparable whole. The spacetime ceased to be empty.

We analyze now the massification of spacetime for the slow motion speed and the slow rotating body in a static and a weak gravitational field.

5. A weak gravitational field approximation

In a weak gravitational field we can decompose \( \rho_{\mu\nu}(x) \) to following simple form \( \rho_{\mu\nu}(x) = \rho_{\mu\nu}^\text{bare} + \rho_{\mu\nu}^*(x) \), where: \( \rho_{\mu\nu}^*(x) \ll 1 \) is a very small perturbation in the effective mass density tensor.

5.1. The equation of motion

At slow motion speeds, in a static, a weak and the spherically symmetric field the equation of motion (6) reduces to

\[
\left(\rho_{\mu\nu}^\text{bare} + \rho_{\mu\nu}^*(r)\right) \frac{d^2 r}{dt^2} \simeq -\frac{c^2}{2} \nabla \rho_{00}^*(r)
\]

where: \( c \) is the speed of light.

The equation (10) is a little different than well-known Newton's equation of the motion for the gravity. It what currently we consider to be the inertial mass density, really is the sum of the bare mass density \( \rho_{\mu\nu}^\text{bare} \) and \( \rho_{\mu\nu}^*(x) \) - \( r \)-component of the very small perturbation in the effective mass density. Note that gravitational mass density does not appear explicitly in the equation (10).
Does it mean that, in our model, during massification of the spacetime, the Equivalence Principle, underlying the GR, lost raison d’être?

According to the Correspondence Principle we expect that there is a relationship between the component \( \rho^\bullet_{00}(r) \) and the gravitational potential \( V(r) \) in the following form [5]

\[
\frac{\rho^\bullet_{00}(r)}{\rho^\text{bare}} \equiv \frac{2V(r)}{c^2}
\]  

(11)

where: \( V(r) = \frac{GM}{r} \), \( G \) is the gravitational constant, \( M \) is the mass and \( r \) is the distance. After substituting (11) into (10), (on the assumption that \( \rho^\bullet_{\mu
u}(r) = 0 \) ), we obtain

\[
\frac{d^2 r}{dt^2} = -\frac{\partial V(r)}{\partial r}
\]

(12)

the well-known Newtonian equation of motion in the gravitational potential \( V(r) \).

### 5.2. The rotating body

Let’s consider the slowly rotating body in a static and weak gravitational field. The equation of motion have the form

\[
\rho^\text{bare} \cdot \frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \frac{\partial \rho^\bullet_{00}(x)}{\partial x^i} + \left( \frac{\partial \rho^\bullet_{0k}(x)}{\partial x^j} - \frac{\partial \rho^\bullet_{0j}(x)}{\partial x^k} \right) \frac{dx^j}{dt}
\]

(13)

For the slowly rotating body the source of inertia are the following expressions: \( \frac{\partial \rho^\bullet_{00}(x)}{\partial x^i} \) and \( \frac{\partial \rho^\bullet_{0k}(x)}{\partial x^j} - \frac{\partial \rho^\bullet_{0j}(x)}{\partial x^k} \). These components we can determine from the matrix [6] below

\[
\rho^\bullet_{\mu
u}(x) = \rho^\text{bare} \cdot \begin{pmatrix}
-\omega^2 \frac{(x^2 + y^2)}{c^2} & \frac{\alpha y}{c} & -\frac{\alpha \kappa}{c} & 0 \\
\frac{\alpha y}{c} & 0 & 0 & 0 \\
-\frac{\alpha \kappa}{c} & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

where \( r^2 = x^2 + y^2 \).

Finally, we get well-known equation

\[
\frac{d^2 r}{dt^2} = -\omega^2 r - 2\omega \frac{dr}{dt}
\]

(14)
which includes an real the centrifugal and Coriolis acceleration.

In the Newtonian approximation the equations of motion (12) and (14) does not depend, explicitly, on the mass density.

6. The rotating bucket with water problem

There are two entirely different measurements of the Earth’s angular velocity, astronomical (from upper culmination to upper culmination of the star) and dynamic (by means of Foucault’s pendulum experiment), which give the same results (in the limit of the experimental errors). In both cases the motion is described with respect to the effective medium and the coincidence of these measurements is the result of massification of the spacetime.

In the famous experiment with the rotating bucket with water [7, 8] the motion of water takes place also to relative of the effective medium, therefore the surface of water takes the shape of the parabolic. So, massification of the spacetime explains both these physical phenomena.

7. What is the bare and the effective mass density?

Each theoretical model must correspond with the real of the physical world. We suppose that the bare mass density $\rho^{bare}$ corresponds with the critical density $\rho_c = \frac{3H^2}{8\pi G}$, where: $H$ is the Hubble constant.

This term is use in the modern cosmology to determine the spatial geometry of the Universe, where $\rho_c$ is the critical density for which the spatial geometry is flat (or Euclidean).

The flat spatial geometry in GR corresponds with the bare medium in our model. The curved spacetime corresponds with the effective medium.

8. Summary

In this paper was applied an alternative attempt to describe gravitational phenomena, using a new idea of the massification of spacetime, which provides the following benefits:

1. During any change in state of motion of the body appears the inertia, which source is the spacetime with the effective mass density.
2. The inertia becomes an intrinsic property of massification of the spacetime.
3. The magnitude of the inertia of any body is determined by massification of the spacetime.
4. Inertial forces, appearing in the non-inertial frames of reference, there are no longer fictitious forces.
5. In the gravitational field clocks and roots indicate the different time and length, than in the absence of the field. This difference results from the change of the effective mass density in a gravitational field [2].

9. Conclusion

Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1909, German mathematician H. Minkowski connected together space
and time into single idea, creating a new the four-dimensional spacetime [9]. Idea of the spacetime enjoyed success in SR and GR correctly describing a range of physical phenomena. The massification of the spacetime gives the old phenomena of the gravity a new physical interpretation.

The idea of massification of the spacetime, although a very attractive, requires experimental confirmation. Predicted the annual relative change of the fluctuation in the effective mass as resulting from ellipticity of the orbit for the Earth, is equal to $6.6 \times 10^{-10}$ [2]. GR does not predicts a such fluctuations.

References